On the quasi-Halley method for solving nonlinear equations

G. Albeanu

Halley's method is a famous iteration for solving nonlinear equations $P(X) = 0$ [1]. Some Kantorovich-like theorems have been given, including extensions for general spaces [2]. The quasi-Halley method was initially discussed in [3]. This paper uses the generalized inverse approach and studies the numerical behaviour of the following iterative scheme:

$$Y_n = X_n - [P'(X_n)]^+ P(X_n)',$$

$$H(X_n, Y_n) = -[P'(X_n)]^+ P''(X_n)(Y_n - X_n),$$

$$X_{n+1} = Y_n - \frac{1}{2}[P'(X_n)]^+ [I - \frac{1}{2}H(X_n, Y_n)]^+ P''(X_n)(Y_n - X_n)^2.$$

Preliminary theoretical results and numerical comparisons are provided.

References

1. J. L. Varona, *Graphic and numerical comparison between iterative methods*, The Mathematical Intelligencer, **24**, 1(2002), 37-46.
2. D. Chen, *Ostrowski-Kantorovich theorem and S-order of convergence of Halley method in Banach spaces*, Comment. Math. Univ. Carolinae, **34**, 1(1993), 153 - 163.
3. Y. Levin, A. Ben, *Directional Halley and quasi-Halley methods in n variables*, in "Inherently Parallel Algorithms in Feasibility and Optimization and Their Applications", D. Butnariu, Y. Censor, S. Reich (eds), Elsevier, Amsterdam, (2001), 345-365.

Algorithmic techniques for several optimization problems regarding distributed systems with tree topologies

M.-I. Andreica

Distributed systems present efficient solutions to many real-life problems and, as a consequence, they are spreading increasingly fast all over the world. As their development progresses, they present more and more challenges and, thus, the need for optimizing such systems from several perspectives becomes more stringent. In this paper we present several novel algorithmic techniques which are applicable to the optimization of distributed systems having tree topologies. We consider optimization problems regarding topics like reliability improvement, partitioning, coloring, content delivery and optimal matchings. Some of the presented techniques are only of theoretical interest, while others can be implemented and used in practical settings.

The motivation for considering distributed systems with tree structures is given by the fact that trees are some of the simplest non-trivial topologies which occur in real-life situations. Many of the existing networks have a hierarchical structure (a tree or tree-like graph), with user devices at the edge of the network and router backbones at its core. Some peer-to-peer systems used for content retrieval and indexing have a tree structure. Multicast content is usually delivered using multicast trees. Furthermore, many graph topologies can be reduced to tree topologies, by choosing a spanning tree or by covering the graph's edges with edge disjoint spanning trees [1]. In a tree network, there exists a unique path between every two nodes. Thus, the network is quite fragile. The fragility is compensated, though, by the simplicity of the topology, which makes many decisions in a distributed system become easier.

Some of the problems considered in this paper are the following:
1. find a minimum weight subset of extra edges to be added to a tree, such that every vertex of the resulting graph belongs to exactly one cycle;
2. partition a tree into almost-connected parts, subject to lower and upper bounds on the number of vertices of each part;
3. partition a tree into a fixed number of connected parts of given sizes;
4. compute the worst-case scenario of the first-fit tree coloring heuristic;
5. find an optimal matching (of maximum size or minimum cost) in a tree or the square of a tree, subject to several constraints.

The proposed algorithmic solutions use both general principles, like *dynamic programming* or *greedy*, and specialized techniques and data structures [2].

References

1. J. Roskind, R. E. Tarjan, *A note on finding minimum-cost edge-disjoint spanning trees*, Mathematics and Operations Research **10**, 4(1985), 701-708.
2. M. A. Bender, M. Farach-Colton, *The LCA problem revisited*, Proc. of the Latin American Symposium on Theoretical Informatics, Lecture Notes in Computer Science, **1776** (2000), 88-94.
3. M. Scortaru, *National olympiad in informatics*, Gazeta de Informatica (Informatics Gazzette), **12 (7)** (2002), 8-13.
4. S. M. Hedetniemi, S. T. Hedetniemi, T. Beyer, *A linear algorithm for the grundy (coloring) number of a tree*, Congressus Numerantium **36** (1982), 351-362.
5. M. I. Andreica, N. Tapus, *Reliability analysis of tree networks applied to balanced content replication*, Proc. of the IEEE Intl. Conf. on Automation, Robotics, Quality and Testing (AQTR) (2008), 79-84.
6. D. J. Chen, T. H. Huang, *Reliability analysis of distributed systems based on a fast reliability algorithm*, IEEE Transactions on Parallel and Distributed Systems, **3 (2)** (1992), 139-154.
7. A. Kumar, A. S. Elmaghraby, S. P. Ahuja, *Performance and reliability optimization for distributed computing systems*, Proc. of the 3rd IEEE Symp. on Computers and Communications (ISCC) (1998), 611-615.
8. H. Abachi, A. J. Walker, *Reliability analysis of tree, torus and hypercube message passing architectures*, Proc. of the 29th IEEE Southeastern Symp. on System Theory (1997), 44-48.
9. G. N. Frederickson, *Optimal algorithms for tree partitioning*, Proc. of the 2nd ACM-SIAM Symp. on Discrete Algorithms (SODA) (1991), 168-177.
10. R. Cordone, *A subexponential algorithm for the coloured tree partition problem*, Discrete Applied Mathematics **155 (10)** (2007), 1326-1335.
11. Y. Cui, Y. Xue, K. Nahrstedt, *Maxmin overlay multicast: rate allocation and tree construction*, Proc. of the 12th IEEE Workshop on Quality of Service (IWQOS) (2004), 221-231.

Theory of oligopolies: dynamics and stability of equilibria

K. Andriopoulos, T. Bountis, N. Papadopoulos

The theory of oligopolies is a particularly active area of research using applied mathematics to answer questions that arise in microeconomics. It basically studies the occurrence of equilibria and their stability in market models involving few firms and has a history that goes back to the work of Cournot in the 19th century. More recently, interest in this approach has been revived, owing to important advances in analogous studies of Nash equilibria in game theory. In this paper, we first highlight the basic ingredients of this theory for a concrete model involving two firms. Then, after reviewing earlier work on this model, we describe our modifications and improvements, presenting results that demonstrate the robustness of the approach of nonlinear dynamics in studying equilibria and their stability properties. On the other hand, plotting the profit functions resulting from our modified model we show that its behavior is more realistic than that of other models reported in the literature.

References

1. A. A. Cournot, *Recherches sur les principes mathématiques de la théorie des richesses*, Hachette, Paris, 1838.
2. A. Matsumoto, F. Szidarovszky, *Delayed nonlinear Cournot and Bertrand dynamics with product differentiation*, Nonlinear Dynamics, Psychology and Life Sciences **11**(2007), 367-395.

Bifurcation and stability in the problem on capillary-gravity waves on a surface of floating deep fluid layer

A. N. Andronov, L. R. Kim-Tyan, B. V. Loginov

Potential flows of incompressible heavy capillary floating fluid in three-dimensional layer of infinite depth with free upper surface are determined. Asymptotics of periodical regimes in spatial layer with free upper boundary close to the horizontal plane $z = 0$ bifurcating from the basic flow with constant velocity V in the Ox-direction are computed. Their stability is investigated. Methods of group-invariant bifurcation theory and group analysis of differential equations are used. Special attention is paid to cases of high-dimensional ($n \geq 4$) degeneration of the linearized operator.

The obtained results are supported by grant RFBR-Romanian Academy, project No. 07-01-91680a.

References

1. A. N. Andronov, *Bifurcational solutions asymptotics in the case of 4-dimensional degeneration of the linearized operator in the capillary-gravity waves problem on the surface of a deep fluid*, Proc. Middle-Volga Math. Soc., **9**, 2(2007). 9-14.

On CH-quasigroups of finite special rank

A. Babiy

CH-quasigroups of finite special rank are characterized by means of various non-medial (or medial) subquasigroups and by means of various non-Abelian (or Abelian) subgroups of its multiplication group.

The study of the stability of some economic models

D. Bălă

The dynamic models of some economic processes are presented. Actually we study the Liapunov stability of some economic processes described by equations and systems of differential equations. There is no general method to study the Liapunov stability and that's why it is necessary a certain experience obtained from the analysis of many concrete models.

Classic and modern in the construction of stellated polygons

R. Bălan

Older and recent views in the construction of stellated polygons are presented together with the corresponding analytical formulae. Some of these formulae are new.

The transvectants and the first integrals for Darboux systems of differential equations

V. Baltag, Iu. Calin

Consider the real systems of differential equations

$$\frac{dx}{dt} = cx + dy + x\,C(x,y), \qquad \frac{dy}{dt} = ex + fy + y\,C(x,y), \tag{1}$$

where c, d, e, f are real coefficients and the homogeneous polynomial $C(x,y)$ has real coefficients and degree $r \in \mathbb{N}^*$. The $GL(2,\mathbb{R})$-invariant S and the $GL(2,\mathbb{R})$-comitants [1] $R(x,y)$ and $C(x,y)$ of the system (1) have the form

$$S = c+f, \quad R(x,y) = -ex^2+(c-f)x+dy^2, \quad C(x,y) = \sum_{k=0}^{r} a_k x^{r-k} y^k.$$

Definition 1. [2] *Let $f(x,y)$ and $\varphi(x,y)$ be homogeneous polynomials in x and y with real coefficients of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, and $k \in \mathbb{N}^*$. The polynomial*

$$(f,\varphi)^{(k)} = \frac{(\rho-k)!(\theta-k)!}{\rho!\theta!}\sum_{h=0}^{k}(-1)^h\binom{k}{h}\frac{\partial^k f}{\partial x^{k-h}\partial y^h}\frac{\partial^k \varphi}{\partial x^h \partial y^{k-h}}$$

is called the transvectant of the index k of polynomials f and φ.

Let $p = \left[\dfrac{r-1}{2}\right]$ and suppose that: if the lower index in the symbol of the sum $\sum$ is greater than the upper index, then the sum is equal to zero; in repeated using of the transvectants a set of the parenthesis of the type $((\ldots($ will be replaced by a single parenthesis of the form $[\![$. In the following theorems the following expressions will be used:

$$F_r = \frac{2^{2p+1}\cdot r!}{(r-2p)!}\cdot R^p\left(\frac{2(r-2p)}{r}[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p},R)^{(1)} - [\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p}\cdot S\right)+$$

$$+\sum_{i=0}^{p-1}\left[\frac{2^{2i+1}\cdot r!}{(r-2i)!}\cdot R^i\left(\frac{2(r-2i)}{r}[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{i},R)^{(1)} - [\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{i}\cdot S\right)\times\right.$$

$$\left.\times\prod_{j=i+1}^{p}\left(2(r-2j)^2(R,R)^{(2)}+r^2S^2\right)\right] - \frac{1}{r^2}\prod_{j=0}^{p}\left(2(r-2j)^2(R,R)^{(2)}+r^2S^2\right);$$

$$\Phi_r(x,y) = -2R^{p+1}I_r + rS\cdot F_r(x,\,y), \qquad I_r = 2^{2p+2}\cdot(r-1)![\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p+1};$$

$$\Psi_r(x,y) = V_r(x,y)\cdot[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p+1} + R^{p+1}\cdot[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p},R)^{(1)}\cdot(C,C)^{(r)};$$

$$H=(R,R)^{(2)}, \quad V_r(x,y) = \frac{r+1}{r}R\cdot[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p+1}+$$

$$+\sum_{i=0}^{p}\left(\binom{r}{2i+1}[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{i}\cdot[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p-i},R)^{(1)} -\right.$$

$$\left.-\binom{r}{2i+2}[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{i},R)^{(1)}\cdot[\![C,\overbrace{R)^{(2)},\ldots,R)^{(2)}}^{p-i}\right);$$

$$G_1 = \exp\left[\frac{2S}{\sqrt{2H}}\arctan\frac{\dfrac{\partial R}{\partial x}-y\cdot\sqrt{2H}}{\dfrac{\partial R}{\partial x}+y\cdot\sqrt{2H}}\right], \qquad G_2 = \left|\frac{\dfrac{\partial R}{\partial x}-y\cdot\sqrt{-2H}}{\dfrac{\partial R}{\partial x}+y\cdot\sqrt{-2H}}\right|^{\frac{S}{\sqrt{-2H}}};$$

$$G_3 = \exp\left[\frac{S[(c-f)x^2+2(d+e)xy-(c-f)y^2]}{4(d-e)\,R}\right];$$

$$G_4 = \arctan\frac{ex+fy}{y\cdot\sqrt{\dfrac{H}{2}}}, \qquad G_5 = \left|\frac{\dfrac{\partial R}{\partial x}-y\cdot\sqrt{-2H}}{\dfrac{\partial R}{\partial x}+y\cdot\sqrt{-2H}}\right|.$$

Let $r = \deg C(x,y) = 2p+1$, where $p \in \mathbb{N}$.

Theorem *1. The system (1) with the conditions $R(x,y) \not\equiv 0$ and $C(x,y) \not\equiv 0$ has the following real first integrals:*

a) *for* $S \neq 0,\ H > 0: \quad |F_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_1 = c_1;$

b) *for* $S \neq 0,\ H < 0: \quad |F_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_2 = c_2;$

c) *for* $S \neq 0,\ H = 0: \quad |F_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_3 = c_3;$

d) *for* $S = 0: \quad |F_r|^{\frac{2}{r}} \cdot |R|^{-1} = c_4,$

where c_1, c_2, c_3 and c_4 are real constants.

Let $r = \deg C(x,y) = 2p+2$, where $p \in \mathbb{N}$.

Theorem 2. *The system (1) with the conditions $R(x,y) \not\equiv 0$ and $C(x,y) \not\equiv 0$ has the following real first integrals:*

a) *for* $S \neq 0,\ H > 0: \quad |\Phi_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_1 = c_5;$

b) *for* $S \neq 0,\ H < 0: \quad |\Phi_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_2 = c_6;$

c) *for* $S \neq 0,\ H = 0: \quad |\Phi_r|^{\frac{2}{r}} \cdot |R|^{-1} \cdot G_3 = c_7;$

d) *for* $S = 0,\ H > 0,\ I_r \neq 0: \quad \sqrt{\dfrac{H}{2}} \cdot \dfrac{1}{I_r} \cdot \dfrac{F_r}{R^{p+1}} + G_4 = c_8;$

e) *for* $S = 0,\ H < 0,\ I_r \neq 0: \quad G_5 \cdot \exp\left(-\dfrac{\sqrt{-2H}}{I_r} \cdot \dfrac{F_r}{R^{p+1}}\right) = c_9;$

f) *for* $S = 0,\ H = 0,\ I_r \neq 0: \quad \Psi_r \cdot R^{-(p+2)} = c_{10};$

g) *for* $S = 0,\ I_r = 0: \quad F_r \cdot R^{-(p+1)} = c_{11},$

where c_5, c_6, c_7, c_8, c_9, c_{10} and c_{11} are real constants.

References

1. K. S. Sibirsky, *Introduction to the algebraic theory of invariants of differential equations*, Manchester University Press, 1988.

2. G. B. Gurevich, *Foundations of the theory of algebraic invariants*, Noordhoff, Groningen, 1964.

On groupoids with a Moufang identity

G. Belyavskaya, A. Tabarov

We consider some classes of groupoids with the following Moufang identity

$$xy \cdot xz = x^2 \cdot yz. \tag{1}$$

In the class of loops this identity characterizes the commutative Moufang loops.

Proposition 1. *A groupoid $(Q,\cdot)$ with the identity (1) and an idempotent 0 such that the translations R_0 and L_0 are permutations is linear from the right over groupoid $(Q,\circ)$ with the identity 0 where $x \circ y = R_0^{-1}x \cdot L_0^{-1}y$.*

Corollary. *If, in addition, a groupoid $(Q, \cdot)$ from Proposition 1 is commutative, then the groupoid $(Q, \circ)$ is commutative, satisfies the Moufang identity (1) and $(Q, \cdot)$ is linear over the groupoid $(Q, \circ)$.*

Proposition 2. *If a groupoid with the identity (1) has a unity element then it is a groupoid with associative powers and satisfies the following identities*

$$(xy)^{2^n} = x^{2^n} y^{2^n}, \ a^{2n}(bx) = a^n b \cdot a^n x,$$

$$(a_1(a_2...(a_{k-1}b)...))^{2^n} = a_1^{2^n}(a_2^{2^n}...(a_{k-1}^{2^n} \cdot b^{2^n})...)$$

f*or any integer* $n \geq 1$.

These results are similar to the results received [1] with respect to the commutative groupoids with the Moufang identity $xy \cdot zx = (x \cdot yz)x$.

References

1. B. V. Novikov, *On decomposition of Moufang groupoids*, Quasigroups and related systems, **16**, 1(2008), 97-101.

A numerical study by FEM and FVM of a problem which presents a simple limit point

C. L. Bichir

A problem, which presents a simple limit point, is approximated by finite element method (FEM) and by finite volume method (FVM). In order to obtain the branch of solutions, an arc-length-continuation method and Newton's method are used. Numerical results obtained by two computer programs, based on FEM and FVM, are presented.

Lubrication model for the flow driven by high surface tension

E.-R. Borşa

The flow of a thin fluid film down an inclined plane driven by a surface tension is considered. Using the Navier-Stokes equations for thin film flow, the continuity equation, the no-slip condition and the boundary conditions, we obtain the fluid pressure, its velocity and the governing equation for the film height. In general, the introduction of surface tension into standard lubrication theory leads to a fourth-order nonlinear parabolic equation. For steady situations this equation may be integrated once and a third-order ordinary differential equation is obtained.

References

1. D. J. Acheson, *Elementary fluid dynamics*, Oxford University Press, 1990.

2. F. Bernis, A. Friedman, *Higher order nonlinear degerate parabolic equations*, J. Diff. Eq., **83**(1990), 179-206.
3. A. L. Bertozzi, *Lubrication approximations for surface tension driven interface: some open problems*, (to appear).
4. C. I. Gheorghiu, *An Unified treatment of boundary layer and lubrication approximations in viscous fluid mechanics*, “Babes-Bolyai”, (to appear).
5. T. G. Myers, *Surface tension driven thin film flows*, (to appear).
6. H. Ockendon, J. R. Ockendon, *Viscous flow*, Cambridge University Press, 1995.

The lattice of bicategory structures

D. Botnaru, A. Ţurcanu

In the category of local convex topological vectorial Hausdorff spaces, there exist multiple relationships between the following three lattices: lattice of reflective subcategory, lattice of coreflective subcategory and lattice of bicategory structures.

1. Some relationships between these lattices are created.
2. Bicategory structures with classes of projections $\mathcal{M}_u$-hereditary are described.
3. The factorization of any morphism according to certain bicategory structure is constructed.
4. The problem of factorization a reflector functor composed of two reflector functors of certain type is examined.

On the problem of geometrical interpretation of groups of $\overline{P}$-symmetry with “indexed” figures

M. Branişte, A. Lungu

The problem of geometrical interpretation of junior, semijunior and pseudojunior groups of $\overline{P}$-symmetry with “indexed” figures is studied. The conditions in which one junior, semijunior and pseudojunior group of $\overline{P}$-symmetry has or not geometrical interpretation are determined. The methodology of construction of “indexed” geometrical figure, which interprets group gave, is explained. The “indexed” figures are constructed and the results are analyzed.

References

1. A. P. Lungu, *On the theory of $\overline{P}$-symmetry.* Manuscript stored in VINITI archives, Moscow, 1709-78(1978), 16(Russian).
2. A. P. Lungu, *On the methods for deriving minor groups of $\overline{P}$-symmetry.* Manuscript stored in VINITI archives, Moscow, **1587**-79(1979), 22(Russian).

3. A. M. Zamorzaev, Yu. S. Karpova, A. P. Lungu, A. F. Palistrant, *P-symmetry and its further development*, Shtiintsa, Chisinau, 1986. (Russian)
4. A. Lungu, *Aplicaţii cvasiomomorfe şi produse semidirecte de grupuri*, Conferinţa Ştiinţifică Jubiliară "Universitatea de Stat din Moldova la 50 ani", Chişinău: USM, 1996, 22-24.
5. A. M. Zamorzaev, *On the groups of quasi-symmetry (P-symmetry).* Soviet Phys. Crystallogr. **12**, 5(1967), 819-825.(Russian)
6. A. M. Zamorzaev, E. I. Galyarskii, A. F. Palistrant, *Colored symmetry, its generalizations and aplications*, Shtiintsa, Chisinau, 1978.(Russian)
7. A. Lungu, M. Branişte, *Structura automorfismelor grupurilor cristalografice de categorie* G_{30}, Studia Universitatis, Seria "Ştiinţe exacte şi economice", Chişinău, CEP USM, **8**(2007), 5-11.

Some properties of semireflexive product of two subcategories

D. Botnaru, O. Cerbu

In the vectorial topological locally convex Hausdroff spaces we study:

1. the proprieties of the semireflexive product;
2. the correlation between the semireflexive product and the right product of two subcategories;
3. the semireflexive product and the c-reflective subcategories.

Operations in the lattice of the reflective subcategories

D. Botnaru, E. Baeş, O. Cerbu

We examine the operation of intersection and union in the class of reflective subcategories with some properties. There are shown two possibilities to construct the intersection and the union. We construct examples in different classes of reflective subcategories of the category of vectorial topological locally convex Hausdorff spaces.

Mathematics education: from intuition to rigor for usefulness

V. Bront

We plead for a differentiation in mathematical instruction according to the future career of the scholar. Some ideae and examples are chosen from the mathematics classes delivered by the author to future artisans in several industrial schools.

Quadratic dynamical systems and commutative algebras

I. Burdujan

Quadratic dynamical systems come both from quadratic differential systems (QDSs) and from quadratic difference systems. A plenty of mathematical models in life sciences (e.g. epidemic models, enzymatic reaction models, the models for the dynamics of microparasitic infections etc.) as well as many models in physics, chemistry, meteorology, engineering are realized as QDSs. There exists a standard procedure for homogenizing any QDS. It is applied in order for the obtained homogeneous quadratic differential systems (HQDSs) could be studied by using some algebraic techniques. In fact, a binary commutative algebra is naturally associated with any HQDS; this association implies the existence of a 1-to-1 correspondence between the classes of affinely equivalent systems and the classes of isomorphic commutative algebras. Consequently, the classification problem of HQDSs (up to an affine equivalence) is proved to be equivalent with the problem of classification (up to an isomorphism) of the commutative binary algebras. Actually, there exists a 1-to-1 mapping between the qualitative properties of a HQDS and the structural properties of the corresponding commutative algebra.

The study of any QDS is realized by using the following scheme

$$QDS \Rightarrow HQDS \iff A(\cdot) \Rightarrow A_{hs} \iff L_A \iff L_{vf},$$

where $A(\cdot)$ denotes the algebra associated with the analyzed system, A_{hs} is the homogeneous system (in the Yamaguti's meaning) associated with A, L_A is the Lie algebra generated by the left multiplications of A, and L_{vf} is a realization of L_A as a Lie algebra of polynomial vector fields.

This scheme is applied to study the mathematical model of a SIR epidemiological model. This time the associated algebra $A(\cdot)$ of the homogenized QDS has $dim_{\mathbb{R}}\ L_A = 6$, while $L_A = S \oplus B$ is its Levi-Maltsev decomposition (with S - its maximal solvable radical and B is a Lie subalgebra isomorphic with $s\ell(2,\mathbb{R})$). Similar results are obtained for Leibnitz and Rösler models.

A classification up to an affinity of HQDSs defined on $\mathbb{R}^3$ and having a derivation with a complex characteristic root is realized. A qualitative study of the corresponding HQDSs is achieved.

On remainders of Wallman compactifications of T_0-Spaces

L. I. Calmuţchi, M. M. Choban

Let X be a T_0-space and $F(X)$ be the family of all closed subsets of X.

The family ξ of subsets of X is called an ultrafilter in X if it has the following properties: U1. $\xi \subseteq F(X)$, $\emptyset \notin \xi$ and $A \cap B \in \xi$ for any $A, B \in \xi$;U2. if $A \in \xi$, $B \in F(X)$ and $A \subseteq B$, then $B \in \xi$; U3. if $B \in F(X)$ and $B \notin \xi$, then $A \cap B = \emptyset$ for some $A \in \xi$.

The ultrafilter ξ is free if $\cap\xi = \emptyset$. For every $x \in X$ we put $\xi(x) = \{H \in F(X) : x \in H\}$. Let ufX be the family of all free ultrafilters in X, $sfX = \{\xi(x) : x \in X\}$. On $\omega X = sfX \cup ufX$ consider the topology generated by the closed base $eF(X) = \{eH = \{\xi \in \omega X : H \in \xi\} : H \in F(X)\}$. The correspondense $x \to \xi(x)$ is an embedding of X into ωX. We identify x and $\xi(x)$. Then $X \subseteq \omega X$ and X is dense in ωX. The space ωX is a compact T_0-space and it is called the Wallman compactification of the space X [1 - 3]. If X is a T_1-space, then ωX is the space of all ultrafilters in X and ωX is a T_1-space.

In ([1], Problem 4) it was formulated the following question.

Question. Let Y be a T_0-space. Under which conditions there exists a T_0-space X such that the spaces Y and $\omega X \setminus X$ are homeomorphic?

Proposition 1. *Let Y be a T_0-space and Y be not a T_1-space. Then there does not exists a T_0-space X for which the spaces Y and $\omega X \setminus X$ are homeomorphic.*

Theorem 1. *Let Y be a T_1-space, τ be an infinite regular cardinal and $\chi(Y) \leq \tau$. Then there exists a τ-compact T_1-space X such that*:

1. *the spaces Y and $\omega X \setminus X$ are homeomorphic;*
2. *if Y is a compact space, then the space X is locally compact;*
3. *if Y is a normal compact space or a Hausdorff locally compact space, then X is a normal space.*

Denote by βX the Stone-Čech compactification of a completely regular T_1-space X. A space $X$is normal if and only if $\beta X = \omega X$.

Theorem 2. *Let Y be a completely regular T_1-space and ω_1 be the first uncountable cardinal. Then there exists a completely regular pseudocompact T_1-space X such that*:

1. *the spaces Y and $\beta X \setminus X$ are homeomorphic;*
2. *if Y is a compact space, then the space X is locally compact;*
3. *the weight $w(X) = w(\beta X) = w(Y) + \omega_1$.*

References

1. L. I. Calmuţchi, *Metode algebrice şi funcţionale în teoria extensiilor spaţiilor topologice,* Editura Pamântul, Piteşti, 2007.
2. L. I. Calmuţchi, M. M. Choban, *On Wallman-Shanin compactification of T_0-spaces,* Analele Universitâţii din Oradea, Fascicola Matematica, **6**(1997-1998), 130-141.
3. R. Engelking, *General topology,* PWN, Warszawa, 1977.

On a differential sandwich theorems associated with multiplier transformations

A. Cătaş

Some subordination and superordination results for certain normalized analytic functions associated with multiplier transformations are presented. The results obtained in this paper are compared with various known results.

References

1. F. M. Al-Oboudi, *On univalent functions defined by a generalized Salagean operator*, Int. J. Math. Math. Sci., **27**(2004), 1429-1436.
2. T. Bulboaca, *Classes of first-order differential superordinations*, Demonstratio Math., **35**, 2(2002), 287-292.
3. N. E. Cho, H. M. Srivastava, *Argument estimates of certain analytic functions defined by a class of multiplier transformations*, Math. Comput. Modelling, **37**, 1-2(2003), 39-49.
4. S. S. Miller, P. T. Mocanu, *Differential subordination. Theory and applications*, Pure and Applied Mathematics, Marcel Dekker, New York, 2000.
5. S. S. Miller, P. T. Mocanu, *Subordinants of differential superordinations*, Complex Var. Theory Appl., **48**, 10(2003), 815-826.
6. G. S. Salagean, *Subclasses of univalent functions*, Complex analysis.Fifth Romanian-Finnish Seminar, Part 1 (Bucharest), LNM, **1013**, 1981, 362-372.
7. D. R. Wilken, J. Feng, *A remark on convex and starlike functions*, J. London Math. Soc., Ser. 2, **21**(1980), 287-290.

Categorial aspects of the semireflexive product

O. Cerbu

We examine the semireflexive product of two subcategories and their correspondence under the bicategory structures. We establish the categorial diapason with each component of the semireflexive product.

On topological groupoids and (n,m)-homogeneous isotopies

L. Chiriac, N. Bobeica

A groupoid G is called medial (paramedial) if it satisfies the law $xy\cdot zt = xz\cdot yt$ ($xy\cdot zt = ty\cdot zx$) for all $x, y, z, t \in G$. A groupoid G is called a primitive groupoid with divisions, if there exist two binary operations $l : G\times G \to G$, $r : G\times G \to G$, such that $l(a, b)\cdot a = b$, $a\cdot r(a, b) = b$ for all $a, b \in G$. Thus a primitive groupoid with divisions is a universal algebra with three binary operations. We study the topological medial groupoids with (n, m)-identities, which are obtained by using special isotopies of topological groupoids. Such groupoids are called the (n, m)-homogeneous isotopies. We investigate some properties of (n, m)-homogeneous isotopies of medial topological groupoids. The relationship between mediality and paramediality, paramediality and associativity is examined too. We extended some affirmations of the theory of topological groups on the class of topological (n, m)-homogeneous primitive goupoid with divisions.

Selections of lower semi-continuous mappings and compactness

M. M. Choban, E. P. Mihaylova, S. I. Nedev

The aim of the present article is to determine conditions on a space X under which for any lower semi-continuous closed-valued mapping $\theta : X \to Y$ of the space X into a complete metrizable (or discrete) space Y there exists a selection $\varphi : X \to Y$ for which the image $\varphi(X)$ is “small” in a given sense. All considered spaces are assumed to be T_1-spaces. Our terminology comes, as a rule, from [1 - 3]. The cardinal number $k(X) = min\{m :$ every open cover of X has an open refinement of cardinality $< m\}$ is *the degree of compactness* of X. The following assertions are considered:

$K1$. $k(X) \leq \tau$;

$K2$. *for every lower semi-continuous closed-valued mapping* $\theta : X \to Y$ *into a complete metrizable space* Y *there exists a lower semi-continuous selection* $\phi : X \to Y$ *of* θ *such that* $k(cl_Y \phi(X)) \leq \tau$*;*

$K3$. *for every lower semi-continuous closed-valued mapping* $\theta : X \to Y$ *into a complete metrizable space* Y *there exists a single-valued selection* $g : X \to Y$ *of* θ *such that* $k(cl_Y g(X)) \leq \tau$*;*

$K4$. *for every lower semi-continuous closed-valued mapping* $\theta : X \to Y$ *into a complete metrizable space* Y *there exists a lower semi-continuous selection* $\phi : X \to Y$ *of* θ *such that* $w(\phi(X)) < \tau$*;*

$K5$. *for every lower semi-continuous closed-valued mapping* $\theta : X \to Y$ *into a complete metrizable space* Y *there exist a compact-valued lower* *semi-continuous mapping $\varphi : X \to Y$ and a compact-valued upper semi-continuous mapping $\psi : X \to Y$ such that $k(cl_Y(\psi(X))) \leq \tau$ and $\varphi(x) \subseteq \psi(x) \subseteq \theta(x)$ for any $x \in X$.*

Theorem 1. *Let X be a regular space and τ be a regular cardinal number. Then the assertions $K1 - K3$ are equivalent. Moreover, if the cardinal number τ is not sequential, then the assertions $K1 - K4$ are equivalent.*

Theorem 2. *Let X be a $\mu -$ complete space and τ be a sequential cardinal number. Then the assertions $K1 - K3$ are equivalent.*

Theorem 3. *Let X be a paracompact space and τ be an infinite cardinal. Then the assertions $K1 - K3$ and $K5$ are equivalent. Moreover, if the cardinal number τ is not sequential, then the assertions $K1 - K5$ are equivalent.*

For a space X put $\omega(X) = \bigcup\{U : U$ is open in X and $dimU = 0\}$ and let $c\omega(X) = X \setminus \omega(X)$ be *the cozero-dimensional kernel* of X. The similar results for the spaces with the property $k(c\omega(X)) \leq \tau$ are proved.

References:

1. M. M. Choban, E. P. Mihaylova, S. Y. Nedev, *On selections and classes of spaces*, Topology and Appl., **155**, 8(2008), 797-804.
2. E. Michael, *A theorem on semi-continuous set-valued functions,* Duke Math. J. **26**, 4(1959), 647-656.
3. D. Repovš, P. V. Semenov, *Continuous selections of multivalued mappings*, Kluwer, Dordrecht, 1998.

The Frattini theory for p - Lie algebras

C. Ciobanu

In this paper we present $F_p(L)$ and $\Phi_p(L)$. The basic properties are given in Section 2. The Section 3 is concerned with the main result $F_p(L) \subset F_p(L) \subset C_L(F(L))$.

References

1. G. P. Hochschild, *Basics of algebraic groups and Lie algebras*, Graduate Texts in Math., **75**, Springer, New York, 1981.
2. J. E. Humphreys, *Introduction to Lie algebras and their representation theory*, Graduate Texts in Math., **9**, Springer, New York, 1972.
3. J. E. Jacobson, *Lie algebras*, Dover, New York, 1979.
4. H. C. Myung, *Lie-admissile algebra*, Soviet Encyclopaedia Math., Kluwer, Dordrecht, **5**, 1990.
5. C. Reutenauer, *Free Lie algebras*, Clarendon, Oxford, 1995.
6. K. W. Roggenkamp, *Cohomology of Lie algebras, Groups and algebras*, Seminar Series in Mathematics, "Ovidius" University, Constanta, 1994.

About some particular classes of bounded operators on pseudo-Hilbert spaces

L. Ciurdariu

In this paper we define n-quasicontractions, n-quasi-isometries, n-quasihyponormal and power bounded operators on pseudo-Hilbert spaces, give some properties and show that a $(T^*T)^\alpha$-contraction with $0 < \alpha < 1$ is a power bounded operator. Some basic properties of a quasi-isometry on pseudo-Hilbert spaces are investigated too.

Local subrings of function fields and some questions related to the problem of the finite generation of subalgebras.III

A. Constantinescu

This talk is dedicated to the memory of Professor Masayoshi NAGATA (1927-2008), a giant creator and a world leader in the fields of Commutative Algebra*, 14-th Hilbert Problem** and Invariant Theory ***, an outstanding algebraic geometer, Member of the Executive Committee and Vice-President of the "International Mathematical Union (IMU)" between 1975 and 1982.

Based on our approach on the local finite generation of subalgebras of algebras of finite type, one presents an idea of proof for the following.

Theorem 1. *Let k be a field, $\mathcal{O}$ a local k-subalgebra of an algebraic function fields K over k with the maximal ideal $\mathfrak{m} \subset \mathcal{O}$. Suppose that: i) $\mathcal{O}$ is a normal Noetherian ring; ii) $dim\,\mathcal{O} + tr.deg._k\,\mathcal{O}/\mathfrak{m} = tr.deg._k\,K$. Then $\mathcal{O}$ is essentially of finite type over k.*

Theorem 1 is a strengthening of our result [1], Lemma 3 - issued also later as the main result in [2], with a different proof. Some developments have been done in [3] and [4].

In low dimensions of $\mathcal{O}$ one establishes some reinforcements of Theorem 1, as follows:

Corollary - *Let $k, \mathcal{O}, K, \mathfrak{m}$ as above. Suppose that: 0) $dim\,\mathcal{O} = 1$ (resp. $dim\,\mathcal{O} = 2$); i) $\mathcal{O}$ is a normal ring (resp. $\mathcal{O}$ is a Krull ring); ii) $dim\,\mathcal{O} + tr.deg._k\,\mathcal{O}/\mathfrak{m} = tr.deg._k\,K$. Then $\mathcal{O}$ is essentially of finite type over k.*

References

*. M. Nagata, *Local rings.* Interscience Tracts in Pure and Applied Mathematics, **13**, John Wiley and Sons, New York, 1962, reprinted by R. E. Krieger Pub. Co., 1975.

**. M. Nagata, *On the fourteenth problem of Hilbert, Proc. of the Int. Congress of Mathematicians*, Cambridge University Press, 1958, 459-462.

***. M. Nagata, *Invariants of a group in an affine ring*, J. Math. Kyoto Univ., **3**(1964), 369 - 377.

1. A. Constantinescu, *Algebraization of the schemes dominated by algebraic varieties.* I, Rev. Roum. Math. Pures et Appl., **31**, 6(1986), 525 - 535.
2. B. Nashier, *When is a regular ring a locality ?*, J. of Algebra, **118**, 2(1988), 464 - 467.
3. W. Heinzer, C. Huneke, J. D. Sally, *A criterion for spots*, J. Math. Kyoto Univ., **26**, 4(1986), 667-671.
4. N. Onoda, *A remark on spots*, Kobe J. Math., **5**, 1(1988), 103-107.
5. A. Constantinescu, *Local subrings of function fields and some questions related to the problem of the finite generation of subalgebras.*I,II, talks at ICTAMI III, Thessaloniki, September 15-18, 2004, resp. CAIM XIV, Chisinau, 17-20, 2006.

Systems of equations involving almost periodic functions

S.-O. Corduneanu

We use the theory of Fourier series for almost periodic functions for solving a system of equations which is linear with respect to convolution by functions with compact support.

The similarity of XML-based documents in finding the legal information

S. Cornoiu

In recent years, W3C's XML (eXtensible Mark-up Language) has been accepted as a major mean for efficient data management and exchange. The use of XML ranges over information formatting and storage, database information interchange, data filtering, as well as web services interaction. Due to the ever-increasing web exploitation of XML, an efficient approach to compare XML-based legal documents becomes crucial in information retrieval (IR). Legal documents play an important role in all activities related to the legal domain. In particular they represent an efficient human communication mean to transmit legal knowledge. Legislations are often complex and prone to change.Organizations that base their daily work on a set of legal documents have to deal with a massive amount of legal and numerous legal updates. Legal documents typically combine structured and unstructured information. The structured information is increasingly tagged with markup languages such as XML (Extensible Markup Language). A range of algorithms for comparing semi-structured data, e.g. XML documents, have been proposed in the literature. All of these approaches focus exclusively on the structure of documents, ignoring the semantics involved. However, in the legal information retrieval systems, estimating semantic similarity between legal documents is of key importance to improving search results. In this paper, we propose integrates semantic similarity assessment in an edit distance algorithm, seeking to amend similarity judgments when comparing XML-based legal documents.

References

1. J. Tekli, R. Chbeir, K. Ytongnon, *A hybrid approach for XML similarity*, 33rd Int. Conf. on Current Trends in Theory and Practice of Computer Science Springer, Harrachov, Czech Republic, 2007, 783-795.
2. D. Buttler, *A short survey of document structure similarity algorithms*, The 5th Int. Conf. on Internet Computing, Las Vegas, NV, June 21-24, 2004.
3. J. Tekli, R. Chbeir, K. Ytongnon, *Semantic and structure based XML similarity-an integrated approach*, 13th Int. Conf. on Management of Data (COMAD'06), Delhi, India, 2006.
4. *Joint Practical Guide for the drafting of Community legislation*, http://reterei.eu/rete/GPC_en.pdf.
5. F. Fady, R. Rousselot, *DARES: Documents annotation and recombining system-application to the European law*, Artificial Intelligence and Law, **15**, 2(2007), 83-102(20), 2007.
6. M. F. Moens, *Retrieval of legal documents: combining structured and unstructured Information*, Proc. ELPUB2005 Conf. on Electronic Publishing, Katholic Univ., Leuven 2005.
7. O. Vasutiu, D. Jouve, Y. Amghar, J.M. Pinon, *XML based legal document drafting information system*, Standards for Legislative XML Workshop, Amsterdam, 15, 2007.
8. T. Pedersen, J. Michelizzi, *Wordnet::Similarity - measuring the relatedness of concepts*, Proc. of the 9th Conf. on Artificial Intelligence (AAAI-04), 2004.

The cubic differential system with two invariant conics

D. Cozma

We consider the system of cubic differential equations

$$\begin{aligned} \dot{x} &= y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3 = P(x,y), \\ \dot{y} &= -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3) = Q(x,y), \end{aligned} \tag{1}$$

in which all variables and coefficients are assumed to be real. The origin $O(0,0)$ is a singular point of a centre or focus type for (1), i.e. a weak focus. An algebraic curve $f(x,y) = 0$ (real or complex) is said to be an invariant curve of system (1) if there exists a polynomial $K(x,y)$ such that $P \cdot \partial f/\partial x + Q \cdot \partial f/\partial y = K \cdot f$. The polynomial K is called the cofactor of the invariant algebraic curve $f = 0$. For cubic system (1) we find coefficient conditions for the existence of two irreducible invariant conics $f(x,y) \equiv c_{20}x^2 + c_{11}xy + c_{02}y^2 + c_{10}x + c_{01}y + 1 = 0$ and study the problem of the centre.

References

1. D. Cozma, A. Şubă, *Solution of the problem of the center for a cubic differential system with three invariant straight lines*, Qualitative Theory of Dynamical Systems, **2**(2001), 129 – 143.

2. A. Şubă, D. Cozma, *Solution of the problem of the centre for cubic differential system with three invariant straight lines in generic position*, Qualitative Theory of Dynamical Systems, **6**(2005), 45-58.

Weak Solutions of the Mixed Boundary Value Problems of Plane Micropolar Elasticity

I. Crăciun

The elastic equilibrium theory of the interior of a material right cylinder B with the generic cross–section $S \subset \mathbb{R}^2$ and the lateral boundary Π is considered. Let $\Gamma = \partial S$ be the boundary of S. Suppose that the cylinder is occupied by a homogeneous, isotropic, and centro–symmetric linearly elastic micropolar (Cosserat) material with elastic constants $\lambda, \mu, \alpha, \beta, \gamma$, and ε.

The Cosserat, micropolar or asymmetric elasticity was established by Eringen in [1] and then it was developed and studied in many papers and books, e.g. [2], [3], [4], [5]. The rectangular Cartesian coordinate frame is supposed to be chosen in such a way that the x_3-axis is parallel to the generators of B. The micropolar elastic plane strain parallel to the x_1, x_2-plane is characterized by the following fields of displacements and rotations $u_\alpha = u_\alpha(x_1, x_2) = u_\alpha(x)$, $u_3 = 0$, $\varphi_\alpha = 0$, $\varphi_3 = \varphi = \varphi(x)$, where the Greek indexes run over $1, 2$, while the Latin indexes take the values 1, 2, 3. These functions defined on $\bar{S} = S \cup \Gamma$ are the unknowns of the equilibrium equation

$$-L(\partial_x)u(x) = f(x), \quad x \in S, \tag{1}$$

where $u(x) = (u_1(x), u_2(x), \varphi(x))^T$, $f = (X_1, X_2, Y_3)^T$, f being the load vector (body forces and couple body force), and $L(\partial_x) = L(\partial/\partial x) = L(\xi) = L(\xi_\alpha)$ is the linear differential operator of the plane asymmetric elasticity theory.

The plane boundary value problems (BVPs) of the Cosserat elasticity represent the BVPs of equation (1). They have been developed by many authors where the regular (classical) solutions have been obtained in the form of integral potentials in $L^2(S)$ space. However, in $L^2(S)$ such solutions can be found only if the boundary Γ is sufficiently smooth and cannot be obtained in the case of reduced boundary smoothness or if the domain contains cracks. In order to obtain solutions for the domains with irregular boundaries, guided [6], we formulate a BVP of plane Cosserat elasticity in a Sobolev space V and introduce the corresponding weak solution. By using the dual space V^* of the Hilbert space V, and the Riesz representation theorem we establish existence, uniqueness, continuously dependence of the data of the corresponding BVP, and stability of the weak solution of this BVP of plane Cosserat elasticity. The same scheme was recently used by us in [7] and [8] to study some BVPs of the steady–state heat conduction equation [9].

The weak or variational form of a classical BVP of plane Cosserat elasticity have the advantageous that it can be approached by different methods, both analytical and computational, than those which apply to the classical form of that BVP. The weak form of a BVP puts much less stringent requirements on the problem functions (such as the right–hand side of (1) and the solution, and therefore applies to a larger set of problems. However, a more important advantage is that the weak form admits new solution techniques, in particular the Galerkin method. The variational form of a BVP and the Galerkin method represent the two theoretical foundations of the finite element method (FEM)[10] for producing an approximate solution to a variational equation from a given finite–dimensional subspace. The FEM is Galerkin's method with a subspace of piecewise polynomial functions. The Galerkin method requires the computation of the stiffness matrix K and the load vector F, and the solution of the system of algebraic linear equations $KU = F$. The Galerkin method produces the best approximation, from a given approximating subspace, to the exact solution of a variational problem. When the approximating subspace in the Galerkin method is chosen to be a subspace of piecewise polynomial functions, the resulting algorithm is both efficient and effective. It is easy to integrate and differentiate polynomials (the main requirements for computing K and F). Moreover, piecewise polynomials lead naturally to a sparse stiffness matrix, allowing $KU = F$ to be solved efficiently. The system $KU = F$ can be formed and solved efficiently even when the number of unknowns is very large and the resulting approximate solution can be highly accurate. Finally, smooth functions can be well–approximated by piecewise polynomials.

References

1. A. C. Eringen, *Linear theory of micropolar elasticity*, J. Math. Mech., **15**(1966), 909-923.
2. W. Nowacki, *Theory of asymmetric elasticity*, Polish Scientific Publishers, Warszaw, 1986.
3. V. D. Kupradze, et al., *Three-dimensional problems of the mathematical theory of elasticity and thermoelasticity*, North-Holland, Amsterdam, 1979.
4. D. Ieşan, *Thermoelastic models of continua*, Kluwer, Dordrecht, 2004.
5. J. Dyszlewicz, *Micropolar theory of elasticity*, Springer, Berlin, 2004.
6. M. S. Gockenbach, *Understanding and implementing the finite element method*, SIAM, Philadelphia, 2006.
7. I. Al. Crăciun, *Weak solutions of the Boundary Value Problems of the Steady-State Heat Equation*, Bul. Inst. Politehnica, Iaşi, **54**(58),(LVIII), 2(2008), 211-218.
8. I. Al. Crăciun, *On the existence, and uniqueness for the weak solution of a boundary value problem of the steady–state heat conduction equation*, ibidem, 219-224.
9. N. M. Özişik, *Heat conduction, 2nd ed.*, John Wiley & Sons, New York, 1993.
10. C. C. Spyrakos, J. Raftoyiannis, *Linear and nonlinear finite element analysis in engineering practice*, Algor, Inc. Publishing Division Pittsburgh, PA, 1997.

The particular invariant $GL(2,\mathbb{R})$-integral for two-dimensional nonlinear differential system

O. V. Diaconescu, M. N. Popa

The nonlinear two-dimensional differential systems are considered. The necessary and sufficient conditions for the existence of partial invariant $GL(n,\mathbb{R})$-integral of the nonlinear two-dimensional differential systems are indicated. The form of the systems which admit centeraffine-invariant quadratic integral are found. This sistem has the particular integral are found if and only if two of parameters of nonlinearities can be expressed by the other parameters.

The work is partially supported by grant CERIM-1006-06 (CRDF-MRDA).

References
1. M. N. Popa, *The center-affine invariant conditions of comitants devidedness for the differential system*, Preprint, Chisinau, **65**(1989)(Russian).
2. K. S. Sibirsky, *Introduction to the algebraic theory of invariants of differential equations*, Shtiintsa, Kishinev, 1982; Manchester University Press, 1988.

On exponential trichotomy of C_0-semigroups in locally-convex spaces

S. Dragan

In this paper we obtain some characterizations for the exponential trichotomy property of C_0-semigroups in locally-convex spaces.

On the stability of viscous flows in curved channels

F. I. Dragomirescu, A. Georgescu

The Taylor-Dean viscous fluid flow between two rotating cylinders is a combination of a circular Couette flow and azimuthal Poiseuille flow. Its linear stability was investigated analytically and numerically, among others, in the case when the size of the gap between the two cylinders was taken into account [1], [2], [3], [4]. When this parameter becomes important, the existence of a large variety of patterns bifurcated from the Taylor-Dean flow depends on the strata determined in the parameter space. The main interest in most of these studies is for the critical instability conditions. In this paper, the eigenvalue problem governing the Lyapunov stability of the basic Taylor-Dean flow against rotationally symmetric perturbations, previously investigated in [5] by using isoperimetric inequalities, is studied along with some other examples from [6], [3], by means of spectral methods based on Legendre polynomials [7]. In each case the critical Taylor number at which the instability sets in is obtained. All our numerical results agree with those existing in the literature.

References
1. F. Chen, M. H. Chang, *Stability of Taylor-Dean flow in a small gap between rotating cylinders*, J. Fluid Mech., **243**(1992), 443-455.
2. P. Laure, I. Mutabazi, *Nonlinear analysis of instability modes in the Taylor-Dean system*, Phys. Fluids, **6**, 11(1994), 3630-3642.
3. I. Mutabazi, C. D. Andereck, *Transition from time dependent to stationary flow patterns in the Taylor-Dean system*, Phys. Rev. **A**, **44**, 10(1991), 6169-6171.
4. C. P. Hills, A. P. Bassom, *The asymptotics of neutral curve crossing in Taylor-Dean flow*, ZAMP, **57**(2006), 76-93.
5. A. Georgescu, *Stability of spiral flow and of the flow in a curved channel*, Rev Roum. de Math. Pures and Appl., **17**, 3(1972), 353-357.

6. W. H. Reid, *On the stability of viscous flow in a curved channel*, Proc. Roy. Soc., **A**, **244** (1958), 186-198.
7. C. I. Gheorghiu, I. Dragomirescu, *Spectral methods in linear stability. Applications to thermal convection with variable gravity field*, http://dx.doi.org/10.1016/j.apnum.2008.07.004.

Analytical and numerical solutions to an electrohydrodynamic stability problem

I.-F. Dragomirescu

A linear hydrodynamic stability problem corresponding to an electrohydrodynamic convection between two parallel walls is considered. The problem is an eighth order eigenvalue one containing homogeneous boundary conditions for the even order derivatives up to the sixth order. By variational arguments it is shown that its smallest eigenvalue is real and positive. The problem is transformed into a second order differential system supplied with Dirichlet boundary conditions. Two classes of methods are used in order to solve the problem, namely, direct methods (based on series of Chandrasekar-Galerkin type and of Budiansky-DiPrima type) and spectral methods (tau, Galerkin and collocation) based on Chebyshev and Legendre polynomials. For certain values of the physical parameters the numerically computed eigenvalues from the low part of the spectrum are displayed in a table. They are accurate and confirm the analytical results.

Computer assisted design of a digital thermostate

A. M. Dumitrescu, M. A. Ghelmez

Simulations using computers and the proper choice of materials represent the ways of improving the performance of many devices [1]. The goal of this work is to design a digital thermostate, having a temperature measuring and display system based on the microcontroller Atmega 128. This device is working with previous imposed parameters, established taking into account the practical applications: thermostating an usual room, or a special room needing a constant temperature maintaining [2].

Advantages of this system are the scalability, fiability and low cost. The work exposes shortly some general and basic idea of the project, bloc diagrams and detailed description of the bias supply, sensors for interior and exterior temperature, display and control system, data processing of the results. Details are given on the microcontroller Atmega 128, programming this microcontroller (AVAR Studio 4), cabling circuits, and a list of components. The bias supply, necessary for transforming the alternative network voltage in a d.c. 5V voltage for the microcontroller, sensors and operational amplifiers supply was designed using Orcad Capture. It contains 4 main parts: transformer, rectification (a nonlinear element), filtering (electrolytic condensers), and stabilizers.

Sensors are analogical devices, and because of that the majority of physical quantities have continuous spectra; the output voltage can be written in terms of the unknown temperature. The display and control devices actually interact with the user. For Atmega 128 programming, the AVR Studio 4 application (ATMEL) has been used. A description of the functions used in program is given, followed by the detailed program. Then, the electric scheme was exported in Orcad Layout program, and the functional zones were created, verified and realized on the cable bay-up (top and bottom layers). Real cabling was printed (TOP, BOTTOM, DRILL, SOLDERING MASK).

References

1. J. Mueller, U. Dilner, *Thermal microsensors. Staking out the future*, Europhys. News, **251**(1994), 207-209.
2. M. Ghelmez (Dumitru), E. Slavnicu, D. Slavnicu, C. Toma, A. R. Sterian, *Studies in laser field of some irradiated fatty acids in liquid crystal state*, Rev. Chim. (Bucuresti), 56, 7, **1**(2005) , 762-765.

Interaction between mathematics and artificial intelligence: fuzzy logic versus Boolean logic

I. Dzitac

Artificial Intelligence (AI) is an important and active actual research field. Interaction between mathematics and AI has been resumed in 1996 [1] in the following phrase: "The role of mathematics in artificial intelligence is pervasive, but controversial; the role of artificial intelligence in mathematics is relatively small, but growing, and could be larger, to the benefit of both fields." In this paper we present a survey of actual aspects of interaction between mathematics and AI realized in some papers published in [2] and [3]. Main part of our work consists in a description of discussions about paper "Fuzzy Set Theory in Boolean Frame" presented by Dr. Dragan G. Radojevic at International Conference on Computers, Communications and Control(ICCCC 2008), and published in [3]. We here present the opinions of Professor Lotfi A. Zadeh, creator of fuzzy logic theory, Dr. Gheorghe Paun, creator of P systems theory and other famous scientists.

References

1. F. Hoffman (ed.), *Mathematical aspects of artificial intelligence*, Proc. of Symposia in Applied Mathematics, AMS, Orlando, Florida, 1996.
2. L. A. Zadeh, D. Tufis, F. G. Filip, I. Dzitac (eds.), *From natural language to soft computing: new paradigms in artificial intelligence*, Romanian Academy Ed., Bucharest, Romania, 2008.
3. I. Dzitac, F. G. Filip, M. J. Manolescu (eds.), *Proc. of ICCCC 2008*, Int. J. of Computers, Communications and Control (IJCCC), III, supplementary issue, (2008) 1841-9836.

Reliability simulation software system for electric plants using fuzzy algorithms

S. Dzitac, T. Vesselenyi, I. Dzitac, M. Pârv

The paper comprises three parts. The first is an introduction of fuzzy logic application in electromagnetic systems and equipments reliability analysis. The second concerns the development of a computer simulation program for complex electric system reliability study fuzzy algorithms, definition of asymmetric Gauss input and output membership functions, rule sets and results display methods. The third part is focused on the development of a case study for the electric station in Voivozi, Bihor county using the developed simulation program in the Matlab environment. The fourth part emphasis on the conclusions which show the importance and efficiency of fuzzy modeling in reliability analysis by comparative evaluation of fuzzy and Monte Carlo methods also shown in equivalent reliability diagrams, highlighting authors contributions. The last part of the paper presents the references which were consulted.

Keywords: *failure tree, reliability, fuzzy simulation*

Numerical approximation of Poincaré maps

R. Efrem

A classical technique for analyzing dynamical systems is due to [4]. It replace the flow of a nth-order continuous-time system by an (n-1)th-order discrete-time system called the Poincaré map. A Poincaré map essentially describes how points on a plane Σ (the Poincaré section), which is transverse to an orbit Γ and which are sufficiently close to Γ, get mapped back onto Σ by the flow. Such construction of the Poincaré map has become one of the basic tools in nonlinear dynamics and is contained in every textbook of nonlinear dynamics [5]. Unfortunately, except under the most trivial circumstances, the Poincaré map cannot be expressed by explicit equations. Here the numerical analysis interposes.

In this paper we will present some algorithms that help us to construct the Poincaré map, we build their Maple code, and implement them on some examples.

References

1. S. H. M. J. Houben, J. M. L. Maubach, R. M. M. Mattheij, *An accelerated Poincaré-map method for autonomous oscillators*, Applied Mathematics and Computation, **140**, 2-3, 2003.
2. W. Just, H. Kantz, *Some considerations on Poincaré maps for chaotic flows*, Journal of Physics A: Mathematical and General, **33**, 1(2000).
3. T. Parker, L. Chua, *Practical numerical algorithms for chaotic systems*, New York, Springer, New York, 1989.
4. H. Poincaré, *Les méthodes nouvelles de la méchanique céleste*, Gauthier-Villars, Paris,1899 .
5. H. G. Schuster, *Deterministic chaos*,VCH, Weinheim, 1989.

Generalized semidirect products

D. Fechete, I. Fechete

In this paper we give a generalization of the semidirect product of a commutative ring with identity R and an Abelian group G and we study some properties of this construction.

Semidirect products and trivial extensions. A categorial overview

D. Fechete, I. Fechete

In this paper we study some categorial properties concerning the trivial extension and the semidirect product of a commutative ring with identity R and an Abelian group G.

Algorithms for finding optimal flows on dynamic networks

M. Fonoberova

Optimal dynamic flows are widely used in modeling processes from different economic, technical and informational systems. Traffic, telecommunication, logistics, evacuation problems, scheduling, personnel assignment are just a tiny sample of the applications that have been posed as network flow problems. We extend and generalize the classical optimal flow problems on networks for the cases of nonlinear cost functions on arcs, multicommodity flows and time- and flow-dependent transactions on arcs of the network. We assume that all parameters of the network depend on time. We also study the dynamic model with transit time functions that depend on the amount of flow and the entering time-moment of flow in the arc. Moreover, we consider the optimal dynamic generalized network flow problem which extends the traditional problem by introducing the gain-factor in the model. To develop algorithms for solving such kind of problems we use special dynamic programming techniques based on the time-expanded networks method together with classical optimization methods. We also investigate multiobjective versions of the optimal multicommodity flow problems by using the concept of cooperative and noncooperative games.

Good and special WPO for Bernstein operators in p variables

L.-F. Galea

In 1969, D. D. Stancu have introduced the Bernstein operators with arguments real functions in p variables. Using weakly Picard operators and contraction principle, C. Bacotiu studied the convergence of these operators iterates. In the present paper good and special weakly Picard convergence for Bernstein operators in p variables is investigated.

Asymmetry level, a new fuzzy measure

A. Garrido

When one works with causal theory the principal trouble is the temporal asymmetry problem. It is a difficult question, with many philosophical and practical consequences. For instance, the natural laws are basically asymmetrical, whereas the physical laws (after modeling) are symmetrical. Along the time, many possible approximations have been proposed attempting to measure the symmetrical/asymmetrical character, but ever as discrete feature: the shapes as totally symmetrical or totally asymmetrical mathematical constructs. However, the real situation is that we found different degrees of such interesting feature in each shape, perhaps modulating about the considered region. So, we need to consider the (a) symmetry as a continuous feature, and then, as a fuzzy concept. Attempting to contribute to solve such question, we propose here a new fuzzy measure, the *(A) symmetry level function,* departing of two well-known fuzzy measures, as entropy and specificity. We also prove their character of normal fuzzy measure.

Keywords: *fuzzy analysis, fuzzy measure theory, knowledge representation, reasoning under uncertainty, A. I.*

MSC (2000): 26E50, 28E10, 68T30, 68T37.

References

1. Dubois, Prade (eds.),: *Fundamentals of fuzzy sets*, Kluwer, Dordrecht, 2000.
2. Garrido, A. *Fusion Modeling....* Electronic International Journal Advanced Modeling and Optimization. **10**, 1(2008), 135-146.
3. Grabisch, Murofushi, Sugeno, *Fuzzy measure of fuzzy events defined by fuzzy integrals.* Fuzzy Sets and Systems **50**(1992), 293-313.
4. Murofushi, Sugeno, *An interpretation of fuzzy measures and the Choquet Integral as an integral with respect a fuzzy measure,* Fuzzy Sets and Systems(1989), 201-227.
5. Schmeidler, *Integral representation without additivity,* Proc. AMS, **97**(1986), 253-261.
6. Xuechang, *Entropy, distance measure and similarity measure of fuzzy sets and their relations*, Fuzzy Sets and Systems **52**, 305-318.

On developing macroeconomic growth model in Visual Basic for Applications framework

V. Geru, E. Naval, L. Smolina, M. Popovscaia

In this paper an information system for developing macroeconomic growth model is presented. The main goal of the examined financial programming model is to achieve such objectives as: sustainable economic growth, low inflation and admissible balance of payments. This model incorporates private sector which produced all economic yield and assured economic growth, government sector promoted such fiscal policy which maintain budget deficit throw tax collecting and income distributing for wages and main social programs, monetary sector which principally objective is low inflation and external sectors main goal of which consists in balance of payment maintaining is considered. Behavioral equations incorporated in the model represent production function of the Cobb-Douglas type. Credit, fiscal and exchange rate policy are employed as instruments for reaching proposed objectives. The model can be used for short and middle term forecasts of the main economic variables. The statistical data for the Republic of Moldova are organized in efficient and convenient way to take advantage of in the process of developing macroeconomic growth model in Visual Basic for Applications framework. Possibilities to extend and to enhance the model are examined.

References

1. M. S. Khan, P. Montiel, N. Haque, *Adjustment with growth*, Journal of Development Economics, **32**(1990), 155-179.

Computer and Internet in scientific research of students

M. A. Ghelmez, E. Slavnicu, A. M. Dumitrescu

The goal of this work is to present a project for studying use of computers and network in improving students' scientific activity. We propose a work team, composed by students and teachers as leaders[1]. In the modern education , based on the international and European standards, students involvment in the scientific research is an important objective for obtaining specialists in modern technologies. Scientific activity of students is traditional in Politehnica University, but every year it has new forms, since the students are every time different. In the paper we present some examples of students works awarded with Politehnica prizes. We believe that offering such a unit to them, they will find a place for developping their activities.

References

1. M. K. Wentland, E. N. Forte, *Proc. of CAEE'93*, Intelligent hyperhelp in a scientific simulation: "overview of the HIPOCAMP research project" 9, 1, **1**(1993), 33-37.

Modelling automate bus suspension with Matlab

M. A. Ghelmez, O. Danila, C. Nenciu, A. Sterian

A simple solution for a bus suspension is presented. The equations of motion and the transfer functions are given in the paper. Practical situations in the real traffic are considered. One considers a virtual medium for simulating and practical solutions are discussed. The scheme of the transfer functions is given. The code lines are written in Matlab[1]. The PID controller is computer embedded, and the graphical answers are presented.Then the initial program is modified for taking into account these considerations.

References

1. C. I. Toma, M. Ghelmez (Dumitru), A. R. Sterian, M. C. PiscureanuD, *Invariance property of differential equetions used for generating practical test functions in averaging procedures*, Rom. J. of Optoelectronics, 9, 1, **1**(1996), 33-37.

Invariant conditions for the dimensions of the $Aff(2,\mathbb{R})$-orbits for quadratic differential system

N. N. Gherstega, V. M. Orlov

The two-dimensional autonomous quadratic differential system is considered concerning the group of affine transformations $Aff(2,\mathbb{R})$. The problem of the classification of $Aff(2,\mathbb{R})$-orbit dimensions is solved for the given system by means of Lie algebra of operators corresponding to the $Aff(2,\mathbb{R})$-group.

The work is partially supported by CSSDT grant Ref. Nr.07.411.05INDF.

References

1. M. N. Popa, *Algebraic methods for differential systems*, Ed. The Flower Power, Univ. Piteşti, Seria Matematică Aplicată şi Industrială, **15**, 2004 (Romanian).
2. K. S. Sibirsky, *Introduction to algebraic theory of invariants of differential equations*, Ştiinţa, Chişinău, 1982 (Russian; published in English in 1988).
3. P. J. Olver, *Classical invariant theory*, London Mathematical Society student texts: **4**, Cambridge University Press, 1999.
4. D. Boularas, Iu. Calin, L. Timochouk, N. Vulpe, *T-comitants of quadratic systems: a study via the translation invariants*, Delft Univ. of Technology, Report 96-90, 1996.
5. D. Schlomiuk, N. Vulpe, *Geometry of quadratic differential systems in the neighborhood of infinity*, J. Differential Equations, **215**(2005), 357-400.
6. D. Boularas, *Computation of affine covariants of quadratic bivariate differential systems.* J. Complexity, **16**, 4(2000), 691-715, .

Error analysis when the boundary element method is used to solve the problem of the compressible fluid flow around obstacles

L. Grecu

The paper presents some aspects about the errors that appear when solving the problem of a compressible fluid flow around obstacles by using the boundary element method. We consider both cases: smooth obstacles and profiles with cusped trailing edge. Different types of boundary elements are used. The paper is focused on the case of lifting profiles because in their case the errors are quite large in the vicinity of the cusped trailing edge. Some techniques for minimizing the errors are discussed too.

Queuing systems with semi-Markov flow in average and diffusion approximation schemes *

Iu. Griza, V. S. Koroliuk, A. V. Mamonova, Gh. K. Mishkoy

We study asymptotic average and diffusion approximation scheme for semi-Markov queuing systems by a random evolution approach and using compensating operator of the corresponding extended Markov process. The specific our queuing system is that series scheme is considered with phase merging procedure. The average algorithm and algorithm of diffusion approximation are established for the queuing process (QP) described by the number of claims at every node and by using the random evolution approach on the Banach space $C^3(R)$. To this end the main tool is the compensating operator of the extended Markov renewal process. These results generalize the Markov and renewal flow queuing systems (QS). The main reason is that such systems are very large and so it is difficult and often impossible to handle them numerically by usual methods of Markov and semi-Markov processes. Stochastic approximation of QS is a very active and interesting method to obtain numerical but also qualitative results for complex systems. For example, for a given QS of the type, with a large number of places, say $N(N \to \infty)$, for a fixed time t, the number $\nu N(Nt)$, which in general is complicated to calculate directly, it can be approximated by the variable $Nqt + N1/2t\sigma Z$, where Z is the standard normal random variable, where the calculus is elementary.

* This work is supported partially by Russian Foundation for Basic Research grant 08.820.08.09RF and partially by Scopes grant IB 7320-110720.

References

1. V. V. Anisimov, *Switching processes in queuing models*, ISTE, Wiley, London, 2008.
2. V. V. Anisimov, E. A. Lebedev, *Stochastic queuing networks. Markov models. K. Lybid.*, 1992. (Ukrainian)
3. A. A. Borovkov, *Asymptotic methods in queuing theory*, Wiley, New York, 1984.
4. W. Feller, *An introduction to probability theory and its applications*, 2 vols., Wiley, New York, 1966.

5. I. I. Gihman, A. V. Skorohod, *Theory of stochastic processes*, II, Springer, Berlin, 1974.
6. J. Jacod, A. N. Shiryaev, *Limit theorems for stochastic processes*, Springer, Berlin, 1987.
7. V. S. Korolyuk, A. V. Swishchuk, *Semi-Markov random evolution*, Kluwer, Dordrecht, 1995.
8. V. S. Korolyuk, V. V. Korolyuk, *Stochastic models of systems*, Kluwer, Dordrecht, 1999.
9. V. S. Korolyuk, A. V. Swishchuk, *Random evolution*, Kluwer, Dordrecht, 1999.
10. V. S. Korolyuk, N. Limnios, *Stochastic systems in merging phase space*, World Scientific, Singapore, 2005.

The impact of limiter control and stochastic limiter in inventory model with rational expectations

A.-D. Gurgui Costea

Prediction of economic states in view of limiting the undesired effects is investigated. Within the study of equilibrium and economic cycles, the inventory model with rational expectations is analyzed by using the bifurcation method. A Hard Limiter Control (HLC) and a stochastic limiter are used to prove that a chaotic dynamics can be modified in a cycle or even in an equilibrium state. The bifurcation diagrams offer an intuitive understanding of the current model.

On polynomial factorization and topology of complement space

H. Shaker

A polynomial P of $deg > 1$ from the polynomial ring $\mathbb{C}[X, Y]$ corresponds to the plane algebraic curve C in $\mathbb{C}^2$. The question under investigations is, how many irreducible factors the polynomial P has? The answer to this question is directly related to the study of the topology of the complement space of C in the complex plane $\mathbb{C}^2$. This study also leads us to extend this problem for more variables.

References

1. A. Dimca, *Singularities and topology of hypersurfaces*, Springer, Berlin, 1992.
2. A. Grothendieck, *On the de Rham cohomology of algebraic varieties*, Publ, Math. IHES, **29**(1966), 95-103.
3. A. Hatcher, *Algebraic topology*, Cambridge, 2002.
4. G. Chèze, A. Galligo, *Four lectures on polynomial absolute factorization, Solving polynomial equations*, Algorithms and Computation in Mathematics, **14**, Springer, 2005.
5. W. Ruppert, *Reducibility of polynomials $f(x, y)$ modulo p*, J. Number Theory, **77**(1999), 62-70.

On (a, d)-vertex-antimagic total labeling of cycle with chords

M. Hussain

Let $G = (V, E)$ be a graph with v vertices and e edges. An (a, d)-*vertex-antimagic total labeling* is a bijection λ from $V(G) \cup E(G)$ to the set of consecutive integers $1, 2, \ldots, v + e$, such that the weights of the vertices form an arithmetic progression with the initial term a and common difference d. If $\lambda(V(G)) = \{1, 2, \ldots, v\}$ then we call the labeling a *super* (a, d)*-vertex-antimagic total.* In this paper we construct an (a, d)-vertex-antimagic total labeling on Harary graphs as well as for the disjoint union of k identical copies of Harary graphs.

References

1. M. Baca, F. Bertault, J. MacDougall, M. Miller, R. Simanjuntak, Slamin, *Vertex-antimagic total labeling of graphs*, Discuss. Math. Graph Theory, **23**(2003), 67-83.
2. J. A. Gallian, *A dynamic survey of graph labeling*, Electron. J. Combin., #DS6 (2007).

About totally geodezical embeddings of compact 2-manifolds in compact 3-manifolds

I. Gutsul, E. Tikhonov

The paper represent the demonstration of the following theorem.

Theorem. *Let n be a natural number and S_i be a finite set of compact 2-dimentional surfaces of genus $3p_i$ for any $i \in \{1, 2, ..., n\}$. Then there exists a 3-dimensional compact hyperbolic manifold M in which entire this set of surfaces can be embedded totally geodezically.*

Some differential games with stochastic pertubation associated with non adapted solutions

D. Ijacu

The main theorem in this paper allow us to convert the stochastic control problem into a deterministic one, with corresponding changes in formulating results. As an applications we give exemples of differential games with stochastic perturbation associated with Nash equilibrum solutions and open loop strategies. A problem of optimal control, as usual, is defined through the same elements as a differential game with open loop strategies, unlike that this time we have just one functional wich must be minimized in comparison with admissible comands. A differential game with stocastic perturbation is determined by a dynamic of state variabile $x \in X \subseteq R^n$ defined by a system of stochastic differential equations

$$(1) \begin{cases} d_t x = f\left(t, x, u_1, .., u_N\right) dt + \sum_{j=1}^{d} g_j(t, x) \otimes dw_j(t) \\ x(0) = x_0, (t, x, u_1, .., u_N) \in [0, t_f] \times X \times U_1 \times .. \times U_N \end{cases}$$

For each $u(\cdot) \in A$ we define the coresponding solution $x = x(t,\omega;u)$, $(t,\omega) \in [0,t_f] \times \Omega$ as the solution for (1) and associate the following pathwise functional

$J_i(\omega,u) = F^i\left(x\left(t_f,\omega;u\right)\right) + \int f_0^i\left(t,x\left(t,\omega;u\right),u\left(t,\omega\right)\right)dt, i \in \{1,2,..,N\}$

The stochastic differential game with N-players and open loop strategies

$\Gamma_N(\omega) \triangleq \{[0,t_f], X = \mathbb{R}^n, U_i, \mathcal{A}_i, f, x_0, J_i(\omega,\cdot)\}_{i=1,2,..,N}, \omega \in \Omega$

An admissible solution $x(t,\omega;u)$ for (1) is represented as follows

$x(t,\omega;u) = G(t,\omega)(y(t,\omega;u)) + \eta(t,\omega)$,

$d_tG = \sum\limits_{j=1}^{d} A_j(t)G\circ dw_j(t), G(0,\omega) = I_n, t \in [0,t_f], \eta(t,\omega) = \sum\limits_{j=1}^{d}\int\limits_0^t b_j(s)\cdot dw_j(s), t \in [0,t_f]$, The vector valued function $y(t,\omega;u) \in R^n$ is defined as a differentiable and non F_t-adapted process fulfilling the following system of differential equations

$$\begin{cases} \dfrac{dy}{dt} = [G(t,\omega)]^{-1} f\left(t, G(t,\omega)\left(y\right)\right) + \eta\left(\left(t,\omega\right), u\left(t,\omega\right)\right) \triangleq \\ \qquad \triangleq \tilde{f}\left(\omega,t,y,u\left(t,\omega\right)\right), t \in [0,t_f] \\ y(0) = x_0 \in \mathbb{R}^n \end{cases}$$

The corresponding deterministic system describing the evolution of the new state variabile $y \in Y = \mathbb{R}^n$ reads as follows

$$\tilde{1})\begin{cases} \dfrac{dy}{dt} = \tilde{f}(\omega,t,y,u), t \in [0,t_f],\ \omega \in \Omega \\ y(0) = x_0 \end{cases}$$

References

1. D. Ijacu, C. Vąrsan, *Stochastic differential equations associated with smooth mappings and non $\mathcal{F}_t-$adapted solutions,* Preprint IMAR, **6**/2003.
2. C. Vârsan, *On evolution system of differential equations with stochastic perturbation*, Preprint IMAR, **4**/2001.
3. B. Oksendals, *Stochastic differential equation. An introduction with application*, 5th ed., Springer, New York, 2000.

The child and teenager ask . . . Young people ask themselves . . . Grown up and old people reflect . . . Mathematics of nowadays, where is it going?

N. Ile, C. Bercovici

The paper is an analysis of the Romanian mathematical education. The ideas it contains are based an the quality (what), the ensemble (how much), the implementation (how), the motivation (why) of the notions delivered to the students from the mathematics teacher's point of view. A few expectations of the quadrilateral student - teacher - family - social partner are discussed too.

The authors propose some solutions.

Copilul şi adolescentul întreabă ... Tinerii se întreabă ... Adulţii şi vârstnicii reflectă ... Matematica, în zilele noastre, încotro?

N. Ile, C. Bercovici

Lucrarea conţine o încercare a autorilor de a analiza educaţia matematică din România. Ideile lansate au la bază **C**alitatea (ce), **A**nsamblul (cât), **I**mplementarea (cum), **M**otivaţia (de ce) noţiunilor predate elevului prin prisma profesorului de matematică. Sunt prezentate câteva aşteptări ale patrulaterului "elev - profesor - familie - partener social". Autorii vă oferă câteva soluţii.

Processing of experimental data from an electrochemical biosensor

M. Ilie, S. Oprişescu, A. Masci, L. Nardi, M. Montereali, V. Wastarella, E. Vasile, V. Foglietti, R. Pilloton

Biosensors, as functional analogs of chemo-receptors, are based on the direct spatial coupling of immobilized biologically active molecules with a signal transducer and an electronic amplifier. The electrochemical indication prevails over all other methods of transduction. Indeed, potentiometric and amperometric enzyme electrodes are at the top of biosensor technology. Screen-printed electrodes are widely used as transducers in electrochemical biosensors. The miniaturization of the electrodes and their integration in a analyte continuous flow micro-cell is useful in automatic control of waters, food or environment.

Experimental research has been performed in order to calibrate a biosensor of this type. The electric current generated by reduction/oxidation reactions in the microcell has been measured in temporal sequences for different concentration of the analyte. A data processing algorithm and a program have been elaborated for the calibration current-concentration-analyte dependence. The data acquisition has been performed using AUTOLAB (LABVIEW); for computing MATLAB has been used. The work is in progress in order to mathematical processing in a real time of a huge volume of data from electrochemical biosensors used in automatic control systems.

Numerical method for viscous free surface flows with a hydrofoil

T. Ioana, T. Petrila

In this paper we purpose a numerical method for viscous free surface flows in the presence of a hydrofoil. The model is based on the steady Navier Stokes equations. By an optimization approach an algorithm is purposed. Then some numerical examples are presented.

References
1. E. H. van Brummelen, A. Segal, *Numerical solution of steady free-surface flows by the adjoint optimal shape design method*, Int. J. for Numerical Methods in Fluids, **41**(2003) 3 - 27.
2. T. Petrila, *Mathematical model for the free surface flow under a sluice gate*, Applied Mathematics and Computation, **125**(2002) 49 - 58.
3. T. Petrila, D. Trif, *Basics of fluid mechanics and introduction to computational fluid dynamics*, Numerical Methods and Algorithms, **3**, Springer, New York, 2005.

Lyapunov stability of the zero solution of a perturbed abstract parabolic non-autonomous equation

A.-V. Ion

We consider a perturbed parabolic non-autonomous abstract equation

$$\frac{dx}{dt} = Ax + Bx + R(t, x), \tag{2}$$

in a Banach space X. The hypotheses on A and R are those of [3], i.e. $A : D(A) \subset X \to X$ is a closed linear operator, with $D(A)$ dense in X, that generates a strongly continuous semigroup $T(t) = exp(At)$, exponentially decreasing, while the nonlinear operator R is continuous, $R(t, 0) = 0$ for all $t \in \mathbb{R}^+$ and there is a $\beta > 0$ and a continuous function $C(t) > 0$ such that for all $t \in \mathbb{R}^+$, x_1, x_2 in a centered in 0 ball of X, inequality
$||R(t, x_1) - R(t, x_2)|| \leq C(t) max^\beta(||x_1||, ||x_2||)||x_1 - x_2||$ holds.

The perturbation operator B is a bounded linear operator on X with the property that there is a constant $C \in (0, 1)$ such that

$$\int_0^t \|BT(s)x\| ds \leq C\|x\|, \; (\forall)t \geq 0, \; (\forall)x \in X.$$

The semigroup of operators generated by $A + B$ is studied in [1]. The Lyapunov stability of the zero solution of eq.(2) is investigated by the methods used in [2] and [3]. The results are applied to the study of a partial differential functional equation.

References
1. V. Casarino, S. Piazzera, *On the asymptotic behaviour of perturbed semigroups*, Ulmer Seminare, **3**, 1998.
2. V. A. Trenogin, *Lyapunov theorem about the stability by linear approximation as the consequence of the implicit operators theorem*, Doklady Academii Nauk, **407**(2006), 742-746. (Russian)
3. V. A. Trenogin, *Asymptotic stability of trivial solution of differential equations with unbounded linearity, in Banach spaces*, ROMAI, **3**, 1(2007), 207-212.

Induced trajectories versus approximate inertial manifolds in modified Galerkin methods

Anca-Veronica Ion

We present the already classical modified Galerkin methods for approximating the solution of a semi-linear parabolic equation. These nonlinear and post-processed Galerkin methods rely on the use of approximate inertial manifolds (a.i.m.s). We also present a method constructed by us that uses approximations of the so-called induced trajectories instead of a.i.m.s. We compare our method to the nonlinear Galerkin method from the computational volume point of view, and show the advantages of using induced trajectories instead of a.i.m.s.

New features in simulating the behavior of 3D turbulent mixing

A. Ionescu

This paper continues the previous work in the turbulent mixing field.

The mixing theory is a modern theory in the field of flow kinematics. Its mathematical methods and techniques developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamical systems with few freedom degrees, the so-called "far from equilibrium systems". These are widespread between the models of excitable media.

Studying a mixing for a flow implies the analysis of successive stretching and folding phenomena for its particles, the influence of parameters and initial conditions. In the previous works, [1,2], the study of the 3D non-periodic models exhibited a quite complicated behavior. In agreement with experiments [4], they involved some significant events - the so-called "rare events". The variation of parameters had a great influence on the length and surface deformations. The 2D (periodic) case was simpler, but significant events can issue for irrational values of the length and surface unit vectors, as was the situation in the 3D case.

The aim of this paper is to get a new approach of numeric simulation of turbulent mixing. Namely, using specific solving procedures of Maple11 soft, the deformation efficiency for 3D mixing model with a different output is taken into account. The numerical results are used for comparing the two output cases, involving different plot types.

Key words: turbulent mixing, stretching, folding, efficiency, dsolve procedure

References

1. A. Ionescu, *The structural stability of biological oscillators.* Analytical contributions, Ph.D. Thesis Bucarest, 2002.
2. A. Ionescu, M. Costescu *Some qualitative features of 2D periodic mixing model*, Acta Universitatis Apulensis, Mathematics-Informatics, **15**(2008), 387-396, Univ. of Alba Iulia.
3. J. M. Ottino, *The kinematics of mixing: stretching, chaos and transport*, Cambridge Univ. Press., 1989.

4. St. N. Savulescu, *A special vortex tube for particle processing flows*, Rev. Roum. Sci. Techn.-Mec. Appl., **43**, 5(1998), 611-615.

Stochastic approximation for identification of convection heat transfer coefficient in the secondary cooling zone of steel continuous casting process

G. Ivanova

In continuous casting the secondary cooling system influences significantly the quality of the cast products. In order to guarantee stability of controlled parameters of quality, it is enough to have an adequate mathematical model establishing interrelation between quality and control parameters.

Due to the absence of necessary values (physical constants or dependences) for coefficients of the equations it is difficult to implement most of mathematical models. While modeling process for specific industrial conditions it is necessary to determine some thermal or physical parameters each time, in particular convective heat-transfer coefficient on a surface of an ingot in the secondary cooling zone which depends on many factors.

The detailed mathematical model of heat and mass transfer of steel ingot of curvilinear continuous casting machine is proposed. The thermal field of a moving steel ingot and mold wall in the system of coordinates attached to motionless construction of CCM is considered. The process of heat and mass transfer is described by nonlinear partial differential equations of parabolic type. Position of phase boundary is determined by Stefan conditions. The temperature of cooling water in mould channel is described by a special balance equation. Boundary conditions of secondary cooling zone include radiant and convective components of heat exchange and account for the complex mechanism of heat-conducting due to airmist cooling using compressed air and water.

Convective heat-transfer coefficient of secondary cooling zone is unknown and considered as distributed parameter. It is necessary to find the convective heat-transfer coefficient based on information about surface temperature of an ingot.

Such problems belong to inverse boundary problems. The solution obtained by the method of direct inverse is unstable and unsuitable for practical application. Therefore other approaches are necessary to solve this problem.

After the initial adjustment according to the algorithm described in [1] it is possible to provide adaptation of model parameters during usual working condition of CCM when the information on thermal condition is limited to indications of temperature sometimes only at one point of the surface of an ingot (with some step of discretization of time). Operative fine-tuning consists in refinement of constant value, which defines distribution of convective heat transfer coefficient that is obtained as a result of solving the problem of initial adjustment of parameters. Moreover, fine-tuning of parameter should be carried out in real time. Such algorithms can be based on the stochastic approximation method.

In this work a new algorithm, which was not used previously and allows obtaining a solution that is stable and convenient for further practical use, is offered.

The stochastic approximation algorithm of operative adjustment when the surface temperature of an ingot is measured only at one point is developed.

The different variations of realization of this proposed algorithm are considered. Numerical results are presented and their comparative analysis is performed.

The advantage of stochastic approximation algorithm is its successful performance for a large enough interval of initial values of the parameter being adjusted.

References

1. A. P. Sage, J. L. Melsa, *System identification*, Academic, New York, **1**(1971).

2. E. Laitinen, R. Nieminen, S. Louhenkilpi, *Calibration of the real-time heat transfer model for the simulation of the continuous casting process*, Transactions on Modelling and Simulation, **4**(1993), 467 - 481.

Some structures of operations

V. Izbash, M. Vrabie

The method of universal algebra and the use of algebraic structures are the appropriate way to look at some combinatorial problems. This is due to the fact that often one can find a relationship between algebraic properties of operations on finite set and combinatorial designs on the set. Many questions of this theme are discussed in [1,2,3]. Our purpose in studying binary operations is to classify them and to obtain a description of some classes of $\Omega_n(Q)$. We introduce and study the notion of type of binary operations and divide the family $\Omega_2(Q)$ into $(n^2+n-1)!/(n-1)!(n^2)!$ classes of same-typed operations, if $|Q| = n$. We study some algebraic structures in $(\Omega_n(Q), \frac{\oplus}{i})$ with respect to multiplication $(\frac{\oplus}{i})$ of operations (Mann's composition), where for $A, B, C \in \Omega_n(Q)$ we put

$$A \frac{\oplus}{i} B = C \Leftrightarrow C(x_1^n) = A(x_1^n B(x_1^n) x_1^n),$$

where $i \in \{1, 2, ..., n\}$ and $(x_1^n) \in Q^n$ [3]. $(\Omega_n(Q), \frac{\oplus}{i})$ is a binary semigroup. In particular it is proved that the semigroup $(\Omega_2(Q), \frac{\oplus}{1})$ is isomorphic to the direct product of $|Q|$ copies of $\Im(Q)$, where $\Im(Q)$ is the semigroup of mappings of Q. Operation $A \in \Omega_n(Q)$ is called i-invertible if the equation $A(c_1^{i-1}, x, c_{i+1}^n) = b$ has a unique solution for all $c_j, b \in Q, j \in \{1, 2, ..., i-1, i+1, ..., n\}$. $(\Lambda_i, \frac{\oplus}{i})$ is a binary group, where $\Lambda_i \subseteq \Omega_n(Q)$ is the class of all i-invertible operations on Q [1,3]. For the group $(\Lambda_1, \frac{\oplus}{1})$ and $\Lambda_1 \subseteq \Omega_2(Q)$ we have

$$\Lambda_1 \cong S(Q) \times S(Q) \times ... \times S(Q).$$

We give a practical construction of left zeros, idempotent elements, invertible elements of $(\Omega_2(Q), \frac{\oplus}{1})$. The orthogonality, isomorphism and automorphism of operations are studied.

References

1. V. D. Belousov, *On the properties of binary operations*, Učen. Zap. Bel'sk. Ped. Inst., **5** (1960) 9-28.(Russian)

2. V. D. Belousov, *Systems of orthogonal operations* , Mat. Sb., **77**(119)(1968), 38-58.(Russian)

3. T. Yakubov , *On $(2, n)$-semigroup of n-ary operations* , Izv. AN MSSR, **1** (1974), 29-46.(Russian)

The use of student-focused strategies in mathematics education

D. Jeder

Many young teachers have difficulties in constructing efficient strategics of presentation of the matter. This must induce a change of optics in the activity with the students and especially with those who want to follow a didactic career. The work presents some possible modalities of configurating and using these strategics in the frame work of mathematical disciplines.

A relatively global optimization approach for nonlinear programming

A. V. Kamyad, A. M. Vaziri, M. Zamirian

In this paper a global optimization approach for approximate solution of nonlinear programming problem (N.L.P) is proposed. This approach is based on parametric linearization applying to the initial nonlinear programming. Then it is reduced to a sequence of linear programming problems defined on the special feasible region. The proposed approach is convergent to the global optimum of original N.L.P as the norm of partitions of the region tends to zero. Numerical examples indicate that the proposed approach is extremely robust and may be used successfully to solve a wide range of global numerical examples, showing the efficiency of our approach.

A Petri nets application in the mobile telephony

M. Kere

Petri nets are used to describe graphically the structure of the distribute systems that need some representations of the concurrent or parallel activities. It represents a modelling language that is applicable to a wide variety of systems thanks to their generality and permissiveness. There are many types of Petri nets; more studied in this paper are CEN(Condition Event Nets), PTN(Place Transition Nets) and CPN (Coloured Petri Nets).
This paper tries to see the differences between these three types of Petri nets using an application on a mobile phone. From the several process that the mobile phone can be use for, it is presented the action of sending the data using infrared or bluetooth system.

References

1. D. Bj.rner, *Software engineering*, 3 vols, Springer, New York, 2006.
2. T. Murata, *Petri nets: properties, analysis and applications*
3. J. L. Peterson, *Petri Nets Theory and the Modeling of Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1981.
4. http://www.cse.fau.edu/ maria/COURSES/CEN4010-SE/C10/10-7.html
5. http://en.wikipedia.org/wiki/Petri_net

Solving higher-order fuzzy differential equations under generalized differentiability

A. Khastan, F. Bahrami, K. Ivaz

Higher-order fuzzy differential equations with initial conditions are considered. The new results are applied to the particular case of a second-order fuzzy linear differential equation.

Keywords: fuzzy differential equations, fuzzy initial value problem, generalized differentiability.

Avoiding bankruptcy at all costs

M. Lefebvre

Assuming that the value of the stock of a company at time t can be represented as a controlled one-dimensional Bessell process, we consider the problem of finding the control that minimizes the mathematical expectation of a cost function with quadratic control costs. Two terminal cost functions, one of which being infinite if the process hits the origin before a given positive boundary, are used and the optimal control is obtained explicitly in each case.

References

1. M. Lefebvre, *Forcing a stochastic process to stay in or to leave a given region*, Ann. Appl. Probab. **1**(1991), 167-172.
2. P. Whittle, *Optimization over Time*, I, Wiley, Chichester, 1982.

On the influence of the type of exams on the performance of students

M. Lefebvre

We analyse the results obtained by engineering students in a probability course over a 10-year period. During this time period, essay exams, multiple choice exams, and sometimes part essay and part multiple choice exams were used to evaluate the students. We are interested in the average obtained by the students, the percentage of A's, and the percentage of students who failed the course.

References

1. M. Lefebvre, *Cours et exercices de probabilités appliquées*, 2e Ed., Presses internationales Polytechnique, Montréal, 2003.
2. M. Lefebvre, *Applied Probability and Statistics*, Springer, New York, 2006.

Pseudoperturbiteration methods in generalized eigenvalue problems

B. V. Loginov, O. V. Makeeva

Based on the determination of special form of pseudoperturbation operator [1, 2] and some results of bifurcation theory [3] four iteration processes for the sharpening of approximately given eigenvalues eigen and Jordan elements in generalized eigenvalue problems in Banach spaces are suggested with investigation of their convergence rates. Many aspects of their application to spectral problems for linear and nonlinear operator-functions of spectral parameter are discussed with results of numerical experiment in concrete examples of various generalized eigenvalue problems [4,5]. The obtained results are supported by grant RFBR-Romanian Academy N 07-01-91680.

References

1. B. V. Loginov, B. Karasözen, V. A. Trenogin. *Pseudoperturbation method for sharpening of approximately given generalized Jordan chains*, Proc. CAIM-X, Bul. Şt. Ser. Mat. Info., Univ. Piteşti, **9**(2003), 183-190.
2. B. V. Loginov, O. V. Makeeva, A. V. Tsyganov, *Sharpening of approximately given Jordan chains for linear operator-function of spectral parameter on the base of perturbation theory,* Interuniv. Sci. Proc. "Functional Analysis", Ulyanovsk State Pedagogical University, **38**(2003), 53-62. (Russian)
3. B. V. Loginov, O. V. Makeeva, E. F. Foliadova *Sharpening of critical spectral parameterr by pseudoperturbation method,* Proc. of CAIM XIII, ROMAI J. **2**, 1(2007), 119-126.
4. M. M. Vainberg, V. A. Trenogin, *Branching theory of solutions of nonlinear equations*, Nauka, Moscow, 1964. (Engl. transl. Noordorf, Leyden, 1974).
5. B. V. Loginov, O. V. Makeeva, *Pseudoperturbation method in generalized eigenvalue problems,* Doklady Mathematics, **77**, 2(2008), 194-197.

Transformations of geometric space well-balanced by scalar "physical" load

A. Lungu

The mixed transformation $\tilde{g}$ of the regular decomposition of geometric space of constant curvature with summary scalar load N is composed of two independent components: $\tilde{g} = gw$. Component g is pure geometrical isometric transformation and w is one complex rule which describes the transformation of the "indexes" $r \in N$ ascribed to the interior points M of a certain fundamental domain. If the rule w is the same for every "indexed" point of space, then the mixed transformation $\tilde{g}$ is exactly a transformation of P-symmetry [1-3]. The "indexes" r_i and r_j, ascribed to the points which belong to distinct fundamental domains, are transformed, in general, by different permutations p_i and p_j. The permutations p_i and p_j are of certain transitive group of permutations P over N. In this case the rule w is composed exactly from $\|G\|$ components-permutations $p \in P$. As a result $\tilde{g} = gw$ is exactly a transformation of the W_p-symmetry [3-6].

References
1. A. M. Zamorzaev, *On the groups of quasi-symmetry (P-symmetry)*, Soviet Phys. Crystallogr., **12**, 5(1967), 819-825. (Russian)
2. A. M. Zamorzaev, E. I. Galyarskii, A. F. Palistrant, *Colored symmetry, its generalizations and aplications*, Shtiintsa, Chisinau, 1978.(Russian)
3. A. M. Zamorzaev, Yu. S. Karpova, A. P. Lungu, A. F. Palistrant, *P-symmetry and its further development*, Shtiintsa, Chisinau, 1986. (Russian)
4. V. A. Koptsik, I. N. Kotsev, *To the theory of W-symmetry groups.* OIYI Reports, R4-8068, Dubna, 1974. (Russian)
5. A. P. Lungu, *Contribution to the theory of W-symmetry groups.* Bul. Acad. Şt. a Rep. Moldova, Matematica., **3**(9)(1992), 72-81. (Russian)
6. A. Lungu, *Discrete groups of W-symmetry*, CAIM-2003, May 29-31, Oradea, Romania, Proceedings, **1**(2003), 156-160.

On a subclass of analytic functions defined by a generalized Sălăgean and Ruscheweyh operator

A. Lupaş (Alb), A. Cătaş

By means of the Sălăgean differential operator and Ruscheweyh derivative we define a new class $\mathcal{BL}(n, \mu, \alpha, \lambda)$ involving functions $f \in \mathcal{A}$. Parallel results, for some related classes including the class of starlike and convex functions respectively, are also obtained.

References
1. A. Alb Lupaş, *Some diferential subordinations using Sălăgean and Ruscheweyh operators*, Proc. of Int. Conf. on Fundamental Sci., (ICFS), Oradea, (2007), 58-61.
2. A. Cătaş, A. Alb Lupaş, *On a subclass of analytic functions defined by differential Sălăgean operator*, (to appear).
3. A. Alb Lupaş, A. Cătaş, *On a subclass of analytic functions defined by differential Ruscheweyh derivative*, (to appear).
4. B. A. Frasin, M. Darus, *On certain analytic univalent functions*, Int. J. Math. and Math. Sci., **25**, 5(2001), 305-310.
5. B. A. Frasin, J. M. Jahangiri, *A new and comprehensive class of analytic functions*, Analele Universităţii din Oradea, **15**(2008).
6. St. Ruscheweyh, *New criteria for univalent functions*, Proc. Amer. Math. Soc., **49**(1975), 109-115.
7. G. St. Sălăgean, *Subclasses of univalent functions*, LNM, **1013**(1983), 362-372.

Financial Analysis and cost of quality

M. Luţă, E. M. Popa, C. A. Drăgan, C. B. Milian

The economic procedures and phenomena are characterised by the fact that their manifestation is very complex, having a wide range of aspects. This feature essentialy distinguishes them from the phenomena from other areas. The remarkable complexity of economic phenomena has as causes lots of factors whose specificity depends on the context. This complexity of the social-economic area also has a cause of informational nature. The informational tides from this area have an heterogeneous nature and a relatively low degree of accuracy and relevance, because of the imperfections characteristic to the measuring process. In the decisional act it is used a lot of information which has to be registered, analysed and interpreted.

The paper stresses upon the importance of financial analysis as an instrument of appreciating the performances and the financial potential of a company, cooperative statistics and new solutions of tackling financial analysis.

Keywords: financial analysis, economic models, indicators, financial diagnosis, function, interpreting

A survey of the three bodies problem

M.-V. Macarie

A survey of the three bodies problem is presented. Some particular cases (Euler case, Lagrange case and others) studied in the literature are shown.

Some characteristics for polling systems with an exhaustive service and semi-Markov switchings *

Gh. Mishkoy, V. Rykov

The mathematical models of polling systems play an important role in the analysis, modeling and design of various modern networks and their components [1,2]. In this work we study the polling system with an exhaustive service and non zero semi-Markov switchings policy. For the mentioned system the distribution of k-periods, their first moments, virtual an steady state probabilities are obtained. The method of investigation is based on a new approach described in [3]. The main characteristics are obtained in the terms of Laplace and Laplace-Stieltjes transforms.

References

1. V. M. Vishnevskii, O. B. Semenova, *Polling systems: the theory and applications in broadband wireless networks*, Moscow, 2007. (Russian)
2. V. M. Vishnevskii, D. V. Lakantsev, O. V. Semenova, C. A. Shpilev, *A model of polling system for analysis of broadband wireless networks*, Avtomatica and Telemechanica, **12**(2006), 123-135. (Russian).
3. V. V. Rykov, Gh. K. Mishkoy, *A new approach for analysis of polling systems*, Int. Conf. on Control Problems, Moscow, 2009 (accepted for presentation).

* This work is supported partially by Russian Foundation for Basic Research grant 08.820.08.09RF and partially by Scopes grant IB 7320-110720.

New developments in queueing analysis: generalized models and algorithms. Results and problems *

Gh. Mishkoy, S. Giordano

The mathematical models of queueing systems, in particular, priority queueing systems, play an important role in the analysis, modeling and design of various modern networks and their components [1,2]. We present and discuss some results regarding a class of priority queueing systems with non-zero switching time, referred to as generalized priority systems [3-5]. This class of systems appeared as a result of mathematical formalization and consideration of the switchover times between priority classes and strategies in the free states. The assumption of non-zero switchings of the service process allows one to take into consideration the various time losses and delays existing in real time systems. Its consideration and analysis is very important from the applied point of view, especially at a stage of modeling and designing. It has appeared that generalized priority systems are more adequate to real processes taking place in real time systems than classical priority systems. On the other hand, the consideration of the switchover times and their formalization inevitably leads to the appearance of a number of new important features in the elaboration of priority queueing models. Among these features we shall point out the appearance of new priority disciplines more flexible than traditional ones. From the theoretical point of view the elaboration and study of the generalized priority systems correspond to the intrinsic logic of the development of the priority queueing systems theory. Namely, the results regarding generalized models are, naturally, more general and contain as particular cases many of the results already obtained in the theory of priority systems and in the classical queueing theory. In particular, it will be shown that the presented results regarding distribution of busy periods and queue length distribution can be viewed as generalization of famous, referred to in most text-books on queueing theory, Kendall -Takacs and Pollaczek - Khintchin equations.

References

1. V. M. Vishnevskii, *Theoretical bases of design of the computer communication networks* , Moscow, 2003. (Russian)

2. V. M. Vishnevskii, O. B. Semenova, *Polling systems: the theory and applications in broadband wireless networks*, Moscow, 2007. (Russian)

3. Gh. Mishkoy, *An analog of Kendall-Takacs equation for generalized queueing systems*, Proceedings of the 31th Annual Congress of the American Romanian Academy of Arts and Sciences, Braşov, Romania, 483-486, 2007.

4. Gh. Mishkoy, *On multidimensional analog of Kendal-Takacs equation and its numerical solution*, Lecture Notes in Engineering and Computer Science. World Congress of Engineering, London, 2008, 928-932.

5. Gh. Mishkoy, *A virtual analog of Pollaczek-Khintchin transform equation*, Bul. Acad. of Science of Moldova, Mat., **2**(2008), 81-91.

* This work is supported partially by Russian Foundation for Basic Research grant 08.820.08.09RF and partially by Scopes grant IB 7320-110720.

Advance statistical methods for web mining

I. Moisil

The World Wide Web has evolved in less than two decades as the major source of data and information for all domains. The Web is today not only an accessible and searchable information source but also one of the most important communication channels, becoming a virtual society. Web mining is a challenging activity that aims to discover new, relevant and reliable information and knowledge by investigating the web structure, its content and its usage. Though the web mining process is similar to data mining, the techniques, algorithms and methodologies used to mine the web encompass those specific to data mining, mainly because the web has a great amount of unstructured data and the changes are frequent and rapid. This paper is structured into two sections. The first one briefly discusses statistical methods used for the different web mining task and the second one is focusing on advanced statistical methods for information retrieval and web search, link analysis, opinion mining and web usage mining. Some of the most important web mining applications in areas like library and information sciences, business and education, are also discussed.

References

1. M. Kantardzic, *Data mining: concepts, models, methods, and algorithms.* John Wiley and Sons., New York, 2003.
2. C. Zhai, *Statistical language model for information retrieval.* Tutorial Notes.
3. J. M. Kleinberg, *Authoritative sources in a hyperlinked environment*, J. of the ACM, **46**, 5(1999), 604-632.
4. B. Liu, *Web data mining. Exploring hyperlinks, contents, and usage data*, Springer, New York, 2007.
5. F. Fouss, A. Pirotte, J. M. Renders, M. Saerens, *Random-walk computation of similarities between nodes of a graph with application to collaborative recommendation*, IEEE Transactions on Knowledge and Data Engineering, **19**, 3(2007), 355-369.

Numerical simulation of the thickness accretions in the primary and secondary cooling zones of a continuous casting machine via phase-field transitions system

C. Moroşanu

The phase field transition system (a nonlinear system of parabolic type) introduced in [1] to distinguish between the phases of the material that is involved in the solidification process is considered. On the basis of the convergence of an iterative scheme of fractional steps type, a conceptual algorithm is constructed in order to approximate the solution of the nonlinear parabolic system. The advantage of such approach is that the new method simplifies the numerical computations due to its decoupling feature. The finite element method in 2D is used to deduce the discrete equations and numerical results regarding the physical aspects of solidification process are reported. Industrial implementation of the software package [3]

developed in this context was made to the primary and secondary cooling zones of a continuous casting machine at ArcelorMittal Steel S. A. Galati.

Keywords: boundary value problems for nonlinear parabolic PDE, fractional steps method, finite element method, solidification process.

References

1. G. Caginalp, *An analysis of a phase field model of a free boundary*, Arch. Rat. Mech. Anal., **92** (1986), 205-245.
2. C. Moroşanu, *Approximation of the phase-field transition system via fractional steps method*, Numer. Funct. Anal. and Optimiz. **18**, 5-6, (1997), 623-648.
3. C. Moroşanu, al. (2006)-(2008), *Report Stage II/2006, III/2007, IV/2008*, CEEX program no. 84/2005.
4. C. Moroşanu, *Approximation of the solid region in the continuous casting process of steel via phase-field transition system*, 6^{th} European Conference on Continuous Casting 2008, Riccione, Italy, June 3-6, 2008, 1-6.
5. C. Moroşanu, Gh. Iorga, S. C. Cocindău, *Numerical simulation of the solid region via phase field transition system,* Metalurgia International, **11**(2008).
6. B. G. Thomas, *Continuous casting: modeling*, The Encyclopedia of Advanced Materials, J. Dantzig, A. Greenwell, J. Mickalczyk (eds.), Pergamon Elsevier Science Ltd., Oxford, UK, **2**, 2001.

Ekman currents on the Romanian Black Sea shore

A. Muntean, M. Bejan

Some analytical solutions for Ekman currents in shallow marine waters are presented. Navier-Stokes equations, with many assumptions, are used as mathematical model. The boundary conditions are particular for some regions on the Romanian Black Sea coast. The magnitude of Ekman currents velocity, its direction, the Ekman net transport and its direction, for a specific function describing the sea basin depth are calculated.

Convergence to stable distributions in Mallows distance

B. Gh. Munteanu

The Central Limit Theorem (CLT) with respect to the Mallows distance is considered. Then we show that these variant of the CLT allows us to prove the convergence to stable laws in the infinite variance setting.
The main result in this article is the following theorem.

Theorem *Fix* $\alpha \in (0,2)$*, and let* $X_1, X_2, \ldots$ *be independent random variables, where* $\mathbb{E}X_i = 0$ *if* $\alpha > 1$*, and* $S_n = (X_1 + \ldots + X_n)/n^{1/\alpha}$*. If there exists an* α *- stable random variable* Y *such that* $\sup_i d_\beta(X_i, Y) < \infty$ *for* $\beta \in (\alpha, 2]$*, then* $\lim\limits_{n\to\infty} d_\beta(S_n, Y) = 0$*. Mose exactly,* $d_\beta(S_n, Y) \leq \frac{2^{1/\beta}}{n^{1/\alpha}} \left(\sum\limits_{i=1}^{n} d_\beta^\beta(X_i, Y) \right)^{1/\beta}$ *, therefore in the identical case the convergence occurs at the rate of* $O(n^{1/\alpha - 1/\beta})$*.*

We denote by $d_\beta(S_n, Y)$ the Mallows β - distance between probability distribution functions F_{S_n} and F_Y.

References

1. B. Bahr, C. G. Esseen, *Inequalities for the rth absolute moment of a sum of independent random variables,* $1 \leq r \leq 2$, Ann. Math. Statist., **36**(1965), 299-303.

2. B. V. Gnedenko, A. N. Kolmogorov, *Limit distribution for sums of independent random variables*, Addison-Wesley, Reading, MA, 1968.

3. J. Mijnheer, *On the rate of convergence to a stable limit law*, Litovsk. Mat. Sb., **26**, 3(1986), 482-487.

4. B.-Gh. Munteanu, *Central limit theorem and convergence of Mallows distance. General case*, Conf. SPSR, Bucharest, (submitted).

5. B. Gh. Munteanu,, *The role of the Mallows distance in the central limit theorem*, Sesiunea de Comunicari Stiintifice cu Patricipare Internationala "Terra Dacica-România Mileniului Trei", Academia Fortelor Aeriene "Henry Coanda" Braşov, **1**(16), 2006. (on CD, ISSN:1453-0139), (Romanian).

6. W. Stout, *Almost sure invariances principles when* $\mathbb{E}X_1^2 = \infty$, Z. Wahrsch. Verw. Gebiete, **49**, 1(1979), 23-32.

Further results on approximate inertial manifolds for the FitzHugh-Nagumo model

S. C. Nartea , A. Georgescu

For two particular choices of the three parameters in the FitzHugh-Nagumo model the equilibrium points are found. The corresponding phase portrait around them is graphically represented allowing us to delimit an absorbing domain. Then the Jolly-Rosa-Temam numerical method [1] is applied in order to study the approximate inertial manifold for the model. To this aim the own numerical code of the first author is used.

References

1. M. S. Jolly, R. Rosa, R. Temam, *Accurate computations on inertial manifold*, SIAM J. Sci. Comput., **22**, 6 (2000), 2216-2238.

2. R. Rosa, *Approximate inertial manifolds of exponential order*, Discrete and Continuous Dynamical Systems, **1**(1995), 421-448.

3. R. Rosa, R. Temam, *Inertial manifolds and normal hyperbolicity*, ACTA Applicandae Mathematicae, **45**(1996),1-50.

4. R. Temam, *Infinite-dimensional dynamical systems in mechanics and physics*, Applied Mathematical Sciences, **68**, Springer, Berlin, 1997.

Balance of payment constrained growth for Moldova in the framework of Thirlwall model

E. Naval

The aim of this paper consists in application of the Thirlwall-Hussain modified model of balance of payments constrained growth to the developing economy, such as economy of the Moldova, in order to examine effect of the net foreign inflows on the constrain imposed to aggregate demand and therefore on the income growth. We illustrate that the long run income growth of the open Moldova economy tends to be conditioned by the income elasticity of the demand for imports and exports. Extended version of the Thirlwall growth model for the Moldova economy, years 1995-2007, was applied.

Balance of payments constrained equilibrium growth determined in [1-2] and the equilibrium growth rate derived from the extended model [3] were calculated and compared with actual growth rate. It is demonstrated that there are sufficient constrains on the income growth rate because income elasticity for import demand is highly in comparison with income elasticity for the exports demand.

References

1. J. S. L. McCombie, A. P. Thirlwall, *Economic growth and the galance-of-payments constraint*, London St. Martin Press, 1994.
2. A. P. Thirlwall, *Reflection on the concept of balance-of-payments constrained growth*, J. of Post Keynessian Economics,**19**(1997).
3. A. P. Thirlwall, M. N. Hussain, *The balance of payments constraint, capital flows and growth rate differences between developing countries*, Oxford Economic Paper, (1997), 498-509.

About some functional integral equation in spaces with perturbated metric

I. M. Olaru, V. Olaru

In this paper we study, in spaces with perturbated metric, the following functional integral equation:

$$u(x) = h(x, u(0)) + \int_0^{x_1} \cdots \int_0^{x_m} K(x, s, u(\theta_1 s_1, \cdots, \theta s_m))ds,$$

$x \in [0, b_1] \times \cdots [0, b_m]$, $\theta_1, \cdots \theta_m \in (0, 1)$.

Modeling thermal radiative properties of nano scale multilayer with coherent formulation

S. A. A. Oloomi, A. Saboonchi, A. Sedaghat

In order to achieve high-accuracy temperature measurements in rapid thermal processing (RTP), it is critical to be able to determine the radiative properties of silicon wafers with thin-film coatings such as silicon dioxide and silicon nitride. In this paper, the directional, spectral, and temperature dependence of the radiative properties for the nanoscale multilayer structures are modeled consisting of silicon and related materials such as silicon dioxide, and silicon nitride.

Solving fuzzy system of linear equations

A. Panahi, T. Allahviranloo, H. Rouhparvar

Systems of linear equations, with uncertainty on the parameters, play a major role in various problems in economics and finance. Fuzzy system of linear equations (f.s.l.e.) has been discussed in [1] using LU decomposition when the matrix A in $Ax = b$ is a crisp matrix. Also the Adomian decomposition method and iterative methods has been studied in [2-8] for f.s.l.e. In this paper we study such a system with fuzzy coefficients, i.e. the matrix A is a fuzzy matrix. We find two fuzzy matrices, the lower triangular L and the upper triangular U such that $A = LU$, and give a procedure to solve the f.s.l.e..

References

1. S. Abbasbandy, R. Ezzati, A. Jafarian, *LU decomposition method for solving fuzzy system of linear equations*, Appl. Math. and Comput. **172**(2006), 633-643.
2. T. Allahviranloo, *Numerical methods for fuzzy system of linear equations* Appl. Math. and Comput. **155**(2004), 493-502.
3. B. Bede, J. Fodor, *Product type operations between fuzzy numbers and their applications in geology*, Acta Polytechnica Hungarica **3**(2006), 123-139.
4. J. J. Buckly, Y. Qu, *Solving systems of linear fuzzy equations*, Fuzzy Sets and Systems **43**(1991), 33-43.
5. M. Dehghan, B. Hashemi, M. Ghatee, *Computational methods for solving fully fuzzy linear systems*, Appl. Math. and Comput. **179**(2006), 328-343.
6. M. Ma, M. Friedman, A. Kandel, *A new fuzzy arithmetic*, Fuzzy Sets and Systems **108**(1999), 83-90.
7. M. L. Puri, D. A. Ralescu, *Differentials of fuzzy functions*, J. Math. Anal. Appl. **91**(1983), 552-558.
8. A. Vroman, G. Deschrijver, E. E. Kerre, *Solving systems of equations by parametric functions-An improved algorithm*, Fuzzy Sets And Systems **158**(2007), 1515-1534.

The finite difference method for objects detecting in a given image

V. Pațiuc, G. Rîbacova

The problem for objects detecting in a given image be can formulated in the following manner. Let us consider the pixel image of rectangular form that is defined in the Grayscale's color model. Mathematically this image might be represented as a reflection function $u_0 : \overline{\Omega} \to \mathbb{R}$, where $\overline{\Omega}$ is a two-dimensional rectangle domain. In accordance with [1] let us denote the looked for contour of object by $C(s)$, $s \in [0,1]$. Then the curve $C(s)$ is defined as a set of points satisfying the condition $C = \{(x,y), \varphi(x,y) = 0\}$. The function $\varphi(x,y)$ takes positive values inside the contour C and negative values outside it. Then the problem of determination of the contour C may be reduced to the problem of minimization of the functional [1]

$$F_\varepsilon(c_1, c_2, \varphi) = \mu \int_\Omega \delta_\varepsilon(\varphi(x,y)) \left|\nabla\varphi(x,y)\right| dxdy + \vartheta \int_\Omega H_\varepsilon(\varphi(x,y))dxdy +$$

$$+\lambda_1 \int_\Omega \left|u_0(x,y) - c_1\right|^2 H_\varepsilon(\varphi(x,y))dxdy +$$

$$+\lambda_2 \int_\Omega \left|u_0(x,y) - c_2\right|^2 (1 - H_\varepsilon(\varphi(x,y)))dxdy. \tag{1}$$

Here constants c_1, c_2 are determined as an average values of the reflection function $u_0(x,y)$ inside and outside the contour C, namely $\begin{cases} c_1 = average(u_0) \text{ in } \{\varphi \geq 0\}, \\ c_2 = average(u_0) \text{ in } \{\varphi < 0\}, \end{cases}$ $\mu, \vartheta, \lambda_1, \lambda_2$ are positive parameters, $H_\varepsilon(z), \delta_\varepsilon(z)$ are the ε-approximations of the Heaviside and Dirac functions

$$H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)\right), \ \delta_\varepsilon(z) = \frac{\partial H_\varepsilon}{\partial z} = \frac{\varepsilon}{\pi(z^2 + \varepsilon^2)}.$$

The problem of minimization of the functional $F_\varepsilon(c_1, c_2, \varphi)$ is equivalent to the solution of the corresponding Euler-Lagrange equation, which in this case is a elliptic nonlinear one. For the sake of simplicity let us introduce the time parameter t and consider that function φ is a function of three variables $\varphi(t,x,y)$. Then the Euler-Lagrange equation can be written in the parabolic form. So the problem may be formulated as follows. In the cylinder $Q = \{(t,x,y)\colon\ t \in [0,\infty), (x,y) \in \overline{\Omega}\}$ find the function $\varphi(t,x,y)$, that satisfies the conditions

$$\frac{\partial\varphi}{\partial t} = \delta_\varepsilon(\varphi)\left[\mu div\left(\frac{\nabla\varphi}{|\nabla\varphi)|}\right) - \vartheta -\right.$$

$$\left. - \lambda_1(u_0 - c_1)^2 - \lambda_2(u_0 - c_2)^2\right], \ t > 0, (x,y) \in \Omega, \tag{2}$$

$$\varphi(0,x,y) = \varphi_0(x,y), \ (x,y) \in \overline{\Omega}, \tag{3}$$

$$\frac{\delta_\varepsilon(\varphi)}{|\nabla\varphi)|}\frac{\partial\varphi}{\partial n} = 0,\, t \geq 0, (x,y) \in \partial\Omega. \tag{4}$$

Here n denotes the exterior normal to the boundary $\partial\Omega$, and $\partial\varphi/\partial n$ denotes the normal derivative of φ at the boundary.

In order to solve the problem (2)-(4) we use the finite difference method [2]. Due to the fact that the given image $u_0(x,y)$ represents the rectangular table of pixels with dimensions $N_1 \times N_2$, for example 100×100, let us consider that the domain $\overline{\Omega}$ have the same dimensions. Then in $\overline{\Omega}$ we introduce the rectangular grid $\overline{\omega}_h$ with the same dimension $N_1 \times N_2$ with the grid step $h = 1 : \overline{\omega}_h = \left\{(x_i, y_j), x_i = ih, i = \overline{0, N_1}, y_j = jh, j = \overline{0, N_2}\right\}$. In a similar way with the time step τ we introduce the time grid: $\overline{\omega}_\tau = \left\{t_n = n\tau, n = \overline{0, M}\right\}$. Let $\varphi^n_{i,j} = \varphi(t_n, x_i, y_j)$ be an approximation of $\varphi(t,x,y)$ on the grid $\overline{\omega}_{h\tau} = \overline{\omega}_h \otimes \overline{\omega}_\tau$. Then in accordance to [2] we can construct the following implicit linearized difference scheme

$$\frac{\varphi^{n+1}_{i,j} - \varphi^n_{i,j}}{\tau} = \delta_\varepsilon(\varphi^n_{i,j}) \left[\mu \left(\frac{\varphi^{n+1}_{i,j,\overline{x}}}{a^n_{i-1/2,j}}\right)_x + \mu \left(\frac{\varphi^{n+1}_{i,j,\overline{y}}}{a^n_{i,j-1/2}}\right)_y - \vartheta - \right.$$

$$\left. - \lambda_1 \left(u_{0,i,j} - c_1(\varphi^n_{i,j})\right)^2 - \lambda_2 \left(u_{0,i,j} - c_2(\varphi^n_{i,j})\right)^2\right], \tag{5}$$

$$i = \overline{1, N_1 - 1},\ j = \overline{1, N_2 - 1}\ .$$

Here $a^n_{i-1/2,j}$ and $a^n_{i,j-1/2}$ are the notations for the approximate values of the gradient module $|\nabla\varphi)|$

$$a^n_{i-1/2,j} = \sqrt{\left(\varphi^n_{i,j,\overline{x}}\right)^2 + 0.25\left(\varphi^n_{i,j,\overline{y}} + \varphi^n_{i,j,y}\right)^2},$$

$$a^n_{i,j-1/2} = \sqrt{0.25\left(\varphi^n_{i,j,\overline{x}} + \varphi^n_{i,j,x}\right)^2 + \left(\varphi^n_{i,j,\overline{y}}\right)^2}.$$

The numerical experiments have been done by means of Mathlab system. The results of consecutive approaching of the initial rounded contour to the boundaries of the objects for the time moments $t = 0$, $t = 0.4$, $t = 0.7$, $t = 1.0$ are presented in the extended version of the paper. Here $\mu = 0.1 * 255\hat{}2$, $\vartheta = 0$, $\lambda_1 = \lambda_2 = 1$.

References

1. T. F. Chan, L. A. Vese, *Active contours without edges*, IEEE Transactions on Image Processing, **10**, 2(2001), 266-277.

2. A. A. Samarski, *Vvedenie v teoriu raznostnyh shem*, Nauka, Moscow, 1977.(Russian)

A generalised mathematical cavitation erosion model

C. Pătrăşcoiu

The most usual methods to estimate cavitation erosion resistance pay a special attention to the velocity erosion curve. Depending on the nature and condition of eroded materials, other kinds of the volume loss rate curve of erosion cavitations progress is proposed. This model gives a new vision of the volume loss rate curve and generalizes some previous mathematical models.

Keywords: erosion, cavitations, loss curve, mathematical model

References

1. I. Bordeasu, C. Patrascoiu, V. Badarau, O. Sucitu, M. O. Popoviciu, *New contribution to cavitation erosion curve modelling*, FME Transaction, Univ. of Belgrade, Fac. of Mechanical Enginering, New Series, **34**, 1(2006), 39-43.
2. F. J. Heymann, *On the time dependence of the rate of erosion due to impingement or cavitation*, 69th Annual Meeting, ASTM, **70-82**, Atlantic City, 1996.
3. M. Szkodo, *Mathematical description and evaluation of cavitation erosion resistance of materials*, AMME 13th Int. Sci. Conf., Gliwice, Poland, (2005), 747-650.
4. J. Noskievic, *Some aspect with regard to cavitation resistance of materials*, HIDROTURBO **76**, C7, (1976), 1-16, Brno.(Polish)
5. J. Steller, P. Kaczmarzyk, *Phenomenological models of cavitation erosion progress*, Int. Cavitation Erosion Test Seminar, Sopot, June 1-2, 2000, 1-9.

Interaction between economic dynamical systems

C. Pătrăşcoiu

Economic dynamical systems, the state spaces of which are the Riemannian manifolds are considend. Between two economic dynamical systems, global feedforward and the feedback interaction is defined and the connection between their linearization or prolongation by derivation is studied.

Keywords: economic dynamical system, feedforward, feedback, linearization.

References

1. M. W. Hirsch, S. Smale, *Differential equations, dynamical system and liniar algebra,* Academic, New York, 1974.
2. V. Ivancevic, T. Ivancevic, *Applied differential geometry. A modern introduction*, World Scientific, Singapore, 2006.
3. C. Patrascoiu, I. Bordeasu, C. Saceanu, *Command and control in interaction between two dynamical systems*, Bul. Şt. Univ. "Politehnica", Timisoara, Seria Mecanica, Transactions on Mechanics, **53**(**67**), 1(2008).
4. C. Udriste, M. Ferrara, D. Opriş, *Economic geometric dynamics*, Geometry Balkan Press, Bucharest, 2004.
5. S. Wiggins, *Introduction to applied dynamical system and chaos,* Springer, New York, 1990.

Compactifications of G-spaces

D. I. Pavel

All spaces are assumed to be completely regular.

Fix $n \geq 2$. An n-groupoid is a non-empty set G together with an n-ary operation $\omega : G^n \to G$, called an n-multiplication, for which there exists an element $e \in G$ such that $\omega(x_1, x_2, ..., x_n) = x_i$ provided $x_j = e$ for any $j \neq i$. The element e is called the identity of G. If $n \geq 3$, then in G the identity is not unique. We assume that the identity e of the n-groupoid G is fixed.

If G is a space and the operation $\omega : G^n \to G$ is continuous, then G is called a topological n-groupoid.

Let $n \geq 2$ and (G, ω) be a topological n-groupoid. A topological space X is called a G-space if it is given an n-action $\alpha : G^{n-1} \times X \to X$ such that:

- $\alpha(e, e, ..., e, x) = x$ for any $x \in X$;

- $\alpha(a_1, ..., a_{n-1}, \alpha(a_n, ..., a_{n-2}), x) = \alpha(a_1, ..., a_i, \alpha(a_{i+1}, ..., a_{i+n}), a_{i+n+1}, ..., a_{n-2}, x)$ for all $i \leq n-1$, $x \in X$ and $a_1, a_2, ..., a_{n-2} \in G$;

- the translation $L_{(a_1,a_2,...,a_{n-1})} : X \to X$, where $L_{(a_1,a_2,...,a_{n-1})}(x) = \alpha(a_1, a_2, ..., a_{n-1}, x)$ for any $x \in X$, is continuous for all $a_1, a_2, ..., a_{n-1} \in G$.

Let X and Y be two G-spaces. A mapping $\varphi : X \to Y$ is a homomorfism if $\varphi(\alpha(a_1, a_2, ..., a_{n-1}, x)) = \alpha(a_1, a_2, ..., a_{n-1}, \varphi(x))$ for all $a_1, a_2, ..., a_{n-1} \in G$ and $x \in X$.

Fix an n-groupoid G and a G-space X. A pair (bX, f) is called a generalized G-compactification of X if bX is a compact G-space, $f : X \to bX$ is a continuous homomorphism and the set $f(X)$ is dense in bX. If f is an embedding, then (bX, f) is called a G-compactification of the G-space X. Denote by $GC(X)$ the set of all generalized G-compactifications of the G-space X. If $(bX, f), (cX, g) \in GC(X)$, then we consider $(bX, f) \leq (cX, g)$ if there exists a continuous homomorphism $\psi : cX \to bX$ such that $f = \psi \circ g$. The set $GC(X)$ is a complete lattice for any topological n-groupoid G and any G-space X. It is well-known that the family $gC(X)$ of all generalized compactifications of the space X is a complete lattice. As a rule, $GC(X)$ is not a sublattice of the lattice $gC(X)$.

Theorem 1. *Let X be a locally compact perfect G-space. Then the one-point Alexandroff compactification aX of X is a G-compactification of X.*

Theorem 2. *Let X be a G-space and G be a groupoid with devisions. Then:*

1. *for every $a_1, a_2, ..., a_{n-1} \in G$ the mapping $L_(a_1, a_2, ..., a_{n-1}) : X \to X$ is a homeomorphism.*
2. *X is a perfect G-space.*
3. *if X is locally compact, then the one-point Alexandroff compactification aX of X is a G-compactification of X.*

Other properties of G-compactifications of the G-spaces are studied too.

References

1. J. F. Berglund, H. D. Junghenn, P. Milnes, *Compact right topological semigroups and generalizations of almost periodicity,* Springer, Berlin, 1978.

2. R. Engelking, *General topology,* PWN, Warszawa, 1977.

3. J. de Vries, *On the existence of G-compactifications,* Bull. Acad. Polon. Sci., Ser. Math. Astronom. Phys., **26** (1978), 275 - 280.

Fast K-means image quantization algorithm and its application to iris segmentation

N. Popescu-Bodorin

The main algorithm presented in this paper, fast K-means image quantization, is an adaptation of the generic k-means algorithm, tuned for fast chromatic clustering of both 24-bit RGB and Grayscale continuous-tone still images. Here we show that, in some specific conditions, the automatic discovery of the area morphology can be done through a non-deterministic algorithm mainly controlled by the image itself. The most important properties of the fast K-means image quantization Algorithm are also presented here in association with some newly proposed principles of optimal K-means downsampling which have been proven to be closely related to the proposed algorithm. Some relevant results in iris segmentation are presented as a practical application of the main algorithm.

References

1. International Telecommunication Union (ITU), The International Telegraph and Telephone Consultative Committee (CCITT), International Standard ISO/IEC, 10918-1: 1993(E), CCITT Recommendation T.81, *Digital compression and coding of continuous still images - Requirements and Guidelines*, Section 3 (Definitions, abbreviations and symbols), 1993.
2. W. K. Pratt, *Digital image processing*, Chapter 6 (Image Quantization), 3rd ed., John Wiley and Sons, New York, 2001.
3. W. Lin, L. Dong, *Adaptive downsampling to improve image compression at low bit rates*, IEEE Transactions on Image Processing, **15**(2006), 2513.
4. D. Arthur, S. Vassilvitskii, *How slow is the k-means method?*, Stanford University, http://www.cs.duke.edu/courses/spring07/cps296.2/papers/kMeans-socg.pdf
5. L. Ma, T. Tan, Y. Wang, D. Zhang, *Efficient iris recognition by characterizing key local variations*, IEEE Transactions on Image Processing, **13**, 6(2004), 739.

Lossless DCT-based image compression

N. B. Popescu-Borodin

This paper presents a lossless approach to single-component still image compression. The main algorithm presented here, lossless DCT-based image compression algorithm, is designed to ensure that decoding a compressed image results in exactly the same image initially encoded by compression. Usually, when using JPEG Baseline codecs, even if the quality factor is set to 100%, compression / decompression cycle leads to so called "transcoding errors" which are mainly round-off errors produced after the JPEG quantization and which can be represented as a full matrix with components randomly valued in { -1,0,1}. The lossless DCT-based image compression algorithm ensures that the matrix containing the transcoding errors is a sparse one, and consequently, it can be efficiently stored within the compressed format.

References

1. G. K. Wallace, *The JPEG still picture compression standard*, Communications of the ACM, 34, 4, Association of Computing Machinery, 1991, 30-44.
2. International Telecomunnication Union (ITU), The International Telegraph and Telephone Consultative Committee (CCITT), International Standard ISO/IEC, 10918-1 : 1993(E), CCITT Recommendation T.81, *Digital compression and coding of continuous still images - Requirements and Guidelines*, 1993.
3. Impulse Adventure, *Index of quantization table sources*, http://www.impulseadventure.com/photo/jpeg-quantization.html
4. N. P. Bodorin, *Comprehensive RGB-to-YCbCr color space conversions script*, http://www.mathworks.com/matlabcentral/fileexchange/
5. G. Ridino, *Simple Huffman code implementation*, http://www.mathworks.com/matlabcentral/fileexchange/
6. N. P. Bodorin, L. State, *Optimal luminance-chrominance downsampling through fast K-means quantization*, Proc. of the 3rd Annual South East European Doctoral Student Conference, South-East European Research Centre, 2(2008), 111-125.

Employment of the Romer model for the analysis of the dynamics of the economic growth in the Republic of Moldova

M. Popovscaia

In the paper the Romer model[1] with respect to economic conditions of the Republic of Moldova is analyzed. In the first half of the 20-th century the western economists made the conclusion that besides such factors as labour and capital technological factor also has an effect on the rates of growth. Paul M. Romer concludes that the rates of growth depend not only on the aggregate stock of labour or the population, but also on the stock of human capital engaged in research: a larger total stock of human capital will experience faster growth. The production function of the Cobb-Douglas type is used in the model. The presence of the technological factor in this function explains why having the given values of the labour and facilities the output grows.

The question of the influence of the technological change on the growth rates is very actual for the Republic of Moldova. The agricultural sector can not satisfy all requirements of the population, the industrial sector is on the development stage. That is why there is a need to implement services in all sectors. For the attendance of high technologies, attracted in the production process, there is a need of highly knowledgeable specialists. Much attention in the Republic is piad to the modernization of the educational system, to the training of the qualified specialists; the programm of the creation of the informational society is launched (the national strategy of the development of the knowledge society E-Moldova[2]).

References

1. P. M. Romer, *Endogenous technological change*, J. of Political Economy (1990), 71-102.
2. http://www.gov.md/

Classification of the cubic differential systems with seven real invariant straight lines

V. Puţuntică, A. Şubă

We consider the cubic differential system $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$, where P, $Q \in \mathbb{R}[x,y]$, $\max\{\deg(P), \deg(Q)\} = 3$ and $GCD(P,Q) = 1$.

The straight line $Ax + By + C = 0$ is said to be invariant for this system if there exists a polynomial $K(x,y)$ such that $A \cdot P + B \cdot Q \equiv (Ax + By + C) \cdot K$ holds. If $K(x,y) \equiv (Ax + By + C)^m \cdot K^*(x,y)$, where $m \in \mathbb{N}$, $K^* \in \mathbb{R}[x,y]$, and $Ax + By + C = 0$ does not divide $K^*(x,y)$, then we shall say that the invariant straight line has the *degree of invariance* $m+1$.

A set of invariant straight lines can be infinite, finite or empty. It is known that if the number of invariant straight lines is finite, then it is not greater than eight.

A classification of cubic systems with exactly eight straight lines was carried out in [1]. We give here a similar classification for cubic differential systems with exactly seven real invariant straight lines.

Theorem. *Every cubic differential system having real invariant straight lines with total degree of invariance equal to seven via affine transformation and time rescaling can be written as one of the following seven systems:*

$$\begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)(y-a), \\ a > 0,\ a \neq 1; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)((2+a)x - (1+a)y - a), \\ a > 0,\ a \neq 1; \end{cases}$$

$$\begin{cases} \dot{x} = x^3, \\ \dot{y} = y^2(dx + (1-d)y), \\ d(1-d)(d-3)(2d-3) \neq 0; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(y+1)((1-a)x + ay - a), \\ a > 0,\ a \neq 1; \end{cases}$$

$$\begin{cases} \dot{x} = x^2(bx + y), \\ \dot{y} = y^2((2+3b)x - (1+2b)y), \\ b(b+1)(2+3b)(1+2b) \neq 0; \end{cases} \qquad \begin{cases} \dot{x} = x(x+1)(a + (2a-1)x + y), \\ \dot{y} = y(y+1)(a + (3a-1)x + (1-a)y), \\ a(2a-1)(1-a)(3a-1)(3a-2) \neq 0; \end{cases}$$

$$\begin{cases} \dot{x} = x^2(x+1), \\ \dot{y} = y^2(y+1). \end{cases}$$

A qualitative investigation for obtained cubic systems is done.

References

1. J. Llibre, N. Vulpe, *Planar cubic polynomial differential systems with the maximum number of invariant straight lines*, Rocky Mountain J. Math., **36**(2006), 4, 1301-1373.

Development of the theme in the process of geometrical problems solution

M. Rodionov, S. Velmisova

The article illustrates how a mathematical task can be developed by means of transformations of its conditions to construct a chain of tasks progressing in difficulty.

There are a lot of methodical approaches that motivate and stimulate the students to show their creative initiative in constructing mathematical problems. One of these approaches consists in transformation of a task which leads to a new task. The initiative is most effective when the result is expressed by the student in terms of conjectures and hypotheses. Within the process of consistent task transformation the students supported by the teacher can construct cycles/series of tasks (dynamical tasks) that are based on a common idea and cover a broad part of a mathematical course.

To illustrate this approach we have selected the topic AREAS of the elementary geometry college course. Consistent constructions of appropriate series of tasks and their solutions are represented in the article.

Let us consider an initial task: Extend the edges of the triangle ABC locating the points A_1, B_1, C_1 so that $AA_1 = AB, BB_1 = CB, CC_1 = AC$. Find the ratio of both triangles $A_1B_1C_1$ and ABC areas.

To transform the task one should understand which elements may by varied or generalized. Among such elements we may consider vertices, directions that identify the points A_1, B_1, C_1, lengths of segments, space dimension and also the question of interest which depends on the way of the task transformation.

After identifying these elements and ratios together with the students, the teacher offers them to point out several concepts:

firstly, transformations of the triangle could result in the following: rectilinear triangle, rectilinear polygon, quadrangle, arbitrary polygon, rectilinear tetrahedron, arbitrary pyramid, simplex, spherical triangle;

secondly, the segments can be selected at bisectors of triangle angles, medians, triangle altitudes, lines with arbitrary angles between them and triangle sides;

thirdly, the lengths of triangle sides can be in different ratios with respect to the lengths of constructed segments or be equal to the lengths of other sides;

fourthly, the task may be formulated in the 3-dimensional space or n-dimensional space.

So we can vary any element or combination of elements listed above and obtain series of tasks different in complexity, didactic and developmental opportunities. Note that each separate result can serve for further dynamical development of the initial task. Our experience shows that such development demonstrates the sense and value of the initial problem and outlines its characteristics which are not revealed in direct and/or trivial analysis.

Here we present some directions of the initial task proposed by the students during practical classes. With the help of some examples we tried to demonstrate how a "creative laboratory" can be "developed" within the framework of traditional mode of teaching and learning mathematics. We are convinced that such investigations allow not only to understand the material but also to develop students' creativity.

Linear discriminant analysis and logit analysis in bankruptcy prediction

P. Sajin

The purpose of this paper is to present linear discriminant analysis (LDA) and logistic regression (LR) in forecasting corporate bankruptcy. Discriminant analysis is one of the data mining techniques used to discriminate a single classification variable using multiple attributes. When the distribution within each group is multivariate normal, a parametric method can be used to develop a discriminant function. In logit analysis, conditional probabilities, lying between zero and one are calculated.

The reason why we focus on LDA and the LR technique is because they are used in most failure prediction research. In the last section the results of our empirical research and attempts to explain the findings are presented.

Keywords: discriminant analysis, logit analysis, principal component analysis, discriminant functions, failure prediction model, decision rule.

References

1. B. Andrew, V. Gertjan, *Stylised facts of UK corporate financial health: evidence from micro-data financial stability review.* June 2000, Bank of England.
2. B. Barbo, L. Teija, S. Kaisa, M. van Wezel, *Choosing bankruptcy predictors using discriminant analysis, logit analysis and genetic algorithms,* Working paper, Turku Centre for Computer Science, 1996.
3. C. Carlos, G.-M. Luis, M. Antonio, T. Carlos, *Analysis of procyclical effects on capital requirements derived from a rating system,* Working paper, Bank of Spain, 2001.
4. D. Denis, *Notation et credit scoring.* CNAM 2000, Working paper.
5. E. O. Robert, *An empirical test of financial ratio analysis for small failure prediction.* The Journal of Financial and Quantitative Analysis, **7**, 2(1972).
6. G. C. J. Fernandez, *Discriminant analysis, a powerful classification technique in data mining.* Working paper, Department of Applied Economics and Statistics, University of Nevada.
7. H. Otto, V. Miklos, *A Hungarian model for predicting financial bankruptcy,* Working paper.
8. J. A. Ohlson, *Financial ratios and the probabilistic prediction of bankruptcy,* Journal of Accounting Research, **18**, 1(1980).
9. R. Gheorghe, *Analiza datelor,* Editura ASE, Bucureşti 2001.
10. S. A. Alessio, *Can firm's default be really predicted?* Working paper, University of Verona.
11. S. C. Yoshiko, *Financial ratios as predictors of bankruptcy in Japan: an empirical research,* Working paper, Tsukuba College of Technology, Tokyo Japan, 1998.
12. Z. Constantin, D. Michael, *A multicriteria discrimination method for the prediction of financial distress: The case of Greece,* Multinational Finance Journal, **3**(1999).

In memoriam professor Eugen Onofraş

L. Sandulescu

Professor Eugen Onofraş was born on October 14, 1931 in the village of Tabalaiesti, commune of Avereşti, Vaslui (former Huşi) county and died on January 22, 1991. His life was dedicated to the study of mathematics, to his activity with collegians and especially to the preparation of exceptional scholars for olympiads. In spite of his hard life, he worked unceasingly in the mathematical research and didactics. He never forgot that a teacher must know very well the Romanian language, express himself correctly and easily in writing as well as viva voce. He was exacting first with himself and then with the scholars. I had the honour to be a former schoolgirl of Professor Eugen Onofraş for a period of four years at the "Mihai Viteazul" College in Ploieşti, where he carried on most of his activity.

Object classifcation methods with application in astronomy

C. Săraru

The need of automated object recognition is of great importance nowadays not only because of the practical impact in almost all fields of information manipulation, but also because of the large quantity of data available for processing. Many approaches have been proposed to this purpose. There are different types of artificial neural network architectures taken into consideration as well as statistical methods to discriminate between classes of objects. Also, there are efficient methods that use fractal dimensions of objects to be classified in order to assign them to a certain class. This paper surveys some of the methods used in practice for object classification and studies the possibility of combining a well-known object classification technique with other image processing methodologies such as edge detection.The edge detection method is used to improve the efficiency of identifying the searched objects in a digital image. An application of this proposed process may be the classification of galaxies, based upon the Hubble classification, which divides them into three main categories. The feature taken into consideration is the shape of the galaxy.

References

1. S. C. Odewahn, *Automated galaxy classification in large sky surveys* , Int. Joint Conf. on Neural Networks, **6**(1999), 3824-3829.
2.http://www.ics.uci.edu/s̃myth/courses/cs175/schedule.html
3. http://terapix.iap.fr/rubrique.php?id_rubrique=91

Mathematical and numerical model of rock/concrete mechanical behavior in multi-plane framework

S. A. Sadrnejad

Among the various mathematical and numerical simulating models of plane concrete, the multi-planes models have an excellent position. These models are not as complicated as microscopic models such as discrete particles models and do not have the shortcomings of macroscopic models based on the stress or strain invariants. The object of this study is the presentation of a developed multi-planes damage based model of plane concrete through a 3D finite elements code to show its abilities in crack/damage analysis of actual rock/concrete/concrete structures such as a double curvature arch dam. The proposed code not only is able to predict the crack line, but also determine which combination of loading conditions occurs on damaged multi-planes.

According to this model, the overall deformation of any small part of the medium is composed of total elastic response and an appropriate summation of sliding, separation/closing phenomenon under the current effective normal and shear stresses on sampling planes. These assumptions adopt overall sliding, separation/closing of inter-granular points of grains included in one structural unit (discrete polyhedron elements) are summed up and contributed as the result of sliding, separation/closing surrounding boundary planes. This simply implies yielding/failure or even ill-conditioning, bifurcation response damage and fragmentation phenomena to be possible over any of the randomly oriented sampling planes. Consequently, plasticity control such as yielding should be checked at each of the planes and those of the planes that are sliding will contribute to plastic deformation. Therefore, the geo-material mass has an infinite number of yield functions usually one for each of the planes in the physical space. Compact and isotropic synthetic media are generated automatically and are used to investigate the mechanical behavior of these low-porosity materials. In the case of micro-mechanics, the model considers the two-phase, aggregate and cement medium, at a macroscopic scale.

The proposed multi-plane based model is capable of predicting the behavior of materials under different orientation of bedding plane, history of strain progression during the application of any stress/strain paths, based on five types of planar behavior of sampling planes. Validity of the proposed model is investigated through a few standard benchmark examples.

Spreadsheet in American school

N. Sămărescu

Electronic tools are presented already from school. This paper presents step by step the way in which spreadsheets are used in American school. Spreadsheets contribute to enhancing understanding and developing creativity.

On a certain differential inequality

R. Şendruţiu

Certain conditions on the complex-valued functions $A, B : U \to \mathbb{C}$ defined in the open unit disc $U = \{z \in \mathbb{C} : \ |z| < 1\}$ such that the differential inequality

$$Re\,[A(z)p^2(z) + B(z)p(z) + \alpha(zp'(z) - a)^3 - 3a\beta(zp'(z))^2 + 3a^2\gamma(zp'(z)) + \delta] > 0$$

implies $Re\,p(z) > 0$, where $p \in \mathcal{H}[1, n]$, $a \geq 0$, $\alpha, \beta, \gamma \in \mathbb{C}$, are presented.

References

1. A. Cătaş, *On certain analytic functions with positive real part*, (to appear).
2. B. A. Frasin, *On a differential inequality*, An. Univ. Oradea, Fasc. Math., **14**(2007), 81-87.
3. S. S. Miller, P. T. Mocanu, *Differential subordinations. Theory and applications*, Pure and Applied Mathematics, Marcel Dekker, New York, 2000.
4. G. I. Oros, A. Cătaş, *A new differential inequality I*, Gen. Math. **11**(2003), 1-2, 47-52.
5. Gh. Oros, G. I. Oros, A. Cătaş, *A new differential inequality II*, Studia Univ. Babeş-Bolyai Math., **49**, 4(2004), 85-90.
6. Gh. Oros, G. I. Oros, A. Cătaş, *On a special differential inequality II*, An. Univ. Oradea Fasc. Math., **11**(2004), 119-122.
7. Gh. Oros, G. I. Oros, A. Cătaş, *On a special differential inequality I*, An. Univ. Oradea Fasc. Math., **10**(2003), 109-113.

On 1a Belousov problem

V. Shcherbacov

Let $(Q, \cdot)$ be a groupoid (be a magma in alternative terminology). As usual, the map $L_a : Q \to Q, L_a x = a \cdot x$ for all $x \in Q$, is a left translation of the groupoid $(Q, \cdot)$ relative to a fixed element $a \in Q$, the map $R_a : Q \to Q$, $R_a x = x \cdot a$, is a right translation [1].

A groupoid $(Q, \cdot)$ is called a *quasigroup*, if there exist unique solutions $x, y \in Q$ to the equations $x \cdot a = b$ and $a \cdot y = b$ for all $a, b \in Q$. In this case any right and any left translation of the groupoid $(Q, \cdot)$ is a bijection of the set Q [1].

A quasigroup $(Q, \cdot)$ with an identity element $e \in Q$ is called a *loop*. An element $e(b)$ of a quasigroup $(Q, \cdot)$ is called a right local identity element of an element $b \in Q$, if $b \cdot e(b) = b$ [1].

A quasigroup $(Q, \cdot)$ is: a *left F-quasigroup*, if $x \cdot yz = xy \cdot e(x)z$ for all $x, y, z \in Q$; *left distributive*, if $x \cdot uv = xu \cdot xv$ for all $x, u, v \in Q$ [1].

A loop $(Q, \cdot)$ is *left special*, if $S_{a,b} = L_b^{-1} L_a^{-1} L_{ab}$ is an automorphism of $(Q, \cdot)$ for any pair $a, b \in Q$.

A groupoid $(Q, \cdot)$ is an *isotope of a groupoid* $(Q, \circ)$ if there exist permutations α, β, γ of the set Q such that $x \cdot y = \gamma^{-1}(\alpha x \circ \beta y)$ for all $x, y \in Q$ [1].

A loop $(Q, \circ)$ is a left S-loop, if and only if there exists a complete automorphism ψ of the loop $(Q, \circ)$ such that $\varphi(x \circ \varphi^{-1} y) \circ (\psi x \circ z) = x \circ (y \circ z)$. Then $(Q, \cdot)$, where $x \cdot y = \varphi x \circ \psi y$, is a left distributive quasigroup which corresponds to $(Q, \circ)$ [2].

Problem. (Belousov Problem 1a.) Find necessary and sufficient conditions that a left special loop is isotopic to a left F-quasigroup [1, 4].

I. A. Florea and M. I. Ursul proved that a left F-quasigroup with IP-property is isotopic to an A-loop [3]. It is proved the following

Theorem. *A finite left special loop* $(Q, \oplus)$ *is isotope of a left F-quasigroup* $(Q, \cdot)$ *if and only if* $(Q, \oplus)$ *is isomorphic to the direct product of a group* $(A, +)$ *and a left S-loop* $(B, \diamond)$.

References

1. V. D. Belousov, *Foundations of the theory of quasigroups and loops*, Nauka, Moscow, 1967.
2. V. D. Belousov, V. I. Onoi, *On loops that are isotopic to left distributive quasigroups*, Mat. Issled., **3(25)**(1972), 135 - 152.
3. I. A. Florea, M. I. Ursul, *F-quasigroups with invertibility property (IPF- quasigroups)*, Questions of the Theory of Quasigroups and Loops (1971), 145 - 156.
4. T. Kepka, M. K. Kinyon, J. D. Phillips, *The structure of F-quasigroups*, Journal of Algebra, **317** (2007), 435 - 461.

A survey on connections between hypergroups and geometry

M. Stefanescu

The concept of hypergroup is a generalization of that of group and appeared in 1934, [10] communicated about it at the Eighth Congress of Scandinavian Mathematicians in Stockholm.

After 1943, W. Prenowitz has developed a common algebraic framework in which descriptive geometries could be axiomatized and then studied. By that time the notion used was “multigroup”, but after some years this algebraic hyperstructure was called “hypergroup”. [14-18] and then with [19,20] have produced a series of papers and a book published in Springer studying the join spaces, - a kind of hypergroup,- in connection with hyperbolic, spherical, projective geometries. In his Ph.D. thesis under J.Mittas as supervisor since 1989, [11] has studied this hypergroups from an algebraic point of view.

In the last years, the join spaces have been used in some constructions of hypergroups connected to fuzzy sets (see for example the papers [2] and [22], as well as the book published [3]).

Here we discussed the algebraic aspects of some geometrical facts appeared in this theory.

References

1. P. Corsini, *Prolegomena of hypergroup theory*, Aviani Editore, Udine, 1993.
2. P. Corsini, *Join spaces, power sets, fuzzy sets*, in: Algebraic Hyperstructure Theory and Applications, M. Ştefănescu, (ed.), Hadronic Press, Palm Harbor, (1994), 45-52.
3. P. Corsini, V. Leoreanu, *Applications of hyperstructure theory, Advances in Math.*, Kluwer, Dordrecht, 2003.
4. J. Jantosciak, *Classical geometries as hypergroups*, in Convegno su ipper- gruppi, altre strutture multivoche e loro applicazioni, P. Corsini, (ed.), Udine, Italy, (1985), 93-104.

5. J. Jantosciak, *A brief survey of the theory of join spaces*, in Algebraic Hyperstructure Theory and Applications, M. Ştefănescu (ed.), Hadronic Press, Palm Harbor, 1994, 1-11.
6. Chr. G. Massouros, *Hypergroups and convexity*, Riv. Mat. Pura Appl., **4**(1989), 7-26.
7. J. Mittas, *Hypergroupes canoniques*, Math. Balcanika, **2**(1972), 165-179.
8. W. Prenowitz, J. Jantosciak, *Join geometries*, Springer, UTM, New York, 1979.
9. M. Ştefănescu, *On constructions of hypergroups*, Contrib. General Algebra, **9**(1995), 297-308.

Bifurcation in a nonlinear business cycle model

M. Sterpu, C. Rocşoreanu

A business cycle model consisting of three delay differential equations is considered. The system is obtained by combining a nonlinear IS-LM model with the Kaldor-Kalecki model for the business cycle with delayed investment. For zero time delay the Hopf bifurcation with respect to one of the parameters is analysed by computing the first Lyapunov coefficient. For positive time delay, the Hopf bifurcation boundaries are found.

The theoretical results are illustrated by numerical simulations for particular cases.

References

1. J. Cai, *Hopf bifurcation in the IS-LM business cycle model with time delay*, Electronic Journal of Differential Equations, **15**(2005), 1-6.
2. L. Cesare, M. Sportelli, *A dynamic IS-LM model with delayed taxation revenues*, Chaos, Solitons and Fractals, 25, 233-244.
3. B. Ermentrout, XPPAUT, http://www.math.pitt.edu/~hard/xpp/xpp.html.
4. G. Gabish, H. W. Lorenz, *Business cycle theory-A survey of methods and concepts*, Springer, New York.
5. Gu Keqin, V. L. Kharitonov, Jie Chen, S*tability of time-delay systems*, Birkhäuser, Boston.
6. A.-V. Ion, *On the Bautin bifurcation for systems of delay differential equations*, Acta Univ. Apulensis Math. Inform., **8**, 235-246.
7. M. Kalecky, *A macrodynamic theory of business cycle*, Econometrica, **3**, 327-344.
8. A. Krawiec, M. Szydłowski, *The Kaldor-Kalecki business cycle model*, Annals of Operations Research, **89**, 89-100.
9. Yu. Kuznetsov, *Elements of applied bifurcation theory*, Springer, New York.
10. G. Mircea, M. Neamţu, D. Opriş, *Bifurcaţii Hopf pentru sisteme dinamice cu argument întârziat şi aplicaţii*, Ed. Mirton, Timişoara. (Romanian)
11. M. Sterpu, C. Rocşoreanu, *Stability and Hopf bifurcations for two advertising systems, coupled with delay*, Int. Conf. on Numerical Analysis and Applied Mathematics, Amer. Inst. of Physics Conf. Proc. - 936, Melville, New York, T. Simos(ed.), 535-538.
12. M. Sterpu, C. Rocşoreanu, *Modelarea si simularea proceselor economice*, Universitaria, Craiova. (Romanian)
13. M. Szydlowski, A. Krawiec, *The Kaldor-Kalecki model of business cycle as a two-dimensional dynamical system*, J Nonlinear, Math. Phys., **8**, 266-71.

14. W. B. Zhang, *Capital and knowledge*, Springer, Berlin.
15. C. Zhang, J. Wei, *Stability and bifurcation analysis in a kind of business.*

The efficiency of using educational software on the teaching process

C.-A. Sultana, A. Florea

This paper was written in order to have a better, clearer image on the methods of using educational software. The emphasis is placed on the way in which this group of software programs can be used, as well as on the level at which such a software should be correlated with the age of the students. For this study a few programs found on the M.E.C.T. site have been used. The chosen software was addressed to both gymnasium and high school students. The programs were given to a number of students, along with a test made out of questions about the subject of the chosen software.

Physics for stern flow improvement

H. Tănăsescu, N. Tănăsescu

A modern ships hull lines are designed to minimize forward resistance, to improve propulsion performance, to reduce propeller cavitation, and to increase global hydrodynamic stability. As a general, recently accepted, opinion, the ships of the future will be designed and built only on the basis of some new original devised concepts. It is well known that stern flow problems are complex and pose many challenges. Ships hull stern flows have received much more attention during these last years, in particular with regard to their modeling and design principles. As a state of the art in this field, the most recently known industrial achievements, focused on flow improvement in the stern region, consist of symmetrically flattening of the stern lateral surfaces towards the central plane. This concept has resulted in a huge amount of drawbacks almost in all practical applications to real ships (unsuitable placing of equipments, lack of necessary spaces for inspections, repairs etc.). For a long time, the authors thought how to redesign the two systems - hydroframe system and propulsion system very important (critical) for ships, so that the hydroframe may meet the propulsion and the propulsion may meet the hydroframe in an optimal way.

The paper tries to draw attention and briefly focuses on ships hullfs stern flow improvements in the light of two new fundamental research concepts (ideas), original in the world of ship hydrodynamics: 1-a new stern hydrodynamic concept, with radial crenellated-corrugated sections, of dynamic - gravitational distributor type, able to improve propulsion; 2-use of an inverse piezoelectric effect [electric current high frequency power generator piezoelectric driver made of a certain ceramic material, which induces an elliptical vibratory movement (high frequency over 20 kHz), into the elastic side plates (15 mm thickness) in the streamlines direction (of the external flowing water)], able to reduce total forward resistance.

On a slow-fast system in R^{2+2}

K. Tchizawa

We analyze its singular solution in a direct way. Especially, through blowing up, we give a local model with an exact solution which describes a relation between the systems when $\epsilon = 0$ and $\epsilon \neq 0$. This implies that a generalized pseudo singular point is effective directly.

Some problems of nonlinear optimization

A. Tkacenko

Problems of calculus of variations presume the optimization of a nonlinear functional. In order to approximate the solution various methods of numerical analysis are applied. In the present paper we provide a procedure of minimization of the discrete analog of the initial problem. The decomposition scheme of continuous functional on the given domain that insures a high degree of accuracy of the discrete analog after a minimum number of calls is proposed. Necessary and sufficient conditions that have to be satisfied by the continuous function so that the discrete analog keeps the property of the convexity that leads to the insurance of the existence and uniqueness of the solution of the problem are found too.

Use of Jordan chains in stability and bifurcation problems

V. A. Trenogin, N. N. Avxentieva, L. R. Kim-Tyan

Considered the Cauchy problem for the abstract parabolic differential equation in the Banach space X

$$\dot{x}(t) + Bx = R(x), x(0) = 0, \tag{1}$$

with the following assumptions:

I. B is a closed linear operator with dense domain D in X;

II. B is a Fredholm operator in the sense [1]: its zero-subspace $N(B)$ is n-dimensional, and its range is closed in X, and has a n-dimensional complement;

III. $-B$ is the generator of bounded on X analytical semigroup $U(t)$;

IV. the nonlinear operator $R(x)$ is analytical in the Fréchet sense at $x = 0$, and $R(0) = R'(0) = 0$.

Note that in [2]-[4] the semigroup $U(t)$ is supposed exponentially decreasing, and B is continuously invertible in the sense of [1], [5]. On the basis of the Schmidt-Trenogin lemma [6] the problem (1) is reduced to the equivalent Cauchy problem in $\Re^n$. It is the analog of the Lyapunov-Schmidt bifurcating equation. In this way we succeed to establish a series of statements about Lyapunov stability and asymptotical stability of trivial solution of differential equation (1). Moreover, for the differential equation (1) the problem of periodical solutions is considered

(Poincaré-Andronov-Hopf bifurcation). These arguments are applying to the differential equation (1) with linear operator (nondegenerate or degenerate) in the derivative [7],[8].

References
[1] V. Trenoguine, *Analyse fonctionnelle*, MIR, Moscou, 1985.
[2] V. A. Trenogin, *Lyapunov theorem about the stability by linear as the implicit operators theorem.* Doklady Academii Nauk, **407**(2006), 742-746.
[3] V. A. Trenogin, *Asymptotic stability of trivial solution of DE in Banach space with unbounded linearity.* CAIM-2007, abstracts, 61-62.
[4] V. A. Trenogin, *Asymptotic stability of trivial solution of DE in Banach space with unbounded linearity,* ROMAI Journal, **3**, 1(2007), 207-212.
[5] M. M. Vainberg, V. A. Trenogin, *Theory of branching of solutions of non-linear equations,* Noordhoff, Leyden, 1974.
[6] N. A. Sidorov, V. A. Trenogin, *Nonlinear analysis and nonlinear differential equations,* Fizmatlit, 2003, 5-49.
[7] B. V. Loginov, Yu. B. Rusak, L. R. Kim-Tyan, Uzbek Math J., **4**(1999), 31-35.
[8] L. R. Kim-Tyan, B. V. Loginov, Yu. B. Rousak., *Nonlinear analysis*, **67**, 5(2007), 1570-1585.

Two reduction schemes in investigation of asymptotic stability and stability by Lyapounov for abstract parabolic differential equations

V. A. Trenogin

For the stability questions investigation two reduction methods of nonlinear equations are suggested going back to A. M. Lyapounov and E. Schmidt. In this way it can be obtained the essential extension of the possibilities to apply the Lyapunov to solution stability of some differential equation (DEq) to linear approximation for this DEq.

Keywords: abstract parabolic differential equations; asymptotic stability; stability by Lyapunov; two reduction methods

2000 MSC: 47J35, 37K45.

Nonhyperbolic singularities of Bogdanov-Takens type in an epidemic model

M. Trifan, G.-V. Cîrlig, D. Lascu

This paper represents a continuation of our previous study [1] on the epidemic model of Kermack and McKendrick involving two individuals populations, namely the susceptibles and the infectives. The normal form for the double zero singularity when some parameter vanishes is deduced by using the method in [2] and the phase portrait for this case is sketched.

Keywords: epidemic model, normal form, degenerated singularity.

References
1. M. P. Trifan, G. V. Cîrlig, *Saddle-node bifurcation in an epidemic model*, The 12-th Conf. on Applied and Industrial Mathematics, Pitesti, 2004.
2. S. N. Chow, C. Li, D. Wang, *Normal form and bifurcation of planar vector fields*, Cambridge University Press, 1994.

Economical problems and conditioned optimization

R. Ţurcan, C. Ţurcan

Many problems on economics concern the optimization of some objective functions, with a multitude of admissible solutions. Problems regarding the condition optimization may be grouped into two categories:

1) conditioned optimization - with the so-called "equality restrictions";

2) conditioned optimization - with the so-called "inequality restrictions". We also refer to the general case together with a concrete application, for the two types of conditioned optimizations - equality and inequality restrictions.

Quasicomplete study of cycloid

R. Ţurcan, C. Ţurcan

Some of the main aspects concerning the study of plane curves - the object of the differential geometry of plane curves are presented. After a brief theoretical presentation, we discuss: the

analytical representation of plane curves;

charting the plane curve;

calculating the main elements when studying the curve.

Then we perform a quasi complete study of a representative plane curve, which have important applications, namely the cycloid.

Mathematical-physical descriptions of the main processes occurring in the transport of pollutants into aquifers

R. Ţurcan, C. Ţurcan

In this paper we are concerned with: underground water pollutions by a serial characteristics magnitudes (porous medium mass - for elementary volume, velocity of filtration, absolute and effective pore, permeability); fundamental equations of fluids flow through a porous medium (Darcy's law and its generalization, equation of continuity for the general case and for porous medium); transport equation of aquifer pollution (equation deduction and its solution at infinity, for the one dimensional case).

Solution principles for simultaneous and sequential games mixture

V. Ungureanu

We study various equilibria principles for strategic form games that involve both sequential decisions (Stackelberg game) and simultaneous decisions (Nash game, multi-criteria Pareto-Nash game) made by independent and interdependent players. Via notion of the *best response mapping graph* we define *unsafe and safe Stackelberg equilibria* for Stackelberg games, *pseudo and multi-stage Nash-Stackelberg equilibria* for Nash-Stackelberg games, and *Pareto-Nash-Stackelberg equilibria* for multi-criteria Pareto-Nash-Stackelberg games.

At every stage (level) of the Nash-Stackelberg game a Nash game is played. Stage profiles (joint decisions) are executed sequentially throughout the hierarchy as a Stackelberg game.

At every stage of the multi-criteria Pareto-Nash-Stackelberg game a multi-criteria Pareto-Nash game is played. Stage profiles are executed sequentially throughout the hierarchy.

Existence theorems are proved. Various properties are revealed. Some illustrative examples are given.

The carried out investigation continues and extends the Nash game research via Nash equilibrium as an element of the *best response mapping graphs intersection* [1,2]. It is significant that this research permits, among others, to look at a classical dynamic programming from a new perspective.

References

1. V. Ungureanu, *Nash equilibrium set computing in finite extended games*, Computer Science Journal of Moldova, **14**, 3(42)(2006), 345-365.

2. V. Ungureanu, *Nash equilibrium conditions for strategic form games*, Libertas Mathematica, **27**(2007), 131-140.

Algebraic concepts in history of mathematics

I. Valuţă

Among the three basic compartments (arithmetic, geometry and algebra) of mathematics, the latter is considered the youngest. This fact can be accepted only if we admit the idea that algebra begins with Diophant (the III-rd century), and Al-Horezmi (787-850) [1] and appears as a theory of equations and not from the point of view of the appearance of the first notions, rudimentary algebraic concepts which refer to the most ancient times, with a wide influence upon the whole mathematics. From the multitude of algebraic concepts - operation (composition laws), numbers of different types, relations etc.- we mention here two, which can be attributed the leading place, namely the concept of a symbol (a fundamental concept in the theory of cognition), implying calculus in the most general meaning of this word, and that of a unknown quantity, implying the notion of equation. In algebra, whose periods of development enframe in the periods mathematics development [2], an important role is played by the evolution of symbolism. In his work [3], C.Boyer mentions three important stages in symbolism development: (1) the rhetorical stage; (2) a syncopated stage in which some abbreviations are adopted; and (3) the symbolic final stage that sees its culmination in the work of Leibniz. Here is required, at least, a completion. No matter how important the role of symbolism is, the biggest turn in the history of mathematics of the XVII-th century is due to the innovations of François Viète, who, with his logistica speciosa made possible the discovery of analytic geometry and mathematic analysis and, later on, that of formalism. Due to the concept of unknown quantity (which is also a symbol), until the XIX-th century, algebra was considered exclusively as a theory of (algebraic) equations and, further on, again due to the algebraic concepts (Evariste Galois being the pioneer), mathematics became the theory of structures axiomatically defined.

References:

1. J. Derbyshire, *The father of algebra*, 2006.
2. M. Cioban, I. Valuta, *Elemente de istorie a matematicii şi matematica în Republica Moldova*, Chişinău, 2006.
3. C. Boyer, *Revival and decline of Greek mathematics*, 1991.

Some classes of the solutions of gas dynamic equations

P. A. Velmisov, J. A. Kazakova

The solutions of gas dynamics equations are constructed in terms of the parametric method, their classification is carried out, and the applications are showed for solutions of some specific physical problems. The differential equation system is considered. In this system decision functions u_k very depending on coordinates x, y and time t

$$F_k(x,y,t,u_1,\dots,u_n,u_{1x},\dots,u_{nx},u_{1y},\dots,u_{ny},u_{1t},\dots,u_{nt})=0, k=1\div n \quad (1)$$

The solution of (1) is formulary $u_k = u_k(\xi,\eta,t), k = 1 \div n, x = x(\xi,\eta,t), y = y(\xi,\eta,t)$. The conversion formulas are

$$u_{kx}=\frac{u_{k\xi}y_\eta-u_{k\eta}y_\xi}{\Delta},\ u_{ky}=\frac{u_{k\eta}x_\xi-u_{k\xi}x_\eta}{\Delta},\ u_{kt}=u_{kt}+\frac{u_{k\xi}(y_tx_\eta-y_\eta x_t)+u_{k\eta}(y_\xi x_t-y_tx_\xi)}{\Delta},$$

where $\Delta = x_\xi y_\eta - x_\eta y_\xi neq 0$. The system (1) is rearranged as

$$F_k(\xi,\eta,x,y,t,x_\xi,x_\eta,x_t,y_\xi,y_\eta,y_t,y_1,\dots,u_n,u_{1\xi},\dots,u_{nt})=0 \quad (2)$$

In (2) x, y, u_k are functions of variables ξ, η, t. The solution of (2) is found in the form of polynomials of η degrees

$$u_k=\sum_{i=0}^{\alpha k}u_{ki}(\xi,t)\eta^i, x=\sum_{k=0}^{\gamma}x_k(\xi,t)\eta^k, y=\sum_{k=0}^{\omega}y_k(\xi,t)\eta^k$$

where $\alpha_k, \gamma, \omega \in N$ (N is the set of natural numbers). The method of definition of parameters $\alpha_k, \gamma, \omega \in N$ is offered for quasi-linear equations of first order, in which coefficients are polynomials of dependent and independent variables. The differential equation systems for $x_k(\xi,t), y_k(\xi,t), u_{ki}(\xi,t)$ is determined or underdetermined for these parameters. The classification is carried out with a view to determinacy, and some solutions are constructed for the following systems of equations

a) $\left\{ u_t + uu_x + vu_y = -\frac{1}{\rho}p_x, v_t + uv_x + vv_y = -\frac{1}{\rho}p_y, u_x + v_y = 0, \right.$

b) $\left\{ \begin{array}{l} u_t + uu_x + vu_y = -\zeta\omega_x, v_t + uv_x + vv_y = -\zeta\omega_y, \\ \omega(u_x + v_y) + \mu(\omega_t + u\omega_x + v\omega_y) = 0, \end{array} \right.$

c) $\left\{ \begin{array}{l} u_t + uu_x + vu_y = -2c\rho_x, v_t + uv_x + vv_y = -2c\rho_y, \\ \rho_t + u\rho_x + \rho u_x + v\rho_y + \rho v_y = 0, \end{array} \right.$

d) $\left\{ \begin{array}{l} \rho(u_t + uu_x + vu_y) = -p_x, \rho(v_t + uv_x + vv_y) = -p_y, \\ \rho_y + (\rho u)_x + (\rho v)_y = 0, \\ \rho(p_t + up_x + vp_y) - \gamma p(\rho_t + u\rho_x + v\rho_y) = 0, \end{array} \right.$

e) $\left\{ u_t + uu_x - vu_y = 0, u_y - v_x = 0. \right.$

Here $u(t, x, y), v(t, x, y)$ are projections of the velocity vector, $\omega(t, x, y) = \rho^{x-1}$, and $\rho(t, x, y)$ is a pressure. Particularly, the solutions of simple and dual wave are derived. The solutions, describing gas flows with local supersonic zones in Laval nozzles, are constructed too.

Stability of elastic elements of wing profile with time delay of bases reactions

P. A. Velmisov, A. V. Ankilov

Dynamic stability of elastic elements (plates in this work) of wing profile in interaction with flow of fluid or gas is studied. Subsonic regime is considered. The definition of elastic body stability corresponds to Lyapunov's concept of dynamical system stability. Aerodynamic load is determined by asymptotic aerohydromechanics equations [1]. Statements and investigation methods offered for dynamical damping elastic bodies, being in contact with subsonic flow of the fluid or gas, lead to the study of linked initial boundary problems to partial differential equation or studied of partial differential equations. The problems are studied with consideration of time delay of the elements bases reactions, so these equations are the equations with retarded argument. Being based on the construction of functionals, corresponding to this system, conditions of solutions stability are obtained for some aerohydroelastical problems, in particular for dynamics of the plate elements; of elements of the plane channel through which fluid flows; of wing profile elements. Similar problems without time delay of the plates bases reactions were earlier considered in [2-4].

In the linear statement corresponding to small perturbations of homogeneous fluid stream directed along the x - axis and to small deviations of the wing elements this problem is formulated as

$$\triangle\varphi \equiv \varphi_{xx} + \varphi_{yy} = 0,\ (x, y) \in R^2 \setminus [a_1, a_{2n}],$$

$$\varphi_y^{\pm}(x, 0, t) = \dot{w}_k(x, t) + V w_k'(x, t),\ x \in (a_{2k-1}, a_{2k}),\ k = 1 \div n,$$

$$\varphi_y^{\pm}(x, 0, t) = V f_k^{\pm\prime}(x),\ x \in (a_{2k}, a_{2k+1}), k = 1 \div n-1,$$

$$|\bigtriangledown\varphi|_\infty^2 = (\varphi_x^2 + \varphi_y^2 + \varphi_t^2)_\infty = 0,$$

$$L_k(w_k) = \rho\left(\varphi_t^+ - \varphi_t^-\right) + \rho V\left(\varphi_x^+ - \varphi_x^-\right),\ y = 0,\ x \in (a_{2k-1}, a_{2k}),\ k = 1 \div n,$$

$$L_k(w_k) \equiv \left[D_k w_k''(x,t) + \beta_{2k}\ \dot{w}_k''(x,t)\right]'' + M_k \ddot{w}_k(x,t) +$$

$$+ \left(N_k(t) w_k'(x,t)\right)' + \beta_{1k}\dot{w}_k(x,t) + \beta_{0k} w_k(x, t-\tau).$$

Here $\varphi(x,y,t)$ is the potential of fluid velocity; $w_k(x,t)$ are the plate oscillations functions; $f_k^{\pm}(x)$ are given functions determining the form of undeformable parts of a wing; τ is time of the delay of the elements bases reactions.

References

1. P. A. Vel'misov, *Asymptotic equations of gas dynamics*, Russia, Saratov University, 1986, 135.
2. P. A. Vel'misov, Yu. A. Reshetnikov, *Stability of viscoelastic plates at aerohydrodynamic effect*, Russia, Saratov University, 1994, 176.
3. A. V. Ankilov, P. A. Vel'misov, *Stability of viscoelastic elements of chanals walls*, Russia, Ulyanovsk Technical University, 2000, 115.
4. A. V. Ankilov, P. A. Vel'misov, N. A. Degtyareva, *Stability of elastic elements of wing profile*, Applied Mathematics and Mechanics, **7**, Russia, Ulyanovsk Technical University, 2007, 9-18.

Petri nets as models for concurrent and distributed systems

C. Vidrașcu

An important area of research in computer science is that of models for concurrent and distributed systems. Finding suitable mathematical models is important in order to cope with the increasing complexity of real systems and for studying their properties. At the international level there is a wide range of studies dedicated to this topic and among the most frequent approaches we can enumerate: Petri nets, process calculi (CSP, CCS, ACP, π-calculus etc.), abstract state machines and temporal logic (LTL, CTL, TLA etc.).

Petri nets were introduced by C. A. Petri in the early 1960s as a graphical and mathematical tool for modelling concurrent/distributed systems. They are a suitable formalism for describing and studying information processing systems that are characterized as being concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic. As a graphical tool, Petri nets can be used as a visual-communication aid similar to flow charts, block diagrams, and networks. As a mathematical tool, it is possible to set up state equations, algebraic equations, and other mathematical models governing the behaviour of systems.

Petri nets have been proposed for a very wide variety of applications because of their generality and adaptability. They have been successfully used for concurrent and parallel systems modelling and analysis, communication protocols, performance evaluation and fault-tolerant systems.

Due to their numerous applications in areas like engineering, economics, medicine, education and science, Petri nets emerged as a very prolific research field soon after their introduction. At international level the Petri nets research is materialized in a large number of publications in prestigious journals and conference proceedings. Among the most important research centers where Petri nets are studied one can mention: University of Hamburg (Germany), Technical University of Munich (Germany), Humboldt University of Berlin (Germany), University of Aarhus (Denmark), Eindhoven University of Technology (The Netherlands) etc.

References
1. J. L. Peterson, *Petri net theory and the modeling of systems*, Prentice-Hall, New Jersey, 1981.
2. W. Reisig, *Petri nets. An introduction*, EATCS Monographs on Theoretical Computer Science, W.Brauer, G. Rozenberg, A. Salomaa (eds.), Springer, Berlin, 1985.
3. J. Desel, W. Reisig, G. Rozenberg (eds.), *Lectures on concurrency and Petri nets*, Advances in Petri Nets, LN in Computer Science, **3098**, Springer, New York, 2004.

Topics in asynchronous systems theory ∩ dynamical systems theory

S. E. Vlad

The asynchronous systems are the models of the asynchronous circuits from the digital electrical engineering. The naturalness of the import in the autonomous asynchronous systems theory (real time, binary spaces, no input and non determinism) of some elementary notions from the dynamical systems theory: flows, orbits, nullclines, fixed points, ω-limit points, reccurent points, periodicity, invariant sets, dependence on the initial states, transitivity, attraction, attractors, chaos, equivalence, static and dynamic bifurcation, symmetry- is given by the existence of a generator function $\Phi : \{0,1\}^n \to \{0,1\}^n$ (that is called network function by Moisil) and of a vector field $f : \mathrm{R}^n \to \mathrm{R}^n$ that play the same role in the two theories. The fundamental ideas of these theories coincide to some extent and the tools of analysis are different, in the sense that the asynchronous systems do not have linlarizations, Jacobians, multipliers, Poincaré maps etc. The purpose of the paper is that of browsing in the context of the autonomous asynchronous systems some usual concepts of the dynamical systems theory.

Tradition and modern in evaluation in mathematics

C.-L. Voica, M. Angheluță

Comparing curricular objectives with evaluation objectives, as a starting point, we present traditional and modern evaluation methods in mathematics. We make a critical analysis concerning the proposed problems in high school national tests. Some improvements on the subject are proposed too.

References
1. ***, *Tezele cu subiect unic pentru clasele a VII-a şi a VIII-a*, MECT, CNCE, 2007, 2008.
2. C. Cucoş, *Teoria şi metodologia evaluării*, Ed. Polirom, Iaşi, 2008.
3. V. Frunză, all., *Programul naţional de dezvoltare a competenţelor de evaluare ale cadrelor didactice. Suport de curs*, CNCIP, 2008.

Difficulties in formulating "good problems"

C.-L. Voica, C. Voica

In our paper we report on difficulties to propose problems. Our research started from the following questions: How to generate new problems? How to reformulate a given problem? How to develop learning contexts? Our analysis shows that, in spite of the common perception, there are a lot of difficulties in generating and formulating problems. A particular difficult point seems to be the question of the problem: it is difficult to formulate interesting questions in a given situation.

References

1. S. I. Brown, M. I. Walter, *The art of problem posing*, 2nd ed., Hillsdale, NJ, Erlbaum, 1990.
2. M. C. Ramirez, *A mathematical problem-formulating strategy*, Int. J. for Mathematics Teaching and Learning, December, 2006.
3. L. D. English, *Children's problem posing within formal and informal contexts*, J. for Research in Mathematics Education, **29**, 1, (1998), 83-106.
4. I. Pelczer, F. Gamboa, *Factores de dificultad de los problemas con sucesiones e implicaciones para el diseo de cursos de clculo en lnea*, Proc. Virtual Educa' 07, 2007, Brasil.
5. I. Pelczer, C. Voica, F. Gamboa, *Problem posing strategies of first year mathematics students*, Proc. of the Joint Meeting of PME 32 and PME-NA XXX, **4**, 97-104.

Romanian research in mathematics education - a cinderella?

C. Voica

Mathematics education refers both to the practice of teaching and learning mathematics, as well as to a field of research on this practice. Mathematics education research has developed into a field of study, with specific theories, methods, and concepts. This article presents some personal points of view concerning research in mathematics education in Romania. We present some possible research themes and results in the area too.

References

1. http://ams.org/msc/
2. A. G. Howson, *Seventy-five years of the International Commission on Mathematical Instruction*, Educational Studies in Mat., **15** (1984) 75-93.
3. M. Singer, C. Voica, *Perception of infinity: does it really help in problem solving?*, in Proceedings of the International Conference "The Decidable and the Undecidable in Mathematical Education", Brno, Czech Republic, 19-25, 2003, 252-256.
4. M. Singer, C. Voica, *Children's perceptions on infinity: could they be structured?*, Proceedings CERME 5, Larnaca, Cyprus, 22- 26(2007), 506-515.
5. M. Singer, C. Voica, *Extrapolating rules: how do children develop sequences?*, Proceedings PME 32, Morelia, Michoacn, Mexico, 17th - 21st, **4**, 2008, 265-272.
6. M. Singer, C. Voica, *Between perception and intuition: thinking about infinity*, to appear in Journal of Mathematical Behavior.

An optimal control problem

N. Vornicescu

In previous papers various forms of the student optimal problem were studied. In the paper we consider the case $min \int_0^1 a(t)u^2(t)dt$ subject to $\int_0^1 b(t)u(t)dt = S$, $0 \leq u(t) \leq B \ fort \in [0,1]$, $u \in C[0,1]$, where a,b are given continuous functions verifying: $a(t) > 0, b(t) > 0$ for $t \in [0,1]$ and $B > 0, S > 0$ are given real numbers.

References

1. M. S. Klamkin, *Mathematical modelling: A student optimal control problem and extensions*, Math. Modelling, **6**(1985), 49-64.
2. I. Muntean, N. Vornicescu, *Estrategias de aprendisaje en tiempo minimo*, Metodos Matematicos Aplicados a las Ciencias, VII, VIII Simposios, Editorial de la Universidad de Costa Rica, 1994, 183-138.
3. G. Ragget, P. Hempson, K. Jakes, *A student related optimal control problem*, Bull. Inst. Math. Appl., **17**(1981), 133-138.
4. N. Vornicescu, *A generalized student optimal problem*, PU.M.A., **15**(2004), 349-356.
5. N. Vornicescu, *A continuous case of student optimal control problem*, ACAM, **15**(2006), 325-330.
6. W. Woodside, *A student optimal control problem or how to pass courses with the minimum expediture of effort*, Appl. Math. Notes, Canad. Math. Soc., **7**(1982), 2-13.

Generalized quaternions, corresponding to them system of equations and applications

M. Zakhirov

In topology it is proved that every two-dimensional compact, connected and oriented manifold is diffeomorphic to the sphere with $n \geq 0$. Three-dimensional manifolds are investigated very little. For example, it was not yet proved if every compact one-connected three-dimensional manifold is diffeomorphic to the sphere S^3 (Poincaré conjecture) or at least homeomorphic to it [1]. Poincaré conjecture [2] about homeomorphism is positively solved [3].

Key words: main curvatures, mixed equations system (*MES*), manifold, fibration, layer, groups, quaternions, quasi-conformal mappings.

References

1. V. I. Arnold, *Obiknovennie differentsialnie uravneniya*, Moscow, 1976. (Russian)
2. P. Scot, The geometries of 3-manifolds. London Mathematics Society 1983; Moscow, Mir, 1986.
3. J. W. Morgan, *Recent progress on the Poincaré conjecture and the classification of 3-manifolds*, Bulletin (New Series) of the Amer. Math. Soc., **42**(2005), 57–78.

A numerical method for obtaining the generalized derivative of nondifferentiable functions

M. Zamirian, A. V. Kamyad, A. M. Vaziri

In this work, a numerical method to achieve generalized derivative for continuous functions which are nondifferentiable, is introduced. First, it is proved that the relation between a continuously differentiable function and its derivative can be shown with an infinite moment problem (IMP). Then, by approximating the IMP by a finite moment problem (FMP), we obtain the relation between a continuous and nondifferentiable function and its generalized derivative. Thus, by solving the FMP by means of a numerical method, the generalized derivative is achieved. Finally, the efficiency of our approach is confirmed by some numerical examples.

Keywords: generalized derivative; nondifferentiable functions; infinite moment problem; finite moment problem.

Equation of the constant of absorption equilibrium for the system aqueous solution naf - aluminium oxide

V. I. Zelentsov, T. Y. Datsko

The equilibrium constants are important characteristics of substances adsorbility and are used for determination of many thermodynamic functions of an adsorption process [1-4].

The adsorption equilibrium constant K_a is related to standard diminution of mole adsorption energy ($-\Delta G^o$) with an expression $K_a = e^{-\Delta G^0/RT}$, where R is the universal gas constant, $kJ/mol \cdot K$, T is the absolute temperature, ^0K. The K_a value can be also represented as ratio of activity in the surface adsorption layer of an adsorbent (a_a) to its activity in the equilibrium solution (a_e) or as the ratio of concentrations to respective activity coefficients $K_a = \frac{a_a}{a_e} = \frac{C_{ai}\cdot\gamma_{ai}}{C_{ei}\cdot\gamma_{ei}}$. Let us denote the equilibrium solution volume by x, sorbent surface adsorption layer by y, and examine the fluorine adsorption from aqueous solution on aluminum oxide [5]. This is a binary solution where water is a solvent (component 1) and NaF molecules - are a solute (component 2). The concentrations of these components (1 and 2) are expressed in mole fraction: in adsorption layer as y_i and in balk solution as x_i. Then the constant K_a reads $K_{ai} = \frac{y_{ai}\cdot\gamma_{ai}}{x_i\cdot y_i}$. For pore sorbents what Al_2O_3 is to calculate adsorption equilibrium constant from experimental data besides adsorption isotherms we need to know the total sorbent adsorption pore volume (V_a). It can be represented as a sum of partial volumes of an adsorbed substance and a solvent $V_a = a_1 \cdot v_1^0 + a_2 \cdot v_2^0$, where a_1 - the water adsorption value, mM/g, a_2 - the solute adsorption, mM/g, v_1 and v_2 - mole volumes of water and adsorbed substance (NaF).

The mole fraction NaF (y_2) in total adsorbed quantity of the solution components is $y_2 = \frac{a_2}{a_1+a_2}$, or, equivalently, $y_2 = \frac{a_2}{\frac{V_a - a_2\cdot v_2^0}{v_1^0} + a_2}$. Mole fraction of dissolved NaF in the equilibrium solution (x_2) is $x_2 = \frac{C_2}{C_1+C_2}$, where C_1 is the water concentration in equilibrium solution, mM/l, $C_1 = 55.5$ - the number of water moles in 1 liter, C_2 - NaF concentration in equilibrium solution, mM/l.

Substituting the values of mole fractions of NaF adsorbed on the sorbent surface (y_2) and in the equilibrium solution (x_2) into the expression of Kai we obtain an expression for adsorption equilibrium constant for binary solution K_{a2}

$$K_{a2} = \frac{a_2 \cdot (55.5 + C_2) \cdot \gamma_{a2}}{\left(\frac{V_a - a_2 \cdot v_2^0}{v_1^0} + a_2\right) \cdot C_2 \cdot \gamma_2}.$$

In practice it is convenient to use a dimensionless quantity θ - the sorbent surface covering degree. It represents a relation of component adsorbed volume to its total volume $\Theta_2 = \frac{a_2 \cdot v_2^0}{V_a}$. Then, taking into account that $55.5 \cdot v_1 = 1$, we get $K_{a2} = \frac{\Theta_2}{\left[1-\Theta_2\left(\frac{v_2^0 - v_1^0}{v_2^0}\right)\right] \cdot C_2 \cdot v_2^0} \cdot \frac{\gamma_{a2}}{\gamma_2}$.

Mole volumes of NaF and water are equal to 0.0217 and 0.018cm^3/mole, respectively [2]. Expressing the concentration of NaF through mM/cm^3 and entering numerical values for moles of water and NaF into the expression of Θ_2 we will obtain the final expression for adsorption equilibrium constant calculation: $K_{a2} = \frac{4.6 \cdot 10^4 \cdot \Theta_2 \cdot \gamma_{a2}}{(1-0.172 \cdot \Theta_2) \cdot C_2 \cdot \gamma_2}$.

At low surface covering degree with molecules of NaF, when $\theta_2 \to 0$ (at low NaF solution concentrations) the activity coefficients ratio γ_{a2}/γ_2 becomes equal to 1. In that case one can determine an actual value of adsorption equilibrium constant with extrapolation of the dependence $\frac{4.6 \cdot 10^4 \cdot \Theta_2}{(1-0.172 \cdot \Theta_2) \cdot C_2}$ on θ_2 to $\theta_2 = 0$. The results of the a calculation of numerical value of adsorption equilibrium constant from experimental data of NaF adsorption on Al_2O_3 is in Table 1, where

Table 1:

C_{in}, mM/l	C_2, mM/l	a, mM/g	θ_2	$\frac{4.6 \cdot 10^4 \cdot \Theta_2}{(1-0.172 \cdot \Theta_2) \cdot C_2}$	Adsorption equilibrium constant, K_{a2}
41.9	5.1	9.2	0.25	2356	3311
50.7	7.5	10.8	0.30	1940	
77.1	12.3	16.2	0.44	1604	
125.8	28.6	24.3	0.67	1217	
148.7	40.7	27.0	0.74	958	
179.2	60.8	29.6	0.81	712	
212.3	87.1	31.3	0.86	533	
234.0	105.0	33.0	0.90	467	
295.0	153.0	35.5	0.97	350	

C_{in} - the NaF solution initial concentration; C_2 - NaF solution equilibrium concentration; a - adsorption value; θ_2 - adsorbent surface covering degree.

The aluminum oxide specific surface is 358m^2/g, pore volume - 0.457cm^3/g. Adsorption capacity of the sample is equal to 36.5mM/g in relation to NaF.

The obtained value of adsorption equilibrium constant can be used for calculation of thermodynamic functions: free mole adsorption energy ($-\Delta G^0$), standard mole enthalpy (ΔH^0), and entropy ($-\Delta S^0$) of adsorption what is of sufficient interest for adsorption technology.

References

1. Y. I. Gerasimov, *Course of physical chemistry*, Moscow: Khimiya, **2** (1973), 623.(Russian)
2. A. M. Koganovskii, T. M. Levchenko, V. A. Kirichenko, *Adsorbtsiya rastvorennykh veshchestv* (Adsorption of Dissolved Materials), Kiev: Naukova Dumka, 1977, 312.
3. Subhashini, Ghorai, K. K. Pant, *Equilibrium, Kinetics,and Breakthrough Studies for Adsorption of Fluorideon Activated Alumina*, Separation and Purification Technology, **42**, 3(2005), 265-270.
4 Ch. Giles , B. Ingram , J. Cluny, *Adsorption from solutions on solid surfaces*, Mir, Moscow, 1986, 488.
5. V. I. Zelentsov, T. Ya. Datsko, E. E. Dvornikova, *Temperature influence on the Equlibrium and kinetics of fluoride-ion adsorption by heat-treated Al Oxyhydrates*, Surface Engineering and Applied Electrochemistry, **44**, 3(2008), 206-213.