CAIM 2017

14th–17th SEPTEMBER, 2017
Iaşi, ROMANIA

Book of Abstracts

Organized by

"Alexandru Ioan Cuza" University of Iaşi	**ROMAI**	**"Gh. Mihoc- C. Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy**
Institute of Computer Science of the Romanian Academy	**AOŞR**	**Iaşi City Hall**

Volume Editor: Stelian Ion

Printed in Romania: PIM Publishing House
Adress: 11, Ştefan cel Mare St., Iaşi, Romania
General Manager: Maria Pietraru
e-mail adress: editura@pimcopy.ro, tel. +40730086676

Scientific Commitee

Organizing Commitee:

President: Costică Moroşanu
Mitrofan Choban
Călin Ioan Gheorghiu
Stelian Ion

Vice President: Tudor Barbu
Anca Veronica Ion
Carmen Rocşoreanu
Şerban Vlad

Local Organizing Commitee: "Alexandru Ioan Cuza" University of Iaşi

Costică Moroşanu
Tudor Barbu
Dorina Ticu
Andrea Grăjdeanu

Anca Croitoru
Silviu Pavăl
Carmina Manole

Contents

Plenary talks

Nonlinear Diffusion Flows

Viorel Barbu

"Alexandru Ioan Cuza" University of Iaşi, Romania
"Octav Mayer" Institute of Mathematics of Romanian Academy, Iaşi Branch, Romania
e-mail: vb41@uaic.ro

This talk surveys a few nonlinear diffusion problems governed by nonlinear parabolic equations.

Parallel Decompositions of Pairs of String and their Applications

Mitrofan M. Choban[1] and Ivan A. Budanaev[2]

[1] *Tiraspol State University, Republic of Moldova*
[2] *Institute of Mathematics and Computer Sciences of ASM*
e-mail: mmchoban@gmail.com, ivan.budanaev@gmail.com

A monoid is a semigroup with an identity element. Fix a non-empty set A. The set A is called an alphabet. Let $L(A)$ be the set of all finite strings $a_1a_2\dots a_n$ with $a_1, a_2, \dots, a_n \in A$. Let ε be the empty string. Consider the strings $a_1a_2\dots a_n$ for which $a_i = \varepsilon$ for some $i \le n$. If $a_i \ne \varepsilon$, for any $i \le n$ or $n = 1$ and $a_1 = \varepsilon$, the string $a_1a_2\dots a_n$ is called a *canonical string*. The set $Sup(a_1a_2\dots a_n) = \{a_1, a_2, \dots, a_n\} \cap A$ is the support of the string $a_1a_2\dots a_n$ and $l(a_1a_2\dots a_n) = |\{i \le n : a_i \ne \varepsilon\}|$ is the length of the string $a_1a_2\dots a_n$. For two strings $a_1\dots a_n$ and $b_1\dots b_m$, their product(concatenation) is $a_1\dots a_nb_1\dots b_m$. If $n \ge 2$, $i < n$ and $a_i = \varepsilon$, then the strings $a_1\dots a_n$ and $a_1\dots a_{i-1}a_{i+1}\dots a_n$ are considered equivalent. In this case any string is equivalent to one unique canonical string. We identify the equivalent strings. In this case $L(A)$ becomes a monoid with identity ε. We put $\bar{A} = A \cup \{\varepsilon\}$. The set $\bar{A}$ is called an alphabet. If the strings a and b are equivalent, then the string b is a representation of the string a.
Let $d : \bar{A} \times \bar{A} \to \mathbb{R}$ be a mapping such that for all $x, y, z \in \bar{A}$ we have: $d(x, y) \ge 0$; $d(x, y) + d(y, x) = 0$ if and only if $x = y$; $d(x, z) \le d(x, y) + d(y, z)$. Then d is called a *quasimetric* on $\bar{A}$.
Let G be a semigroup and d be a pseudo-distance on G. The pseudo-distance d is called invariant if $d(xa, xb) \le d(a, b)$ and $d(ax, bx) \le d(a, b))$ for all $x, a, b \in G$.
Theorem 1. *For any quasimetric d on the alphabet $\bar{A}$ there exists an invariant quasimetric d^* on the monoid $L(A)$ with following properties:*

1. *$d^*(x, y) = d(x, y)$ for any $x, y \in \bar{A}$.*
2. *If ρ is an invariant quasimetric on $L(A)$ and $\rho(x, y) \le d(x, y)$ for all $x, y \in \bar{A}$, then $\rho(x, y) \le d^*(x, y)$ for all $x, y \in L(A)$.*
3. *If $a = a_1a_2...a_n$ and $b = b_1b_2...b_n$ are representations of the two strings $a, b \in L(A)$, then $d^*(a, b) \le \Sigma\{d(a_i, b_i) : i \le n\}$*
4. *For any two strings $a, b \in L(A)$ there exist $n \ge 1$ and the representations $a = a_1a_2...a_n$ and $b = b_1b_2...b_n$ such that $d^*(a, b) = \Sigma\{d(a_i, b_i) : i \le n\}$.*
5. *If d is a metric, then d^* is a metric too.*

Let $a, b \in L(A)$. The representations $a = a_1a_2...a_n$ and $b = b_1b_2...b_m$ are called parallel decompositions if $n = m$. The representations $a = a_1a_2...a_n$ and $b = b_1b_2...b_m$ are called parallel d-optimal decompositions of the pair of strings a, b if $n = m$ and $d^*(a, b) = \Sigma\{d(a_i, b_i) : i \leq n\}$. There exist algorithms of construction parallel d-optimal decompositions of the given pair of strings $a, b \in L(A)$ [1, 2].
The set $M_d(a, b) = \{x \in L(A) : d^*(a, b) = d^*(a, x) + d^*(x, b)\}$ is called the set of weighted means of the pair of strings $a, b \in L(A)$. Fix d-optimal decompositions $a = a_1a_2...a_n$ and $b = b_1b_2...b_n$. Let B be a non-empty subset of the set $\{i \leq: a_i \neq b_i\}$. Put $c = c_1c_2...c_n$, where $c_i = a_i$ for $i \notin B$ and $c_i = b_i$ for $i \in B$. Then $d^*(a, b) = d^*(a, c) + d^*(c, b)\}$ and $c \in M_d(a, b)$. All elements from $M_d(a, b)$ can be constructed in this way.
The set $B_d(a, b) = \{x \in L(A) : d^*(a; x) = d^*(x, b)\}$ is called the bisector of the pair of strings $a, b \in L(A)$. We propose method of construction of representations of the strings from $B_d(a, b)$ with respect to the optimal parallel decompositions of the strings a and b. As in [2] we examine some penalties of the given d-optimal decompositions $a = a_1a_2...a_n$ and $b = b_1b_2...b_n$.

Bibliography

[1] I. Budanaev. *On Hamming type Distance Functions.* ROMAI J., v.12, no.2(2016), 25-32

[2] M.M. Choban, I.A. Budanaev. *About Applications of Distances on Monoids of Strings.* Computer Science Journal of Moldova, vol. 24, no 3 (2016), pp. 335-356.

European collaboration in industrial and applied mathematics

Dietmar Hoemberg

Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstr. 39, 10117 Berlin Germany

e-mail: `hoemberg@wias-berlin.de`

In the first part of my presentation I will give an overview of recent activities of the mathematics community for a European collaboration in Industrial and Applied Mathematics. After a short review of mission and interaction of key institutions like ECMI and EU?MATHS?IN, I will discuss activities and funding opportunities of the Cost Action "Mathematics for industry network (MI-NET)".
In the second part of my talk I will present some scientific results from the European Industrial Doctorate network "MIMESIS- Mathematics and Materials Science for Steel Production and Manufacturing".

Multi-level Monte Carlo methods for Uncertainty quantification two-phase flow in porous media

Rolf Jeltsch

SAM, ETH Zrich, Switzerland
Mathematics Hong Kong Baptist University
e-mail: `rolf.jeltsch@sam.math.ethz.ch`

Two-phase flow models are used to understand the extraction of oil or gas in porous media or store Carbon Dioxide in empty reservoirs. Mathematically the models consist of a system of nonlinear partial differential equations. The saturation equation is a nonlinear hyperbolic conservation

law if the capillary pressure is neglected while the pressure is modeled using an elliptic equation. Some open problems in the theory will be mentioned. Many physical input variables, e.g. rock permeability, realtive permaebilty are determined by measurement processes and hence are prone to uncertainty. Hence one needs to model uncertainty to be able to compute the effects. For an efficient code one uses the Multi-level Monte Carlo method. This is a joint work with Vitor Alves Pires at ICMC USP.

The Cahn-Hilliard equation in image inpainting

Alain Miranville

University of Poitiers, France
e-mail: miranv@math.univ-poitiers.fr

Our aim in this talk is to discuss variants of the Cahn-Hilliard equation in view of applications to image inpainting. We will present theoretical results as well as numerical simulations.

A Survey on the Best Approximation by Splines

Vasile Postolică

"Faculty of Sciences, Vasile Alecsandri" University of Bacău, Romania
Romanian Academy of Scientists
e-mail: vpostolica@ambra.ro

As a part of the Approximation Theory and its Applications in Vector Spaces, approaches to achieve the Best Approximation by splines are reviewed in this research work, starting from the background of the linear spaces, followed by Hausdorff locally convex spaces, the normed linear spaces and the particular case of Hilbert spaces. Special attention is given to the most significant Best Approximation Problems in Separated Locally Convex Spaces and to their straight connections with the Vector Optimization, that is, with the general Efficiency. Remarkable consideration is given to these problems in H - locally convex spaces where the splines introduced by an original method represent the best approximation simultaneous and vectorial solutions as optimal interpolation elements. The survey covers illustrative references.
Mathematics Subject Classification 2010 : Primary: 41A65; Secondary: 41A99.
Keywords : *locally convex space, best approximation, general efficiency, Isac's cone, spline function, H – locally convex space.*

Bibliography

[1] Isac, G., Postolică, V. – *The Best Approximation and Optimization in Locally Convex Spaces,* Verlag Peter Lang GmbH, Frankfurt am Main, Germany, 1993.

[2] Postolică, V. – *A Method which Generates Splines in H - Locally Convex Spaces and Connections with Vectorial Optimization,* Positivity, 2 (4), 1998, p. 369 - 377.

[3] Postolică V. – *Spline Approximation of Countable Additive Set Functions of Bounded 2 – Variation.* Communication to The Third International Symposium on Orthogonal Polynomials and Their Applicatonsb Erice, Italy, June 1 - 8, 1990. Published by "Babeş – Bolyai" University, Cluj – Napoca, Romania, Faculty of Mathematics and Informatics, Seminar on Mathematical Analysis, Preprint 7/1990, p. 63 – 76.

[4] Postolică V. – *On the Best Approximation of Set Functions.* Proceeding of The Fourth International Conference on Functional Analysis and Approximation Theory, Acquafreda di Maratea, Potenza - Italy, September 22 - 28, 2000. Published in Supplemento ai Rendiconti del Circolo Matematico di Palermo, serie II, no 68, 2002, p. 761 - 775.

[5] Postolică, V. – *Isac's Cones in General Vector Spaces.* Published as Chapter 121 of Encyclopedia of Business Analytics and Optimization 5 Vols, IGI Global, 2014, p. 1323 – 1342.

[6] Precupanu, A. M., Precupanu, T. – *Some Kinds of Best Approximaion Problems in a Locally Convex Space.* An. Univ. "Al. I. Cuza" Iaşi, Romania, XLVIII, 2002, p. 409 - 421.

Approximators and interpolators with fuzzy logic systems. Applications

Horia-Nicolai L. Teodorescu

Dept. ETTI, Gheorghe Asachi Technical University of Iasi, Iasi, Romania
Romanian Academy, Iasi Branch, Iasi, Romania
e-mail: `hteodor@etti.tuiasi.ro`

There are plentiful classes of *universal approximators*, that is, classes of functions that can approximate with any error a specified class of functions. The class of approximated functions is frequently chosen as continuous, bounded functions of a bounded interval of the real axis. Generalizations of the approximations to R^n are usually trivial. Probably, among the best known classes of universal approximators are trigonometric functions (Fourier series), Legendre and Chebyshev polynomials, spline functions, and wavelets. With the advent of machine learning (ML), the interest in universal approximators was revived, with the attention turned toward approximating function classes (approximators) that can be used in supervised and unsupervised algorithms, instead of tediously solving systems of equations. Many ML methods were first used without much care for their ability to approximate functions from general classes; that was theoretically unsatisfactory, because of the lack of guarantee that the algorithm can solve the problems they were trained to solve. Although it is seldom stressed in textbooks, it is clear now that the issue of universal approximators for ML algorithms is a cornerstone problem in ML.
Surprisingly, it seems that the most successful ML approach to date, *deep learning* networks, a class of NNs with neurons with rectifying (non-sigmoid) function is not yet proved to be an universal approximator. An early result has shown that artificial neural networks with sigmoid functions for the neurons are universal approximators. A second set of results, published since the early 1990s until recently proved that there are several subclasses of fuzzy logic systems (FLSs) that are equally universal approximators. The issue of exact interpolators through numerable sets of points is also recollected, showing how to automatically built interpolators with FLSs. In this talk, I will review several results on universal approximators pertaining to the field of ML, revise and extend some of them, especially for FLSs, compare approximators based on FLSs with other classes from the viewpoint of ML, and provide examples of applications from the literature and some new ones in engineering and bio-medical engineering.

Treatise on Differential calculus and Integral calculus for mathematicians, physicists, chemists and engineers in ten volumes

Constantin Meghea, Irina Meghea

This treatise written by Constantin Meghea and Irina Meghea appeared in Old City Publishing, Philadelphia and Éditions des Archives Contemporaines (Vol. 1-3 in 2013, Vol. 4-8 in 2014 and Vol. 9 and 10 in 2015). It is originated from the course of *Mathematical Analysis* delivered by Constantin Meghea along 10 years at the University POLITEHNICA of Bucharest and also from his scientific seminar *Functional Analysis and Applied Mathematics* held for teachers and engineers at the same university.

Based on the Zermelo - Fraenkel axiom system while the natural numbers are defined by Peano axioms, this treatise represents a modern, rigorous and unitary exposure of differential and integral calculus in $\mathbb{R}^n$ and in $\mathbb{C}$, with flights to general topology and functional analysis, massively implemented, and to integration on σ-algebras as well. The treatise is also self-contained: except for the chapter O and for some isolated cases, every concept used is defined, every proposition used is proven. A special and continuous attention is paid to the concepts, propositions and methods which are at the basis of applied mathematics in natural sciences and engineering.

The Lebesgue measure and integral are, probably, the most important creation in the mathematics of the XXth century. Together with their axiomatization − the integration with respect to a measure on a σ-algebra, they have directly, and indirectly through integration on differential sub-manifolds of $\mathbb{R}^n$, a huge impact on natural sciences. Thus, presentation of these theories takes into account for this impact. Moreover, we note that "integration on differential manifolds" has been a ticklish problem. To get together rigor, intuition and at all simple possible proofs and to balance them within the above mentioned subject is a difficult task.

The abstract character of the treatise is strongly tempered on one hand by a torrent of solved exercises and also by figures, these clarifying the concepts and being their intuitive support, and, on the other hand, by numerous applications. To some of them the functional analysis itself offers simple and powerful techniques.

This book in English completes and deepens, over 3500 pages, contained in ten volumes, the unique original approach of the mathematical analysis through its prism of "key that opens the door to the secrets of natural sciences". It is a work of large scale, expressing a special position toward the mathematical analysis, its approach and revealing its crucial role in understanding and solving the problems that arose from reality. The work is distinguished by clarity of explanations, the ease with which the most difficult concepts are introduced, the abstract character being tempered by the impressive number of applications in mathematics, physics, chemistry and engineering.

Abbreviated contents:

FIRST PART: DIFFERENTIAL CALCULUS

Chapter O. Preliminaries to the treatise

Chapter I. Series of complex numbers

Chapter II. Elements of general topology

Chapter III. Elements of functional analysis

Chapter IV. Differential calculus in $\mathbb{R}^n$ and in normed spaces

SECOND PART: INTEGRAL CALCULUS

Chapter V. Uniform limit, generalized Riemann integral, Cauchy integral and complex analysis (one variable)

CHAPTER VI. Riemann integral in $\mathbb{R}^n$

CHAPTER VII. Curvilinear integral in $\mathbb{R}^n$ and integral on smooth surface from $\mathbb{R}^3$

CHAPTER VIII. Lebesgue integral in $\mathbb{R}^n$, abstract integral, integral on differential manifold from $\mathbb{R}^n$, spaces $L^p(X, \mu)$ and harmonic analysis

CHAPTER IX. Completions on general topology, functional analysis and harmonic analysis

Mini Symposium: Biomathematics

Mini-Symposium: Biomathematics

organizers

Gabriel Dimitriu and Paul Georgescu

Reaction-Diffusion Waves with Nonlinear Boundary Conditions in Connection with Inflammatory Diseases

Narcisa C. Apreutesei

Technical University of Iasi Department of Mathematics and Informatics 700506 Iasi, Romania
e-mail: napreut@gmail.com

We are concerned with the existence of travelling wave solutions for reaction-diffusion equations with nonlinear boundary conditions. Such equations have been proposed to model the development of atherosclerosis or other inflammatory diseases. Monostable and bistable equations are approached. In the bistable case one employs the Leray-Schauder method, that is based on a topological degree for elliptic problems in unbounded domains and on a priori estimates of solutions. Fredholm property for associated linearized operators is proved and properness of the semilinear operators is studied in weighted Holder spaces.

A clinical interpretation and the Lyapunov function for the blood-glucose control system

Larisa Cheregi

Technical University of Cluj-Napoca
e-mail: cheregi.larisa@yahoo.com

In this paper the approximation analysis and a Lyapunov function are developed to obtain the phase portrait for the blood-glucose control. The modeling of the measured concentrations has the goal of providing information on the state of the subject's glucose control system. The phase portraits are obtained for different sets of parameter values and each result has clinical interpretation.

Reduced order modeling applied to predator–prey systems in the presence of generalist predators

Gabriel Dimitriu

"Grigore T. Popa" University of Medicine and Pharmacy,
Department of Medical Informatics and Biostatistics, Iaşi, Romania
e-mail: dimitriu.gabriel@gmail.com, gabriel.dimitriu@umfiasi.ro

In a large diversity of engineering and biological applications we are interested in fast and reliable characterization of the relationship between inputs and outputs of a model, in which the outputs are defined as the solution of a system of input-parametrized partial differential equations. The basic idea of model reduction is that this relationship between inputs and outputs can often be reasonably well approximated by a much lower dimensional model generating nearly the same response characteristics, but leading to significantly reduced simulation times.

The interactions between predators and preys, extensively studied in the invasion theory and biological control, represent dominant factors of the species distribution and composition in a community. These interferences are strongly influenced by the preferences and type of predators, predator mediated apparent competition, as well as the vulnerabilities of different prey types. Generalist predators, that use a possibly rich variety of food sources, have profound impacts on the the dynamics of such communities.
This work focusses on the numerical implementation of reduced order techniques – Proper Orthogonal Decomposition (POD) method, and the hybrid variant: POD combined with DEIM (Discrete Empirical Interpolation Method) – applied to spatial predator–prey systems in the presence of generalist predators. Comparative numerical results with respect to CPU time and errors of the approximate solutions in both high-fidelity and reduced models are presented.

Bibliography

[1] S. Chakraborty, *The influence of generalist predators in spatially extended predator-prey systems*, Ecological Complexity, 23 (2015), 50–60.

[2] G. Dimitriu, I.M. Navon, R. Ştefănescu, *Application of POD-DEIM Approach for Dimension Reduction of a Diffusive Predator-Prey System with Allee Effect*, Lecture Notes in Comput. Sci., 8353 (2014), 373–381.

[3] G. Dimitriu, R. Ştefănescu, I.M. Navon, *Comparative numerical analysis using reduced-order modeling strategies for nonlinear large-scale systems*, J. Comput. Appl. Math., 310 (2017), 32–43.

The Feedback Linearization for Dynamical Systems Associated to Evolution Epidemics. The Case of Kermack McKendrick Model

Adela Ionescu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: adela0404@yahoo.com

In the present paper it is continued the recent work started on the dynamical system associated to evolution epidemics modelling. Some new results on the qualitative behavior of the Kermack McKendrick model are presented. Using the feedback linearization method, we search for a state feedback control for the $3D$ ODE system associated to this model. The special form of the inverse model allows some further detailed analysis of the model from the stability standpoint.

Modelling Neural Crest Cell Migration in Early Embryonic Development

Lingyun Xiong

University of Oxford
e-mail: lingyun.xiong@dtc.ox.ac.uk

During vertebrate development, several embryonic cell populations undergo persistent and directed migration in coordinated groups. Failure of these migratory cells to reach their targets can lead to dire consequences ranging from improper cell differentiation to uncontrolled cell proliferation. Among these migratory populations, cells emerging from the neural crest (NC cells) need to

migrate and fully colonize the growing domain. This process is crucial in the formation of peripheral tissues, and it is dictated by the spatiotemporal dynamics of cell movement and differentiation, which is determined by cell signalling and response to interacting chemical and mechanical cues. However, it remains unclear how NC cells respond to different microenvironments to maintain direction and cohesion in multicellular streams during migration. My research is mainly focused on investigating the effect of non-uniform distribution of chemical cues on NC cell migration, but I am also interested in the influence of heterogeneous domain growth on NC cell migration.

Schistosomiasis in Ghana: modelling and prevention

Hong Zhang[1], Prince Harvim[1,2], Paul Georgescu[3]

[1] *Jiangsu University, P.R.China*
[2] *Kwame Nkrumah University of Science and Technology, Ghana*
[3] *Technical University of Iaşi, Romania*
e-mail: v.p.georgescu@gmail.com

The goal of a future free from schistosomiasis in Ghana can be achieved through integrated strategies, targeting simultaneously several stages of the life cycle of the schistosome parasite. In this presentation, the transmission of schistosomiasis is modeled as a multi-scale 12-dimensional system of ODEs that includes vector-host and within-host dynamics of infection. An explicit expression for the basic reproduction number R_0 is obtained via the next generation method, this expression being interpreted in biological terms, as well as in terms of reproductive numbers for each type of interaction involved. After discussing the stability of the disease-free equilibrium and the existence and uniqueness of the endemic equilibrium, the Center Manifold Theory is used to show that for values of R_0 larger than 1, but close to 1, the unique endemic equilibrium is locally asymptotically stable. A sensitivity analysis indicates that R_0 is most sensitive to the natural death rate of the vector population, while the most effective strategy for the control and possible elimination of schistosomiasis should combine sanitary measures (access to safe water, improved sanitation and hygiene education), large-scale treatment of infected population and vector control measures (via the use of molluscicides), for a significant amount of time.

Mini Symposium: Dynamical Systems

Mini-Symposium: Dynamical Systems

organizers

Dana Constantinescu

Asimptotic behavior of some dynamical processes

Bucur Maria-Liliana

University of Craiova
e-mail: lilianabucur@yahoo.com

In this work we are studying some dynamical processes. The dynamical processes represent the natural generalisation of the dynamical systems. A discrete time dynamical process having the generators $(f_n)_{n\in\mathbb{N}}$ is given by the difference equation $x_{n+1} = f_n(x_n)$. The discrete dynamical systems are particular cases of dynamical processes corresponding to a constant sequence of generators, i.e. $x_{n+1} = f(x_n)$.
The asymptotic behaviour of some dynamical processes of logistic type is analysed in this paper. We will consider $f_n(x) = x^\alpha(a_n - x^\beta)$ and we will study the processes having the generators $(f_n)_{n\in\mathbb{N}}$. For some α and β we find the fixed points and for some sequence $(a_n)_{n\in\mathbb{N}}$ the pre-equilibrium points, the basin of attraction, the bifurcations points. Many cases will be illustrated in this paper and we will discussed the effect of the parameters α and β on the number of fixed and pre-equilibrium points and their stability.

Intrinsic versus numerical chaos in discrete models associated to continuous dynamical systems

Dana Constantinescu

University of Craiova
e-mail: constantinescu.dana@ucv.ro

A widespread technique in the study of continuous systems is their association with discrete systems. This happens, for example, when numerical methods are used to integrate the differential equations describing the continuous system or when the Poincaré map is considered for discretization. In this procedure, the analogy of the dynamic properties of the two systems is crucial. For classical numerical methods, this means to use a small enough integration step but the problem is more complicated for Poincaré maps.
In this paper we point out some problems that occur in the study of Poincaré models associated with Hamiltonian systems with 3/2 degrees fo freedom. These models are obtained using a symmetric mapping technique, which is a classical method in the study of Hamiltonian systems.
It is observed that some orbits are chaotic or not, depending on the mapping step. A local criterion for determining the optimal mapping step for the preservation of the dynamical characteristics (regular or chaotic) of an orbit is proposed. A global criterion is also derived. These criteria are applied for the study of a Hamiltonian system modeling the magnetic field configuration in tokamak.

Hidden attractor. Localization problem

Raluca Efrem

Department of Mathematics, University of Craiova, Romania
e-mail: ra_efrem@yahoo.com

The classification of attractors, from a computational point of view, can be made using as a criterion the simplicity of detection their basin of attraction. Taking into account this classification criterion, recently a concept of hidden and self-excited attractors was proposed.
An attractor is called a hidden attractor if its basin of attraction does not intersect with small neighborhoods of equilibria, otherwise it is called a self-excited attractor.
Self-excited attractors can be localized numerically by a standard computational procedure, in which after a transient process a trajectory, starting from a point of unstable manifold in a neighborhood of unstable equilibrium, is attracted to the state of oscillation and traces it.
For localization of hidden attractors it is necessary to develop special procedures, since there are no similar transient processes leading to such attractors.
This survey is dedicated to efficient methods for the study of hidden attractors.

The Feedback Linearization for the Dynamical System Associated to the Mixing Flow Model

Adela Ionescu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: adela0404@yahoo.com

In the present paper there are presented some new results concerning the behavior of the mixing flow dynamical system. The feedback linearization of this dynamical system is taken into account. From analytical standpoint, this technique issue special interpretations. It contains two fundamental nonlinear controller design techniques: input-output linearization and state-space linearization. The approach is usually referred as input-output linearization or feedback linearization and is based on concepts from nonlinear systems theory. The resulting controller includes the inverse of the dynamic model of the process, providing that such an inverse exists.

About Symmetries, Pseudosymmetries and Conservation Laws in k-Cosymplectic Geometry

Florian Munteanu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: munteanufm@gmail.com

The k-cosymplectic formalism was introduced in [1, 2] and is a generalization to the first order field theories of the standard cosymplectic formalism from non-autonomous dynamical systems. The k-Cosymplectic Geometry provides the simplest geometric framework for describing a certain class of non-autonomous dynamical systems in the first-order classical field theories. It will be extended the study of symmetries, pseudosymmetries and conservation laws from cosymplectic formalism to the k-cosymplectic formalism with the purpose of obtaining new kinds of conservation laws for k-cosymplectic Hamiltonian systems and k-cosymplectic Lagrangian systems.
2010 Mathematics Subject Classification: 70S05, 70S10, 53D05.
Key words and phrases: symmetry, pseudosymmetry, conservation law, k-cosymplectic formalism

Bibliography

[1] de León, M., Merino, E., Oubiña, J.A., Rodrigues, P., Salgado, M., *Hamiltonian Systems on k-Cosymplectic Manifolds*, J. Math. Phys. **39**(2) (1998), 876–893

[2] de León, M., Merino, E., Salgado, M., *k-Cosymplectic Manifolds and Lagrangian Field Theories*, J. Math. Phys. **42**(5) (2001), 2092–2104

Osserman generalized locally symmetric spaces with the generalized 2-nilpotent Jacobi operator

Catalin Sterbeti

University of Craiova
e-mail: sterbetiro@yahoo.com

In this paper we construct a family of semi-Riemannian metrics with neutral signature. These are metrics defined as

$$g_\Phi(x,y) = \sum_{i=1}^{p} dx^i \otimes dy^i + \sum_{i,j=1}^{p} \Phi_{ij}(x,y) dx^i \otimes dx^j$$

and generalize the complete deformed lift of usual Riemannian metric on $\mathbb{R}^n$ and they generates k-Osserman spaces for each admissible k.
In the following we will study important properties of these spaces.

Mini Symposium: Numerical Analysis, an art or merely a science?

Mini-Symposium: Numerical Analysis, an art or merely a science?

organizers

Călin-Ioan Gheorghiu

A class of non-homogeneous hyperbolic systems with source terms. Analytical and numerical solution

Stefan Gicu Cruceanu, Dorin Marinescu, and Stelian Ion

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania

e-mail: gcruceanu@yahoo.com, marinescu.dorin@gmail.com, ro_diff@yahoo.com

In this paper we focus on a hyperbolic system that models the water flow over the soil surface with vegetation in a hydrographic basin. The model takes into account the plant cover using the porosity fuction that is defined over a hydrological basin. In order to define a shock wave solution of the system, we introduce based on physical arguments a family of paths that connect two constant states. We discuss and investigate several physical relevant properties of this model and finally, we present some numerical results to illustrate the capability of the model.

Accurate spectral solutions to a class of singular nonlinear boundary value problems on unbounded domain

Călin-Ioan Gheorghiu

Romanian Academy, "T. Popoviciu" Institute of Numerical Analysis, Cluj-Napoca

e-mail: ghcalin@ictp.acad.ro

We are concerned with accurate collocation solutions to some genuinely *nonlinear* and *singular* problems of the form:

$$\begin{gathered} u''(x) = f(x,\ u(x),\ u'(x)),\ x \in (0,\infty), \\ \alpha u(0) - \beta u'(0) = r, \\ \lim_{x\to\infty} u(x) \quad exists, \end{gathered} \tag{1}$$

where $\alpha > 0$, $\beta \geq 0$ and r is a given constant.

Our collocation methods are based on Hermite, Laguerre and sinc functions. The singularities in the origin are resolved by *removing technique of independent boundary conditions*. The boundary conditions at infinity are satisfied by above mentioned shape functions due to the weighting factors involved.

We consider some benchmark problems like the modified Troesch's problem, the Thomas-Fermi's problem, the Kidder's problem and some other problems from electromagnetic theory. Our main aim is to avoid the empiric and arbitrary domain truncation which is frequently used along with shooting. Some p. d. e. such as the radial NLS are taken into account.

The effectiveness of our approach has been confirmed by some numerical experiments.

Numerical Approximation of the Solution to one Nonlocal Problem for a Parabolic Equation

Olga Danilkina[1,2], Gilbert Danso Acheampong[2]

[1] *The University of Dodoma, Dodoma, Tanzania*
[2] *African Institute for Mathematical Sciences (AIMS) Tanzania*
e-mail: olga.danilkina@udom.ac.tz[1]

In this talk, we consider a boundary-value problem with the nonlocal integral condition of the first kind for the parabolic equation with a singularity and discuss the question of numerical approximation of the solution. First, we prove existence and uniqueness of the solution to the nonlocal problem in the appropriate weighted functional space using ideas of the apriori estimates approach. Then we describe the homotopy analysis method (HAM) and study its optional applications to the nonlocal parabolic problem. We derive nth order deformation equations and obtain HAM-approximations to the solution. Finally, we consider the application of the homotopy analysis method to boundary-value problems with nonlocal boundary conditions of other types.

Stability and errors analysis of two iterative schemes of fractional steps type associated to a nonlinear reaction-diffusion equation

Costică Moroşanu

Faculty of Mathematics, "Al. I. Cuza" University, Iaşi, Romania
e-mail: costica.morosanu@uaic.ro

The paper concerns with the approximation of unique solution to a nonlinear reaction-diffusion equation endowed with a cubic nonlinearity and, non-homogeneous boundary conditions. Using energy methods, we prove L^∞ stability and establish error estimates for two implicit-explicit schemes of fractional steps type. Taking the Allen-Cahn equation as particular case of our nonlinear equation, some numerical experiments are done in order to validates the theoretical results as well as to compare the accuracy of that different fractional steps methods.
MSC: 35K55, 35K57, 65M06, 65M12, 65Y20, 80Axx.
Keywords. nonlinear PDE of parabolic type, reaction-diffusion equations, finite difference methods, fractional steps method, stability and convergence of numerical methods, performance of numerical algorithms, thermodynamics, phase-changes.

Bibliography

[1] C. Moroşanu, *Qualitative and quantitative analysis for a nonlinear reaction-diffusion equation*, ROMAI J., Vol. 12, No. 2 (2016), pp. 85-113, https://rj.romai.ro/arhiva/2016/2/Morosanu.pdf

[2] C. Moroşanu, S. Pavăl and C. Trenchea, *Analysis of stability and errors of three methods associated to the nonlinear reaction-diffusion equation supplied with homogeneous Neumann boundary conditions*, Journal of Applied Analysis and Computation, Vol. 7, No. 1 (2017), pp. 1-19, DOI:10.11948/2017001

A FEM approach to fcc-single crystal subjected to large strains

Raisa Pașcan

University of Bucharest, Faculty of Mathematics and Computer Science
e-mail: pascanraisa@yahoo.com

Using the finite crystal plasticity as a framework, we study the behavior of a single fcc-crystal. The plastic deformation of the crystal is generated by the move of the atoms in the planes of maximum atomic density, i.e. slip systems. The dislocations have a crucial place in the deformation mechanism. The hardening depends on the dislocation density in each slip system. The dislocation densities and hardening variables are described by the differential type equations. Our problem is based on the incremental equilibrium equation given by the associated variational equality, coupled with the differential system describing the evolution of the fields involved in the model. The variational problem is solved using the finite element method (FEM) modeled with FreeFem++, together with an update algorithm based on the Runge-Kutta method for solving the differential type equations. The activation condition chooses in every point of the crystal which slip systems are active at a given time and this is also implemented numerically in our model.

Finite-differences for convection-diffusion equation

Vladimir Pațiuc, Galina Rîbacova

Moldova State University, Chişinău, Republic of Moldova
e-mail: ribacus@yahoo.com

The solution of convection-diffusion problems is a challenging task for numerical methods owing to the fact that the governing equation includes the diffusion component and drift or convection component. In case when the coefficients of these terms are comparable, the problem can be solved numerically using a traditional finite-difference approximation. However, in many applications the ratio between the convection and the diffusion coefficients is very large. In this case, the solution has an exponential character and standard approaches to the construction of difference schemes lead to the numerical solutions with high error on coarse grids. We consider several diverse approaches for construction the finite-difference schemes and carry out a comparative analysis of the effectiveness of obtained schemes by solving a specific problem.

Partial Differential Equations

Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and Translating Sphere

Indeewar Kumar

Department of Mathematics and Statistics, Manipal University Jaipur, Jaipur-303007, Raj.(India),
e-mail: indeewar.kumar@jaipur.manipal.edu

Exact solutions are important not only in its own right as solution of particular flows, but also serve as accuracy check for numerical solution. Exact solution of the Navier-Strokes equation are, for example, those of steady and unsteady flows near a stagnation point, Stagnation point flows can either be viscous or inviscid, steady or unsteady, two dimensional or three dimensional, normal or oblique and forward or reverse. The classic problems of two dimensional and three dimensional stagnation point flow are associated with the names of Hiemenz and Homan A novel radial stagnation point flow impinging axi symmetrically on a circular cylinder was reported by Wang. The present paper deals with the laminar boundary layer flow and heat transfer in the stagnation region of a rotating and translating sphere with uniform magnetic fields. The governing equations of flow are derived for $\xi = 0(t^* = 0)$ and $\xi = 1(t^* \to \infty)$ and solutions in the closed form are obtained. The temperature and velocity fields for $\xi = 0$ are numerically computed. This shows that the thermal boundary layer thickness decreases as Prandtl number Princreases.The surface heat transfer (28) increases with the Prandtl number Pr. The surface heat transfer (28) at the starting of motion is found to be strangely dependent on the Prandtl number Pr. But it is dependent of magnetic field, buoyancy force Bp and Rotation Parameter Ro.
Key words : Temperature field, velocity field,uniform magnetic field, buoyancy force, Rotation Parameter.

Existence of Minimal and Maximal solutions for Quasilinear Elliptic Equation with Nonlocal Boundary Conditions on Time-Scales

Mohammed Derhab[1], Mohammed Nehari[2]

[1]*Department of Mathematics, Faculty of Sciences, University Abou-Bekr Belkaid Tlemcen, Algeria*
[2]*Department of Mathematics, Faculty of Sciences, E.S.M.Tlemcen, Algeria*
e-mail: derhab@yahoo.fr, nehari_72@yahoo.fr

The purpose of this work is the construction of minimal and maximal solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

$$\begin{cases} -\left(\varphi_p\left(u^{\Delta}\right)\right)^{\Delta} = f\left(x,u\right), \text{ in } \left(a,b\right)_T, \\ u\left(a\right) - a_0 u^{\Delta}\left(a\right) = g_0\left(u\right), \\ u\left(\sigma\left(b\right)\right) + a_1 u^{\Delta}\left(\sigma\left(b\right)\right) = g_1\left(u\right), \end{cases} \tag{1}$$

where $p > 1$, $\varphi_p\left(y\right) = |y|^{p-2} y$, $\left(\varphi_p\left(u^{\Delta}\right)\right)^{\Delta}$ is the one-dimensional $p-$Laplacian, $f : [a,b]_T \times \mathbb{R} \to \mathbb{R}$ is a rd-continuous function, $g_i : C_{rd}\left([a,b]_T\right) \times C_{rd}\left([a,b]_T\right) \to \mathbb{R}$ $(i = 0$ and $1)$ are rd-continuous

and a_0 and a_1 are a positive real numbers.

Key words: Quasilinear elliptic equation; Time-Scale, Nonlocal boundary conditions; upper and lower solutions; monotone iterative technique.
AMS Classification: 34B10, 34B15
Acknowledgments. The authors acknowledge a partial support of MESRS-DRS grant B02020100075 and PNR project N^0 43/28/2011.

Approximation of the controls for the wave equation with a potential

Laurențiu Emanuel Temereancă

Department of Applied Mathematics, University of Craiova, Romania
e-mail: `temereanca_laurentiu@yahoo.com`

We consider the semi-discrete space approximation of the one dimensional controlled linear wave equation with a potential. We analyze the conditions which ensure the convergence of the numerical scheme. We show that if the high frequencies of the discretization of the initial data are filtered out, then the convergence of the approximating family of controls can be ensured. Moreover, we prove that there exists a space of regular initial data which are directly uniformly controllable. The main difficulty consist of the fact that the corresponding discrete operator does not posses an explicit family of eigenvalues, which is an important issue in our strategy (which reduces the controllability problem to a moment problem solved by constructing a biorthogonal sequence to a family of exponential functions).

ODEs; Dynamical Systems

On solving the variational problem

S. Aisagaliev, A. Kabidoldanova

al-Farabi Kazakh National University
e-mail: kabasem@mail.ru, Serikbai.Aisagaliev@kaznu.kz

A method for solving the Lagrange problem with state variable constraints for processes described by ordinary differential equations without involvement of the Lagrange principle is proposed. Namely, dynamical system described by the equation

$$\dot{x} = A(t)x + B(t)f(x,u,t), \quad t \in I = [t_0, t_1] \tag{1}$$

with the boundary conditions

$$(x(t_0)) = x_0, x(t_1) = x_1) \in S_0 \times S_1 = S \subset R^{2n} \tag{2}$$

the state variable constraints

$$x(t) \in G(t) : G(t) = \{x \in R^n / \omega(t) \leq F(x,t) \leq \varphi(t), \quad t \in I\}, \tag{3}$$

the constrained control

$$u(\cdot) \in L_2(I, R^m). \tag{4}$$

and the perfomance criterion

$$J(u(\cdot), x_0, x_1) = \int_{t_0}^{t_1} F_0(x(t), u(t), x_0, x_1, t)dt \to inf \tag{5}$$

is considered. A necessary and sufficient condition for existence of a solution to the variational problem is obtained, an admissible control is found and an optimal solution is constructed by narrowing a set of admissible controls. The basis of the proposed method for solving the variational problem is an imbedding principle. An essence of the imbedding principle is that the original variational problem with boundary conditions and state variable constraints is replaced by equivalent free end point optimal control problem. This approach is possible due to finding a general solution of a class of the first kind Fredholm integral equations.

Multifrequency systems of differential equations with transformed argument

Yaroslav I. Bihun

"Yurii Fedkovych" Chernivtsi National University, Ukraine
e-mail: yaroslav.bihun@gmail.com

Multifrequency systems of ODE were considered in [1] by averaging method, and systems with delay of argument — in [2], [3] and others.
We consider a system of differential equations of the form

$$\frac{dx}{d\tau} = X(\tau, x_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, x_\Lambda, \varphi_\Theta) \tag{1}$$

with initial conditions, multipoint boundary and integral conditions. Here $0 \leq \tau \leq L$, $x \in D \subset \mathbb{R}^n$, $\varphi \in \mathbb{T}^m$, $\Lambda = (\lambda_1, \ldots, \lambda_p)$, $\Theta = (\theta_1, \ldots, \theta_q)$, $\lambda_i, \theta_j \in (0,1)$, $x_{\lambda_i}(\tau) = x(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$, $\varepsilon \in (0, \varepsilon_0]$.
The complexity of the research of the problem is the existence of resonances. The resonance condition in point $\tau \in [0, L]$ is [2]

$$\sum_{\nu=1}^{q} \theta_\nu (k_\nu, \omega(\theta_\nu \tau)) = 0, \quad k_\nu \in \mathbb{R}^m, \quad \|k\| \neq 0.$$

Averaging in system (1) is carried out on fast variables φ_Θ on the torus T^m. The averaged problem takes the form

$$\frac{d\overline{x}}{d\tau} = X_0(\tau, \overline{x}_\Lambda,), \quad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y_0(\tau, \overline{x}_\Lambda).$$

The condition provides that solution of system goes through small neighborhood of the resonance is Wronski determinant $V(\tau) \neq 0$, built with system of functions $\{\omega(\theta_1 \tau), \ldots, \omega(\theta_r \tau)\}$ is satisfied for $\tau \in [0, L]$.
The existence and uniqueness of solution of the problem and the estimation error $\|x(\tau, \varepsilon) - \overline{x}(\tau)\| \leq c_1 \varepsilon^\alpha$, where $\alpha = (qm)^{-1}$, $c_1 = const > 0$ of averaging method is obtained. When we imposed the conditions obtained error estimates averaging method is not improved.

Remark. If the condition Wronski determinant $V(\tau) \neq 0$ is not satisfied in points τ_ν, $\nu = \overline{1, q}$, and the multiplicity of roots of the equation $V(\tau) = 0$ is limited by the number $r \geq 1$, then the estimation can be proved also in this case. Let us apply the proving scheme suggested [1] for ordinary differential equations, [1] for system of differential equations with linearly transformed arguments. Herewith the error of averaging method for slow variables takes the form $\|x(\tau, \varepsilon) - \overline{x}(\tau)\| \leq c_2 \varepsilon^\beta$, where $\beta = (qm + r)^{-1}$, $c_2 = const > 0$.

Bibliography

[1] A. M. Samoilenko, R. I. Petryshyn, Multifrequency Oscillations of Nonlinear Systems, Kluwer, Dordrecht V-Boston V-London, Netherlands, 2004, 317 p.

[2] Ya. I. Bihun, "Existence of a Solution and Averaging of Nonlinear Multifrequency Problems with Delay". *Ukr.Math.Zh.*, **59**, No. 4 , 485–499 (2007).

[3] Ya. Bihun, R. Petryshyn, I. Krasnokutska, "Averaging in Multifrequency Systems with Linearly Transformed Arguments and with Point and Integral Conditions", *Bukovinian Math. Journal*, **4**, No. 3–4 , 30–35 (2016).

Optimal Tuning of Robust Controllers Using Interval Analysis

Calistru Cătălin

Department of Automatic Control,
"Gheorghe Asachi" Technical University of Iaşi, Romania
e-mail: calistru@ac.tuiasi.ro

Controller tuning is one of the major problems engineers must solve, having in view the complexity and efficiency of the control loop. Since there are different approaches to controller tuning,

the integral ones (i.e. Integral Squared Error ISE and similar) are particularly suitable for this action. This paper presents the tuning methodology for the proportional integral derivative (PID) type controllers based on interval analysis in order to obtain the robustness of the closed loop system. That means that the stability and stationary performances of the closed loop system are preserved for the parameters of the differential system varying within certain limits. An optimization algorithm is developed and illustrative results obtained by numerical simulations are shown. Some practical experimental results are presented.

Existence and approximation of a solution of the boundary value problems for neutral delay integro-differential equations

Ihor Cherevko, Andrew Dorosh, Larysa Piddubna

Yuriy Fedkovych Chernivtsy National University, Ukraine

e-mail: `i.cherevko@chnu.edu.ua`

Solution existence and uniqueness for delay boundary value problems in various function spaces and its construction using the spline-collocation method were considered in [1]-[2]. Application of spline functions allows to create an algorithm with simple implementation and at the same time suitable for a wide range of boundary value problems.
In this work we study boundary value problems for neutral delay integro-differential equations and suggest an iterative scheme for approximating their solution [3].
We consider the following boundary value problem

$$y''(x) = f(x,[y(x)]) + \int_a^b g(x,s,[y(s)])\,ds, s \in [a;b], \tag{1}$$

$$y^{(p)}(x) = \varphi^{(p)}(x), \quad p = 0,1,2, \quad x \in [a^*;a], \quad y(b) = \gamma, \tag{2}$$

where

$$[y(x)] = (y(x), y(x-\tau_0(x)), y'(x), y'(x-\tau_1(x)), y''(x-\tau_2(x))),$$
$$a^* = \min\left\{\min_{x\in[a;b]}(x-\tau_0(x)), \min_{x\in[a;b]}(x-\tau_1(x)), \min_{x\in[a;b]}(x-\tau_2(x))\right\},$$

$\tau_0(x), \tau_1(x), \tau_2(x) \geq 0$ are scalar continuous functions on $[a;b]$, $\varphi(x) \in [a^*;a]$, $\gamma \in R$. Let the functions $f(x,u_0,u_1,v_0,v_1,w)$, $g(x,s,u_0,u_1,v_0,v_1,w)$ be continuous by the set of variables on $G = [a;b]\times G_1 \times G_2 \times G_3$ and $Q = [a;b]\times G$ where $G_1 = \{u \in R: |u| < P_1\}$, $G_2 = \{v \in R: |v| < P_2\}$, $G_3 = \{w \in R: |w| < P_3\}$, $P_1, P_2, P_3 > 0$.
We assume that the delays $\tau_1(x), \tau_2(x)$ are such that the following sets of points are finite

$$E_1 = \{x_i \in [a,b]: x_i - \tau_1(x_i) = a, i = 1,2,...\},$$
$$E_2 = \{x_j \in [a,b]: x_0 = a, x_{j+1} - \tau_2(x_{j+1}) = x_j, j = 1,2,...\},$$
$$E = E_1 \cup E_2.$$

Let us enumerate all points of the set E in ascending order and introduce the notations:

$$a = x_0 < x_1 < ... < x_k < b,$$

$$B\left([a^*;b]\right) = \left\{ y\left(x\right) : y \in C\left[a^*;b\right] \cap C^1\left[a;b\right] C^2\left[a^*;a\right] \cap \left(\bigcup_{j=1}^{k+1} C^2\left(\delta_j\right) \right), \right.$$

$$\left. \left|y\left(x\right)\right| \leq P_1,\ \left|y'\left(x\right)\right| \leq P_2,\ \left|y''\left(x\right)\right| \leq P_3 \right\}.$$

A function $y(x) \in B\left([a^*;b]\right)$ is called a solution of the problem (1)-(2) if it satisfies the equation (1) on $[a;b]$ (with the possible exception of the set of points E) and boundary conditions (2).
Sufficient conditions for the existence of a unique solution of the boundary value problem (1)-(2) in the domain $B\left([a^*;b]\right)$ are obtained.
Similarly to [4], an iterative scheme with usage of a sequence of cubic splines with defect two is suggested for finding the approximate solution of the problem (1)-(2).

Bibliography

[1] Nikolova, T.S., Bainov, D.D., *Application of spline-functions for the construction of an approximate solution of boundary problems for a class of functional-differential equations*, Yokohama Math. J, **29** (1981) 1, pp. 108–122.

[2] Biga, A., Gaber, R. *Existence, Uniqueness and Approximation for the Solution of a Second Order Neutral Differential Equation With Delay in Banach Spaces*, Mathematica, **49** (2007) 2, pp. 117–130.

[3] Dorosh, A., Cherevko, I., *Solution existence for boundary value problems for neutral delay integro-differential equations*, Bukovynian Mathematical Journal, **4** (2016) 3-4, pp. 43–46.

[4] Cherevko, I., Dorosh, A., *Existence and Approximation of a Solution of Boundary Value Problems for Delay Integro-Differential Equations*, J. Numer. Anal. Approx. Theory, **44** (2016) 2, pp. 154–165.

Center conditions and phase portraits in a cubic differential system with invariant algebraic curves

Dumitru Cozma, Anatoli Dascalescu, Vadim Repeșco

Tiraspol State University, Chişinau, Republic of Moldova
e-mail: dcozma@gmail.com

We consider the cubic system of differential equations

$$\begin{aligned} \dot{x} &= y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3, \\ \dot{y} &= -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3), \end{aligned} \tag{1}$$

in which variables and coefficients are assumed to be real. The origin $O(0,0)$ is a singular point of a center or focus type for (1). In this talk we present conditions for the origin to be a center for system (1) with a certain number of invariant algebraic curves and construct the global phase portraits.
Let for cubic system (1) the following coefficient conditions hold [1]

$$f=-2,\ l=-b,\ n=-d-1,\ p=b-c,\ q=[s(ch-k+g)-gh^2]/h^2,$$
$$r=1,\ m=[h^2(d+1)-(g-k)^2+h(ck-cg-s)-h^3]/h^2,\ h=a-1,$$

then (1) has two invariant straight lines of the form

$$l_{1,2}\equiv 2h+(k-g\pm\sqrt{(g-k)^2+4hs}\,)x-2hy=0.$$

Assuming that $l_{1,2}=0$ are invariant straight lines for (1), we determine the conditions under which the system (1) has an irreducible invariant cubic curve of the form $\Phi\equiv x^2+y^2+a_{30}x^3+a_{21}x^2y+a_{12}xy^2-y^3=0$.

We solve the problem of the center for (1) with invariant algebraic curves $l_{1,2}=0,\ \Phi=0$ and obtain nine sets of necessary and sufficient conditions for the origin to be a center.

The necessary conditions for $O(0,0)$ to be a center are determined by calculating the Lyapunov quantities. It was proved

Theorem. *A singular point $O(0,0)$ is a center for cubic system (1) with invariant algebraic curves $l_{1,2}=0,\ \Phi=0$ if and only if the first three Lyapunov quantities vanish.*

The sufficiency of these conditions is confirmed by constructing Darboux first integrals of the form $l_1^{\alpha_1}l_2^{\alpha_2}\Phi^{\alpha_3}\Psi^{\alpha_4}=C$, where α_j are complex numbers and $\Psi=0$ is an invariant conic.

We carry out the global qualitative investigation of cubic systems (1) with a center having the invariant algebraic curves $l_{1,2}=0$, $\Phi=0$, $\Psi=0$ and construct the phase portraits on the Poincaré disc.

Bibliography

[1] D. Cozma, *Darboux integrability and rational reversibility in cubic systems with two invariant straight lines.* EJDE, 2013, vol. 2013, no. 23, p. 1–19.

Solution of a Bratu-like equation arising in electrospinning process by the variation of parameters method

Vedat Suat Erturk

Department of Mathematics, Ondokuz Mayis University, Samsun, Turkey
e-mail: vserturk@omu.edu.tr

The aim of this work is to solve a Bratu-like nonlinear differential equation arising in electrospinning and vibration-electrospinning process that was recently introduced in [1] by the variation of parameters method[2]. Electrospinning process has been associated to Bratu equation through thermo-electro-hydrodynamics balance equations. Following the proposed method, the approximate solution of the considered problem is obtained for certain selected values of the parameters that appear in the modeled equation.

Bibliography

[1] H. Y. Liu, P. Wang: A short remark on Bratu-like equation arising in electrospinning and vibration-electrospinning process, *Carbohydrate Polymers*, (ARBPOL-D-13-02650), 2014.

[2] D.G. Zill, M.R. Cullen: *Advanced Engineering Mathematics*, 2nd ed., Jones and Bartlett Publishers, 2000.

Positive solutions for a system of fractional differential equations with multi-point boundary conditions

Johnny Henderson[1], Rodica Luca[2], Alexandru Tudorache[3]

[1] *Baylor University, Department of Mathematics, Waco, Texas, 76798-7328 USA*
[2] *Gh. Asachi Technical University, Department of Mathematics, Iasi 700506, Romania*
[3] *Gh. Asachi Technical University, Faculty of Computer Engineering and Automatic Control, Iasi 700050, Romania*
e-mail: `Johnny_Henderson@baylor.edu, rlucatudor@yahoo.com, alexandru.tudorache93@gmail.com`

We investigate the system of nonlinear ordinary fractional differential equations

$$(S)\qquad \begin{cases} D_{0+}^{\alpha}u(t)+\lambda f(t,u(t),v(t))=0, & t\in(0,1),\\ D_{0+}^{\beta}v(t)+\mu g(t,u(t),v(t))=0, & t\in(0,1),\end{cases}$$

with the multi-point boundary conditions

$$(BC)\qquad \begin{cases} u^{(j)}(0)=0,\ j=0,\dots,n-2;\ \ D_{0+}^{p_1}u(t)|_{t=1}=\displaystyle\sum_{i=1}^{N}a_iD_{0+}^{q_1}u(t)|_{t=\xi_i},\\ v^{(j)}(0)=0,\ j=0,\dots,m-2;\ \ D_{0+}^{p_2}v(t)|_{t=1}=\displaystyle\sum_{i=1}^{M}b_iD_{0+}^{q_2}v(t)|_{t=\eta_i},\end{cases}$$

where λ, $\mu>0$, α, $\beta\in\mathbb{R}$, $\alpha\in(n-1,n]$, $\beta\in(m-1,m]$, n, $m\in\mathbb{N}$, n, $m\geq 3$, p_1, p_2, q_1, $q_2\in\mathbb{R}$, $p_1\in[1,n-2]$, $p_2\in[1,m-2]$, $q_1\in[0,p_1]$, $q_2\in[0,p_2]$, ξ_i, $a_i\in\mathbb{R}$ for all $i=1,\dots,N$ ($N\in\mathbb{N}$), $0<\xi_1<\dots<\xi_N\leq 1$, η_i, $b_i\in\mathbb{R}$ for all $i=1,\dots,M$ ($M\in\mathbb{N}$), $0<\eta_1<\dots<\eta_M\leq 1$, and D_{0+}^{k} denotes the Riemann-Liouville derivative of order k.
By using the Guo-Krasnosel'skii fixed point theorem, under some assumptions on f and g, we give intervals for the parameters λ and μ such that positive solutions of $(S)-(BC)$ exist. The nonexistence of positive solutions for the above problem is also studied.

Second order dynamical systems with penalty terms associated to monotone inclusions

Szilárd László, R.I. Boţ, E.R. Csetnek

Technical University of Cluj-Napoca
e-mail: `laszlosziszi@yahoo.com`

In this paper we investigate in a Hilbert space setting a second order dynamical system of the form

$$\ddot{x}(t)+\gamma(t)\dot{x}(t)+x(t)-J_{\lambda(t)A}\big(x(t)-\lambda(t)D(x(t))-\lambda(t)\beta(t)B(x(t))\big)=0,$$

where $A:\mathcal{H}\rightrightarrows\mathcal{H}$ is a maximal monotone operator, $J_{\lambda(t)A}:\mathcal{H}\longrightarrow\mathcal{H}$ is the resolvent operator of $\lambda(t)A$ and $D,B:\mathcal{H}\to\mathcal{H}$ are cocoercive operators, and $\lambda,\beta:[0,+\infty)\to(0,+\infty)$, and $\gamma:[0,+\infty)\to(0,+\infty)$ are step size, penalization and, respectively, damping functions, all depending on time. We show the existence and uniqueness of strong global solutions in the framework of the Cauchy-Lipschitz-Picard Theorem and prove ergodic asymptotic convergence for the generated

trajectories to a zero of the operator $A + D + N_C$, where $C = \operatorname{zer} B$ and N_C denotes the normal cone operator of C. To this end we use Lyapunov analysis combined with the celebrated Opial Lemma in its ergodic continuous version. Furthermore, we show strong convergence for trajectories to the unique zero of $A + D + N_C$, provided that A is a strongly monotone operator.

Systems Studies

Mircea Lupu[1], Gheorghe Radu[2]

[1] *"Transilvania University of Brasov, Romania, Corresponding member of the Academy of Romanian Scientists*
[2] *"Henri Coanda" Air Force Academy, Brasov, Romania*
e-mail: m.lupu@unitbv.ro, gh.radu@gmail.com

In this paper there are presented interesting problems from cinematic or dynamic fields, solved by the authors using the inverse methods. These methods are important for systems optimization and control applications in industrial and/or economics area.

1. **Plan robot problem.** Considering the cinematic of roto-translation plate,there are considered the trajectory equations (T), the base (B) and the described curve (R) reported to the axial system. The robot is determined by two (contact-connected) plates ? the fixed base (B) and the known (T) trajectory (the robot ?execute/fallow? the T trajectory and determine the R curve). Same time,corresponding problems exists: (R, T) given and B to be calculated. As solution, the Cauchy problem can be solved for (B, T) or (R, T) cases. Another plan robot that can turn off the floral trajectories can be obtained composing of two cinematic periodical moves.

2. **The problem of heavy rigid solid with a fixed point located in elastic field.** In this case, the prime integrals are determined in linear form and are influenced by the choosing of inertial moments, of heavy center?s coordinates and of the initial conditions. The study is made for the giro-tachometer, in this case we havethe fourth degree Lagrange polynomial $\dot{u} = \sqrt{P_4(u)}$.

3. **The problem of automatic control with absolute stabilization and optimal command.**The purpose of the studies is the automatic pilot stabilization based on Popov and Lurie methods. The optimal control is implemented using Pontreaguine extreme principle for rockets and multiple engines (propellants) ships.

4. **The problem of the optimal ship?s design profiles from hydro-aero-dynamics fields used on extreme resistance / bearing.**The inverse analytical problem leads to Riemann-Hilbert planarity problem and the integral singular equations and dynamics and geometric parameters determination is made by externality procedure of nonlinear functional.

Stability of unperturbed motion for a differential system with nonlinearities of degree four

Natalia Neagu[1,2], Victor Orlov[3,4], Mihail Popa[4]

[1] *Tiraspol State University,*
[2] *Ion Creangă State Pedagogical University,*
[3] *Technical University of Moldova,*
[4] *Institute of Mathematics and Computer Science*
e-mail: neagu_natusik@mail.ru, orlovictor@gmail.com, mihailpomd@gmail.com

We consider the differential system of unperturbed motion with nonlinearities of degree four

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta} x^\alpha x^\beta x^\gamma x^\delta, \tag{1}$$

where $a^j_{\alpha\beta\gamma\delta}$ is a symmetric tensor in lower indices in which the total convolution is done. Suppose for (1) the center-affine invariant conditions

$$I_1^2 - I_2 = 0, \quad I_1 < 0 \quad (I_1 = a^\alpha_\alpha, I_2 = a^\alpha_\beta a^\beta_\alpha) \tag{2}$$

hold. Then by a center-affine transformation, the system (1) can be brought to the form

$$\dot{x}^1 = a^1_{\alpha\beta\gamma\delta} x^\alpha x^\beta x^\gamma x^\delta, \quad \dot{x}^2 = a^2_\alpha x^\alpha + a^2_{\alpha\beta\gamma\delta} x^\alpha x^\beta x^\gamma x^\delta. \tag{3}$$

Let us introduce the following notation

$$A = (a^2_2)^4 a^1_{1111} - 4a^2_1 (a^2_2)^3 a^1_{1112} + 6(a^2_1)^2 (a^2_2)^2 a^1_{1122} - 4(a^2_1)^3 a^2_2 a^1_{1222} + (a^2_1)^4 a^1_{2222},$$

$$B = -(a^2_2)^4 a^2_{1111} + 4a^2_1 (a^2_2)^3 a^2_{1112} - 6(a^2_1)^2 (a^2_2)^2 a^2_{1122} + 4(a^2_1)^3 a^2_2 a^2_{1222} - (a^2_1)^4 a^2_{2222},$$

$$C = (a^2_2)^3 a^1_{1112} - 3a^2_1 (a^2_2)^2 a^1_{1122} + 3(a^2_1)^2 a^2_2 a^1_{1222} - (a^2_1)^3 a^1_{2222},$$

$$D = (a^2_2)^2 a^1_{1122} - 2a^2_1 a^2_2 a^1_{1222} + (a^2_1)^2 a^1_{2222}, \quad E = a^2_2 a^1_{1222} - a^2_1 a^1_{2222}. \tag{4}$$

Then, taking into account the Lyapunov Theorem [1, §32] and the expressions (4), we have

Lemma 1. *The stability of unperturbed motion in system (3) is described by one of the following nine possible cases:* I. $A \neq 0$, *then the unperturbed motion is unstable;*
II. $A = 0$, $BC > 0$, *then the unperturbed motion is unstable;*
III. $A = 0$, $BC < 0$, *then the unperturbed motion is stable;*
IV. $A = C = 0$, $BD \neq 0$, *then the unperturbed motion is unstable;*
V. $A = C = D = 0, BE > 0$, *then the unperturbed motion is unstable;*
VI. $A = C = D = 0$, $BE < 0$, *then the unperturbed motion is stable;*
VII. $A = C = D = E = 0$, $a^1_{2222} B \neq 0$, *then the unperturbed motion is unstable;*
VIII. $A = B = 0$, *then the unperturbed motion is stable;*
IX. $a^1_{1111} = a^1_{1112} = a^1_{1122} = a^1_{1222} = a^1_{2222} = 0$, *then the unperturbed motion is stable.*
In the last two cases, the unperturbed motion belongs to some continuous series of stabilized motion, moreover, it is also asymptotic stable [2] in Cases III, VI.

Remark 1. Using Lemma 1, the invariants and comitants from [3], there were obtained the center-affine invariant conditions of stability of the unperturbed motion for system (1) with the invariant conditions (2).

Bibliography

[1] Liapunov A. M. *Obshchaia zadacha ob ustoichivosti dvijenia,* Sobranie sochinenii, II – Moskva-Leningrad: Izd. Acad. Nauk SSSR, 1956 (in Russian).

[2] Malkin I. G. *Teoria ustoichivosti dvijenia.* Izd. Nauka, Moskva, 1966 (in Russian).

[3] Ciubotaru S. *Rational bases of* GL(2,R)*-comitants and* GL(2,R)*-invariants for the planar systems of differential equations with nonlinearities of the fourth degree.* Bul. Acad. Ştiinţe Repub. Moldova. Mat., 2015, No. 3(79), p. 14-34.

Qualitative investigation of cubic differential systems with degenerate infinity and six real invariant straight lines along four directions

Vitalie Puţuntică

Tiraspol State University, Chişinău, Republic of Moldova

e-mail: `vitputuntica@mail.ru`

We consider the real cubic differential system

$$\begin{cases} \dot{x} = p_0 + p_1(x,y) + p_2(x,y) + q_3(x,y) \equiv P(x,y), \\ \dot{y} = q_0 + q_1(x,y) + q_2(x,y) + q_3(x,y) \equiv Q(x,y), \end{cases} \tag{1}$$

where p_j, q_j $(j = \overline{0,3})$ are homogeneous polynomial of degree j, $gcd\{P,Q\}=1$ and $|p_3(x,y)| + |q_3(x,y)| \not\equiv 0$. If $yp_3(x,y) - xq_3(x,y) \equiv 0$, then the infinity for (1) is degenerate, i.e. consists only of singular points.

The straight line $l \equiv \alpha x + \beta y + \gamma = 0$ is said to be *invariant* for (1) if there exists a polynomial $K(x,y) \in \mathbb{C}[x,y]$ such that the identity in x and y: $\alpha \cdot P(x,y) + \beta \cdot Q(x,y) \equiv (\alpha x + \beta y + \gamma) \cdot K(x,y)$ holds.

In this paper is classified cubic differential systems with degenerate infinity having six real invariant straight lines along four directions. The following result is obtained:

Theorem. *Any cubic differential systems with degenerate infinity having six real invariant straight lines along four directions, via affine transformation and times rescaling can by written as one of the two systems:*

I) $\begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = (y-1)(ay+x^2),\ a>0; \end{cases}$ II) $\begin{cases} \dot{x} = x(x+1)(1+2y), \\ \dot{y} = y(y+1)(1+2x). \end{cases}$ The invariant straight lines of system (I) are: $x=0$, $x+1=0$, $x-a=0$, $y-1=0$, $x-ay=0$, $x+y=0$ and for system (II): $x=0$, $x+1=0$, $y=0$, $y+1=0$, $y-x=0$, $x+y+1=0$.

Systems (I) and (II) has a Darboux first integral of the form respectively

$$F(x,y) = \frac{(x-a)(y+ax+a)}{xy}, \quad F(x,y) = \frac{y(y+1)}{(y-x)(x+y+1)}.$$

For systems (I) and (II) are built phase portraits in the Poincaré disc respectively:

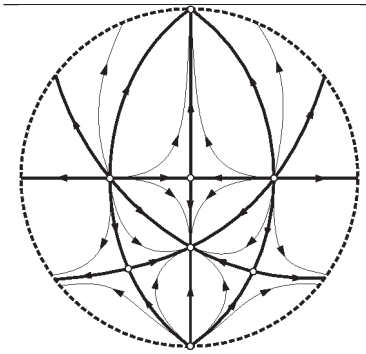


Fig. 1

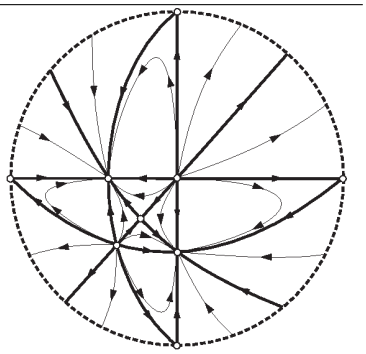


Fig. 2

Bibliography

[1] Şubă A., Repeşco V., Puţuntică V., *Cubic systems with invariant affine straight lines of total parallel multiplicity seven.* Electronic Journal of Differential Equation, Vol. 2013 (2013), No. 274, p. 1-22. SUA Texas.

[2] Şubă A., Repeşco V., Puţuntică V., *Cubic systems with seven invariant straight lines of configuration (3, 3, 1).* Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica, no. 2(69), 2012, p. 81-98.

[3] Puţuntică V., Şubă A., *Classification of the cubic differential systems with seven real invariant straight lines.* ROMAI Journal, 2009, vol. 5, no. 1, p. 121-122.

[4] Puţuntică V., Şubă A., *The cubic differential system with six real invariant straight lines along three directions*, Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica, no. 2(60), 2009, p. 111-130.

[5] Puţuntică V., Şubă A., *The cubic differential system with six real invariant straight lines along two directions*, Studia Universitatis. Seria Ştiinţe exacte şi economice, no. 8(13), 2008, p. 5-16.

Canonical forms of cubic differential systems with invariant straight lines of total multiplicity seven along one direction

Vadim Repeșco

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: repescov@gmail.com

Consider the general cubic differential system $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where $P, Q \in \mathbb{R}[x, y]$, $\max\{\deg P, \deg Q\} = 3$ and $GCD\,(P, Q) = 1$.
According to [1], we can construct a Darboux first integral for a cubic differential system, if this system has sufficiently many invariant straight lines considered with their multiplicities.
In this paper we obtain 26 canonical forms of cubic differential systems which possess invariant straight lines of total multiplicity at least seven (including the invariant straight line at the infinity) along one direction.

Bibliography

[1] Llibre J., Xiang Zhang, *On the Darboux Integrability of Polynomial Differential Systems*, Qual. Theory Dyn. Syst., 2012, pp. 129–144.

On the solution set of differential equations driven by measures

Bianca Satco

Stefan cel Mare University of Suceava
e-mail: bisatco@usm.ro

Existence results and properties of the solution set of measure differential equations in Banach spaces will be described. Studying this very general class of equations gives us the possibility to get results for impulsive equations or equations on time scale domains. Examples will also be presented.

Heteroclinic orbits corresponding to a double-zero with symmetry of order two bifurcation

Mihaela Sterpu, Carmen Rocșoreanu

University of Craiova, Romania
e-mail: msterpu@yahoo.com, rocsoreanu@yahoo.com

A dynamical system possessing an equilibrium point with two zero eigenvalues is considered. We assume that a degenerate Bogdanov-Takens bifurcation with symmetry of order two is present and, in the parameter space, a curve of double heteroclinic bifurcation values emerges from the codimension two bifurcation point. Using a blow-up transformation and a perturbation method, we obtain second-order approximations both for the heteroclinic orbits and for the curve of heteroclinic bifurcation values. Finally, some numerical simulations illustrating the accuracy of our results are performed.

Bibliography

[1] S.N. Chow, C. Li, D. Wang, *Normal Forms and Bifurcation of Planar Vector Fields*, (Cambridge University Press, Cambridge and New York), 1994.

[2] Y.A. Kuznetsov, *Elements of Applied Bifurcation Theory*, 3rd Ed. (Springer, New York), 2004.

[3] Y.A. Kuznetsov, H.G.E. Meijer, B. Al-Hdaibat, W. Govaerts, *Improved homoclinic predictor for Bogdanov-Takens bifurcation*, IJBC, 24(4), 2014.

[4] C. Rocsoreanu, M. Sterpu, *Approximations of the homoclinic orbits near a double-zero with symmetry of order two bifurcation*, IJBC, 27(7), 2017.

Cubic differential systems with a weak focus and an affine real invariant straight line of maximal multiplicity

Alexandru Şubă and Silvia Turuta

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: alexandru.suba@math.md, poderioghin_silvia@yahoo.com

We consider the real cubic differential system

$$\begin{aligned} &\dot{x} = y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3 \equiv P(x,y), \\ &\dot{y} = -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3) \equiv Q(x,y), \\ &\quad \gcd(P,Q) = 1 \end{aligned} \tag{1}$$

and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to the system (1). For the system (1) the origin of coordinates $(0,0)$ is a weak focus, i.e. is a singular point of focus or center type.
An algebraic curve $f(x,y) = 0,\ f \in \mathbb{C}[x,y]$ is called invariant algebraic curve of the system (1) if there exists a polynomial $K_f \in \mathbb{C}[x,y]$ such that the identity $\mathbb{X}(f) \equiv f(x,y)K_f(x,y),\ (x,y) \in \mathbb{R}^2$ holds.

An invariant algebraic curve f of degree d for the vector field $\mathbb{X}$ has algebraic multiplicity k when k is the greatest positive integer such that the k-th power of f divides $E_d(\mathbb{X})$, where

$$E_d(\mathbb{X}) = det \begin{pmatrix} \upsilon_1 & \upsilon_2 & ... & \upsilon_l \\ \mathbb{X}(\upsilon_1) & \mathbb{X}(\upsilon_2) & ... & \mathbb{X}(\upsilon_l) \\ ... & ... & ... & ... \\ \mathbb{X}^{l-1}(\upsilon_1) & \mathbb{X}^{l-1}(\upsilon_2) & ... & \mathbb{X}^{l-1}(\upsilon_l) \end{pmatrix},$$

and $\upsilon_1, \upsilon_2, ..., \upsilon_l$ is a basis of $\mathbb{C}_d[x,y]$ (see [1]). If $d = 1$ then $\upsilon_1 = 1, \upsilon_2 = x, \upsilon_3 = y$ and $E_1(\mathbb{X}) = P \cdot \mathbb{X}(Q) - Q \cdot \mathbb{X}(P)$. Let the system (1) has a real invariant straight line l. By a transformation of the form

$$x \to \alpha \cdot (x\cos\varphi + y\sin\varphi), y \to \alpha \cdot (y\cos\varphi - x\sin\varphi),\ \alpha \neq 0 \tag{2}$$

we can make l to be described by the equation $x = 1$. Then, $k = -a, m = -c-1, p = -f, r = 0$. Denote $\gamma = c + 2$.

Theorem 1. *In the class of cubic differential systems* (1) *the maximal algebraic multiplicity of an affine real invariant straight line is equal to 4. Via a transformation* (2) *each cubic system* (1) *which has an affine real invariant straight line of algebraic multiplicity 4 can be written in the one of the following three forms*:

$$\dot{x} = y(x-1)(-1+x-fy),\ \dot{y} = -x + 2x^2 - x^3 - y^2 + xy^2 - fy^3,\ \ f \neq 0; \tag{3}$$

$$\begin{aligned} &\dot{x} = (1-x)(4ay + 4a^2x^2 + 4a(\gamma-1)xy + \gamma^2y^2)/(4a), \\ &\dot{y} = -(4a\gamma^2x + 8a(a^2(\gamma-2) - \gamma^2)x^2 + 8a^2\gamma^2xy + 2a\gamma^2(\gamma+2)y^2 + \\ &\quad 4a(4a^2+\gamma^2)x^3 + 4a^2\gamma^2x^2y + 4a(\gamma-1)\gamma^2xy^2 + \gamma^4y^3)/(4a\gamma^2); \end{aligned} \tag{4}$$

$$\dot{x} = y(x-1)^2,\ \dot{y} = -x + 2x^2 - dxy - 2y^2 - x^3 - qx^2y + 2xy^2. \tag{5}$$

It is well known that the singular point $(0,0)$ is of the center type for the system (1) if and only if (1) has in a neighborhood of $(0,0)$ an analytic first integral $F(x,y)$ (integrating factor $\mu(x,y)$).

Theorem 2. *If the system* (1) *has a real affine invariant straight line of the algebraic multiplicity four then the origin* $(0,0)$ *is a center for* (1) *if and only if the first Lyapunov quantity vanish.*

Indeed, the systems (3) and (4) have the first integrals respectively:

$$\begin{aligned}
&F(x,y) = (x-1)^6 \exp[(6-12x+6x^2+3y^2-3xy^2+2fy^3)/(1-x)^3];\\
&F(x,y) = (x-1)^{-48a^2(4a^2+\gamma^2)} \exp[4a(8a^3(\gamma+9)+12a\gamma^2-\\
&\quad 24a(a^2(\gamma+7)+\gamma^2)x+12a(2a^2(\gamma+4)+\gamma^2)x^2+6a\gamma^2y^2+\\
&\quad 12a^2\gamma^2x^2y+6a(\gamma-1)\gamma^2xy^2+\gamma^4y^3)/(x-1)^3].
\end{aligned}$$

For the system (5) the first Lyapunov quantity vanish if and only if $q=0$. If $q=0$ then (5) has the integrating factor

$$\mu(x,y) = \frac{1}{(x-1)^6} \exp \frac{d(-d+3dx+6y-6xy)}{6(-1+x)^3}.$$

Acknowledgments. The authors acknowledge the support of project 15.817.02.03F.

Bibliography

[1] C. Christopher, J. Llibre and J.V. Pereira, *Multiplicity of invariant algebraic curves in polynomial vector fields*, Pacific Journal of Mathematics, **329** (2007), no. 1, 63–117.

Quartic differential systems with an affine real invariant straight line of algebraic multiplicity four

Alexandru Şubă and Olga Vacaraş

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: alexandru.suba@math.md, vacaras_olga@mail.md

We consider the differential system of the fourth degree

$$\begin{aligned}
\dot{x} &= P_0 + P_1(x,y) + P_2(x,y) + P_3(x,y) + P_4(x,y) \equiv P(x,y),\\
\dot{y} &= Q_0 + Q_1(x,y) + Q_2(x,y) + Q_3(x,y) + Q_4(x,y) \equiv Q(x,y),
\end{aligned} \tag{1}$$

where P_k and Q_k, $k=1,2,3,4$ are homogeneous polynomials in x and y of degree k.
Suppose that

$$yP_4(x,y) - xQ_4(x,y) \not\equiv 0, \qquad \gcd(P,Q)=1, \tag{2}$$

i.e. at infinity the system (1) has at most five distinct singular points and the right-hand sides of (1) do not have the common divisors of degree greatest that 0.
Let the system (1) has a real invariant straight line l. By an affine transformation we can make l to be described by the equation $x=0$. Then, the system (1) looks as:

$$\begin{aligned}
\dot{x} &= x(a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 +\\
&\quad + a_8x^2y + a_9xy^2 + a_{10}y^3),\\
\dot{y} &= b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y +\\
&+ b_8xy^2 + b_9y^3 + b_{10}x^4 + b_{11}x^3y + b_{12}x^2y^2 + b_{13}xy^3 + b_{14}y^4.
\end{aligned} \tag{3}$$

Theorem. *For quartic differential system* $\{(3),\ (2)\}$ *the algebraic multiplicity of the invariant straight line* $x=0$ *is greater or equal to four if and only if at least one of the following fourteen sets of conditions:*

1) $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_8 = a_9 = a_{10} = 0,\ a_7 \neq 0;$

2) $a_1 = a_2 = a_3 = a_5 = a_6 = a_9 = a_{10} = b_5 = b_9 = b_{14} = 0,\ b_0 = a_4 b_2/a_8,\ b_2 \neq 0;$

3) $a_1 = a_3 = a_6 = a_{10} = b_9 = b_{14} = 0,\ a_5 = a_2 b_2/b_0,\ a_8 = (-2a_2^2 b_0 + a_4 b_0 b_2 - a_2 b_1 b_2 + a_2 b_0 b_4)/b_0^2,\ a_9 = b_{13}/2,\ b_5 = b_0 b_{13}/(2a_2),\ b_8 = (-a_4 b_0^2 b_{13} + a_2 b_0 b_1 b_{13} + 4a_2^3 b_2)/(2a_2^2 b_0);$

4) $a_1 = a_3 = a_6 = a_{10} = b_9 = b_{14} = 0,\ a_4 = (2a_2^2 a_5 + a_5^2 b_1 + a_2 a_8 b_2 - a_2 a_5 b_4)/(a_5 b_2),\ a_9 = a_5 b_5/b_2,\ b_0 = a_2 b_2/a_5,\ b_8 = (2a_5^2 b_2 - 2a_2 a_5 b_5 - a_8 b_2 b_5 + a_5 b_4 b_5)/(a_5 b_2),\ b_{13} = 2a_5 b_5/b_2;$

5) $a_1 = a_2 = a_3 = a_5 = a_6 = a_{10} = b_0 = b_2 = b_9 = b_{13} - 2a_9 = b_{14} = 0,\ b_1 = a_4 b_5/a_9,\ b_4 = a_8 b_5/a_9,\ b_5 \neq 0;$

6) $a_1 = a_2 = a_3 = a_5 = a_6 = a_8 = a_9 = a_{10} = b_2 = b_5 = b_9 = b_{14} = 0,\ a_4 \neq 0;$

7) $a_3 = a_6 = a_{10} = b_2 - a_1 = b_5 = b_9 = b_{12} - a_8 = b_{13} - a_9 = b_{14} = 0,\ a_2 = (-a_5 b_0 + a_1 b_4)/a_1,\ b_1 = b_0(-a_5 b_0 + a_1 b_4)/a_1^2,\ b_3 = a_4 b_0/a_1,\ b_7 = (a_1 a_4 + a_8 b_0)/a_1,\ b_8 = (a_1 a_5 + a_9 b_0)/a_1;$

8) $a_6 = a_{10} = b_5 - a_3 = b_9 = b_{12} - a_8 = b_{13} - a_9 = b_{14} = 0,\ b_0 = a_1(b_2 - a_1)/a_3,\ b_1 = a_2(b_2 - a_1)/a_3,\ b_3 = a_4(b_2 - a_1)/a_3,\ b_4 = (a_2 a_3 - a_1 a_5 + a_5 b_2)/a_3,\ b_7 = (a_3 a_4 - a_1 a_8 + a_8 b_2)/a_3,\ b_8 = (a_3 a_5 - a_1 a_9 + a_9 b_2)/a_3;$

9) $a_{10} = 2b_5 - 3a_3 = b_9 - a_6 = b_{12} - a_8 = b_{13} - a_9 = b_{14} = 0,\ a_1 = a_3^2/(4a_6),\ b_0 = a_3^3/(8a_6^2)$ $b_1 = a_2 a_3/(2a_6),\ b_2 = 3a_3^2/(4a_6),\ b_4 = (a_3 a_5 + 2a_2 a_6)/(2a_6),\ b_7 = (2a_4 a_6 + a_3 a_8)/(2a_6)$ $b_8 = (2a_5 a_6 + a_3 a_9)/(2a_6);$

10) $a_{10} = 2b_5 - 3a_3 = b_9 - a_6 = b_{12} - a_8 = b_{13} - a_9 = b_{14} = 0,\ b_0 = a_3^3/(8a_6^2),\ b_1 = a_3(-2a_3 a_5 a_6 + 8a_2 a_6^2 + a_3^2 a_9)/(8a_6^3),\ b_2 = 3a_1 = 3a_3^2/(4a_6),\ b_4 = (8a_2 a_6^2 + a_3^2 a_9)/(4a_6^2),\ b_7 = (4a_4 a_6^3 + 2a_3 a_6^2 a_8 - 2a_3 a_5 a_6 a_9 + 4a_2 a_6^2 a_9 + a_3^2 a_9^2)/(4a_6^3),\ b_8 = (2a_5 a_6 + a_3 a_9)/(2a_6);$

11) $a_{10} = b_9 - a_6 = b_{12} - a_8 = b_{13} - a_9 = b_{14} = 0,\ b_0 = a_1(b_5 - a_3)/a_6,\ b_1 = a_2(b_5 - a_3)/a_6,\ b_2 = (-a_3^2 + a_1 a_6 + a_3 b_5)/a_6,\ b_3 = a_4(b_5 - a_3)/a_6,\ b_4 = (-a_3 a_5 + a_2 a_6 + a_5 b_5)/a_6,\ b_7 = (a_4 a_6 - a_3 a_8 + a_8 b_5)/a_6,\ \ b_8 = (a_5 a_6 - a_3 a_9 + a_9 b_5)/a_6;$

12) $b_0 = a_1(b_9 - a_6)/a_{10},\ b_1 = a_2(b_9 - a_6)/a_{10},\ b_2 = a_1 a_{10} - a_3 a_6 + a_3 b_9)/a_{10},\ b_3 = a_4(b_9 - a_6)/a_{10},\ b_4 = (a_{10} a_2 - a_5 a_6 + a_5 b_9)/a_{10},\ b_5 = (a_{10} a_3 - a_6^2 + a_6 b_9)/a_{10},\ b_7 = (a_{10} a_4 - a_6 a_8 + a_8 b_9)/a_{10},\ \ b_8 = (a_{10} a_5 - a_6 a_9 + a_9 b_9)/a_{10},\ b_{12} = a_8,\ b_{13} = a_9,\ b_{14} = a_{10};$

13) $a_1 = (3a_6 - 2b_9)(a_6 - b_9)^2/a_{10}^2,\ a_3 = (5a_6 - 3b_9)(b_9 - a_6)/a_{10},\ b_0 = -(3a_6 - 2b_9)(a_6 - b_9)^3/a_{10}^3,\ b_1 = a_2(b_9 - a_6)/a_{10},\ b_2 = (8a_6 - 5b_9)(a_6 - b_9)^2)/a_{10}^2,\ b_3 = (-4a_{10} a_4 a_6 + 3a_6^2 a_8 + 3a_{10} a_6 b_7 + 3a_{10} a_4 b_9 - 5a_6 a_8 b_9 - 2a_{10} b_7 b_9 + 2a_8 b_9^2)/a_{10}^2,\ b_4 = (a_{10} a_2 - a_5 a_6 + a_5 b_9)/a_{10},\ b_5 = -3(a_6 - b_9)(2a_6 - b_9)/a_{10},\ b_8 = (a_{10} a_5 - a_6 a_9 + a_9 b_9)/a_{10},\ b_{12} = a_8,\ b_{13} = a_9,\ b_{14} = a_{10};$

14) $a_1 = (3a_6 - 2b_9)(a_6 - b_9)^2/a_{10}^2,\ a_2 = -(5a_{10} a_5 a_6 - 3a_6^2 a_9 - 4a_{10} a_6 b_8 - 4a_{10} a_5 b_9 + 5a_6 a_9 b_9 + 3a_{10} b_8 b_9 - 2a_9 b_9^2)/a_{10}^2,\ a_3 = -(5a_6 - 3b_9)(a_6 - b_9)/a_{10},\ b_0 = (3a_6 - 2b_9)(b_9 - a_6)^3/a_{10}^3,\ b_1 = (a_6 - b_9)(8a_{10} a_5 a_6 - 6a_6^2 a_9 - 7a_{10} a_6 b_8 - 6a_{10} a_5 b_9 + 10a_6 a_9 b_9 + 5a_{10} b_8 b_9 - 4a_9 b_9^2)/a_{10}^3,\ b_2 = (8a_6 - 5b_9)(a_6 - b_9)^2/a_{10}^2,\ b_3 = (-2a_{10}^2 a_5^2 - 4a_{10}^2 a_4 a_6 + 3a_{10} a_6^2 a_8 + 5a_{10} a_5 a_6 a_9 - 3a_6^2 a_9^2 + 3a_{10}^2 a_6 b_7 + 3a_{10}^2 a_5 b_8 - 4a_{10} a_6 a_9 b_8 - a_{10}^2 b_8^2 + 3a_{10}^2 a_4 b_9 - 5a_{10} a_6 a_8 b_9 - 4a_{10} a_5 a_9 b_9 + 5a_6 a_9^2 b_9 - 2a_{10}^2 b_7 b_9 + 3a_{10} a_9 b_8 b_9 + 2a_{10} a_8 b_9^2 - 2a_9^2 b_9^2)/a_{10}^3,\ b_4 = -(8a_{10} a_5 a_6 - 5a_6^2 a_9 - 6a_{10} a_6 b_8 - 6a_{10} a_5 b_9 + 8a_6 a_9 b_9 + 4a_{10} b_8 b_9 - 3a_9 b_9^2)/a_{10}^2,\ b_5 = -3(a_6 - b_9)(2a_6 - b_9)/a_{10},\ b_{12} = a_8,\ b_{13} = a_9,\ b_{14} = a_{10}$

holds.

Acknowledgments. The authors acknowledge the support of project 15.817.02.03F.

How the (anti)morphisms of Boolean (anti)evolution functions act on invariant sets

Serban E. Vlad

Oradea City Hall
e-mail: serban_e_vlad@yahoo.com

The Boolean evolution functions consist in functions $\Phi : \{0,1\}^n \longrightarrow \{0,1\}^n$ that iterate (in discrete or continuous time) their coordinates Φ_i independently on each other, $i \in \{1, ..., n\}$ and they can be considered as representing Boolean dynamical systems with a variable structure. The concept of morphism is the usual one from the dynamical systems theory, adapted to the discrete, variable structure. A dual perspective is given by the Boolean antievolution functions, that differ from the evolution functions by the fact that time runs backwards. The concept of antimorphism reproduces the idea of morphism, corresponding to the situation when two systems run in opposite senses of time. Several definitions of invariance of a set $A \subset \{0,1\}^n$ are given, for Boolean evolution and antievolution functions. The purpose of the paper is that of showing in which manner the morphisms and the antimorphisms bring invariant sets in invariant sets.

Limits of solutions to a nonlinear second-order ODE

Cristian Vladimirescu

Department of Applied Mathematics, University of Craiova
e-mail: vladimirescucris@yahoo.com

In this talk the existence of solutions to Eq.

$$x'' + 2f(t)\,x' + \beta(t)x + g(t,x) = 0, \qquad t \in \mathbb{R}_+,$$

is discussed. Our approach allows us extension to the case of the whole real line, when the existence of homoclinic solutions having zero limit at $\pm\infty$, is deduced. The result is obtained through the method of Lyapunov function and differential inequalities.

2010 Mathematics Subject Classification: 34A40, 34C37.
Key words and phrases: second-order ODE, Lyapunov function, solutions having zero limit at ∞, homoclinic solution.

Mathematical Modeling

Fourier analysis of steady vibrations of a biharmonic oscillator

Dmytro I. Anpilohov

Zaporizhzhya National Technical University, Ukraine
e-mail: anpilogov@ua.fm

The motion law of a biharmonic oscillator modeling a mechanical structure with one degree of freedom with stepwise varying stiffness is considered. We have constructed dependences of the amplitudes of the harmonics of the signal on the relative change in stiffness on half-cycles of vibrations. The difference in the orders of the smallness of the amplitudes of the even and odd harmonics is revealed in the Taylor expansion in terms of the parameter of the relative change in the stiffness.

Algorithm for increasing the quality of optical trap images

Alexandra Băluță, Diana Rotaru, Mihaela Ilie, Drago Fălie and Marta Zamfir

University Politehnica of Bucharest
e-mail: mihaela_ilie18@yahoo.com, milie@imag.pub.ro, dfalie@imag.pub.ro, alexandraa.baluta@gmail.com

The interaction between the electromagnetic field of an IR laser beam and dielectric microspheres is studied in our laboratory as basic phenomenon of an optical trap experimental set-up. A microscope objective Nokia x 100/1,25 (oil) and a CCD camera is used in order to reveal the trapping effect. Due to the extremely low depth of focus (1μm) of the objective difficulties were encountered when interpreting the images of the obtained movie. An image processing algorithm, using MATLAB computing environment, is developed and applied in order to improve the quality of the images. The bilateral filtering succeeds to smooth the images while preserving the edges: it combines both domain and range filtering. Afterwards, the intensity values of the image are adjusted to increase the contrast. The values are rescaled and improvements noticeable by human perception were obtained. The algorithm was successfully applied to several movies obtained with our optical trap.

Analytic and numeric temporal analysis of the transition probability between two states of a quantum system

Alexandra Baluta, Diana Rotaru, Mihaela Ilie, Dragos Falie, Eugen Vasile

University Politehnica of Bucharest
e-mail: mihaela_ilie18@yahoo.com, milie@imag.pub.ro, dfalie@imag.pub.ro, alexandraa.baluta@gmail.com

The phenomenon of optical assisted transition in crystals in the case of a weak optical radiation has been experimentally studied in our quantum laboratory; both its mathematical modeling and numerical simulation are approached.
The formalism of quantum physics has been used in order to describe the emission or absorption of the optical radiation in an atomic system, in the single photon regime. The Schrdinger equation in

its operator format and state vectors has been used; the matrix format with the Dirac formalism was reserved exclusively for the calculations of scalar vector products.
The atomic system is considered as a chosen system and the photon is considered as a perturbing system. The photon is considered a wavelet with finite spatial and temporal extension. The energy levels of the crystal are considered. The interaction between the two systems is studied on a basis similar with that one used in the information transmission theory. The structural algebraic formalism has been put in evidence for the description of the interaction of the two systems; the previous study of the authors has been reformulated and deepened.
A quasi-complete analytic and numeric temporal analysis of the transition probability between two eigen states of a quantum system in interaction with a perturbation system, in this case an electromagnetic one, is performed. A general formalism has been elaborated based on the Hamiltonian of interaction concept and a rigorous treatment of both the terminology and the notations.
In order to perform the temporal analysis of the transition probability, the first order perturbation theory was used. The calculus relations were standardized. The complex spectral density (Fourier transform) for a perturbation with compact temporal support has been used in order to obtain the transition probability.
An analytical study has been performed in order to validate the numerical simulation. Limit cases, specific cases have been taken into account, graphical representation using MathCAD have been obtained, for numeric evaluation in SI (corresponding to the laser domain). Four cases of the temporal perturbation have been analyzed: monopuls, rectangular monopuls, sinusoidal monopuls modulated by a rectangular envelope, sinusoidal monopuls modulated by a sinusoidal envelope. A basis for the analysis, numerical calculus and simulations for emission or absorption of single photon has been created, useful for future analysis of two- and three- photonic processes.

Mathematical Models in the Risk Assessment in the Decision-Making Process

Inga Bulat

Academy of Sciences of Moldova
e-mail: bulat.inga124@gmail.com

At present, risk analysis in the decision-making process is the discipline of management, which establishes a connection with mathematics and computer technology that deals with the substantiation of the managerial decision under conditions of efficiency in the economic-financial field. The modeling of a decision-making process under conditions of uncertainty leads us to specify its elements, namely: the risk, the formulation of the problem and the specification of the objectives of minimizing / maximizing some technical-economic indicators, the number of possible alternatives / alternatives that characterize a decisional situation, the set of anticipated consequences For each variant, - independent decision-makers and short-term factors. From the set of possible variants, calculated by one or more mathematical methods, the decision maker is to choose only one, that is, the most convenient solution. The evaluation of the decisions in risk and uncertainty in the process of knowledge and analysis of the current situation is accomplished by a multitude of methods, ways and means that make it possible to identify, scientifically determine the economic-financial activity. The method of mathematical models or modeling, the significance of which is described and applied in the analysis and evaluation of scientific research, which does not constitute a new discovery, but which has the effect of reproducing certain aspects of the studied objectives in order to facilitate its scientific research.

Today it is impossible to conceive of an economic or financial discipline that does not use in its process of knowledge methods of quantification, numerical expression, of laws, of interdependencies, of measuring the tendencies in the decision making process.
It is characteristic that the microeconomics itself in its theoretical generalizations, in deducting theses of maximal generality, in its analyzes, often abstract, resorts to statistical-mathematical methods, to the elaboration of economic-mathematical models, to the numerical, rigorous expression of some Processes and phenomena.
Moreover, specific economic disciplines are obliged to resort to this modern instrument which guarantees not only the precision of formulating the conclusions but also the efficiency of the decisions taken on this basis in the concrete economic activity.

A model of the Universe with adiabatic particle creation mechanism

Subenoy Chakraborty[1], Subhajit Saha[2]

[1] *Department of Mathematics, Jadavpur University, Kolkata 700032, India*
[2] *Department of Mathematics, Panihati Mahavidyalaya, Kolkata 700110, India*
e-mail: `scahkaraborty.math@gmail.com, subhajit1729@gmail.com`

Schrodinger initiated the microscopic description of the gravitationally induced particle creation mechanism (PCM) in an expanding universe. After a long gap, investigations carried out by Parker, and Zeldovich and Starobinsky showed that due to quantum effects particles are created by the expansion of spacetime. Particle creation by the changing time-dependent spacetime is also the driving mechanism behind Hawking radiation. Although these investigations were carried out in the last century, gravitational particle creation still remains as an active area of current research. In this talk, we shall deal with the PCM in the framework of irreversible thermodynamics. To that effect, a bulk viscous pressure Π, associated with entropy production, is introduced in the energy-momentum tensor of the cosmic fluid which gives rise to a dissipative effect. In the background of a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological model, we assume adiabatic (or, isentropic) creation of particles so that the entropy per particle remains constant and consequently the dissipative pressure Π can be expressed linearly in terms of the particle creation rate Γ. Further, using the Friedmann and the Raychaudhuri equations and by assuming phenomenological, yet thermodynamically motivated choices of Γ as a function of the Hubble parameter H, it is possible to show (separately) a transition from the inflationary phase to the radiation era and also from the matter dominated era to the observed late time acceleration of the Universe. With the help of the *piecewise* command of the Maple software, a continuous cosmic evolution from inflation to late time acceleration can be observed by adjusting the parameters involved. In a nutshell, this mathematical model of the Universe can successfully serve as an alternative to dark energy as well as the modified gravity models found in the literature.

Bibliography

[1] E. Schrodinger, Physica (Amsterdam) **6**, 899 (1939).

[2] L. Parker, Phys. Rev. Lett. **21**, 562 (1968).

[3] L. Parker, Phys. Rev. **183**, 1057 (1969).

[4] Y.B. Zeldovich and A.A. Starobinsky, Soviet Phys. JETP **34**, 1159 (1972).

[5] Y.B. Zeldovich and A.A. Starobinsky, JETP Lett. **26**, 252 (1977).

[6] S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[7] J.D. Barrow, Phys. Lett. B **180**, 335 (1986).

[8] J.D. Barrow, Nucl. Phys. B **310**, 743 (1988).

[9] J.D. Barrow and T. Clifton, Phys. Rev. D **73**, 103520 (2006).

[10] E. Gunzig, R. Maartens, and A.V. Nesteruk, Class. Quantum Gravit. **15**, 923 (1998).

[11] S. Chakraborty and S. Saha, Phys. Rev. D **90**, 123505 (2014).

A case study of optimal portfolio composition. Critical lines

Fitim Deari[1], Carmen Rocșoreanu[2]

[1] *South East European University, Tetovo, Macedonia,*
[2] *University of Craiova, Romania*
e-mail: `f.deari@seeu.edu.mk, rocsoreanu@yahoo.com`

A portfolio composition, where the weights are supposed to depend on the investor's risk tolerance, is considered. The portfolio's certainty equivalent return has to be maximized. The mathematical model leads to a convex problem. The attached Kuhn-Tucker system is solved using the critical line method.

Bibliography

[1] Bailey, H. David; Lpez de Prado, Marcos. *An Open-Source Implementation of the Critical-Line Algorithm for Portfolio Optimization*, Algorithms, 2013, 6, pp. 169-196.

[2] Calvo, Clara; Ivorra, Carlos; and Liern, Vicente. *On the Computation of the Efficient Frontier of the Portfolio Selection Problem*, Journal of Applied Mathematics, 2012.

[3] Kwan, Clarence C. -A. *Simple Spreadsheet-Based Exposition of the Markowitz Critical Line Method for Portfolio Selection*, Spreadsheets in Education (eJSiE): Vol. 2: Iss. 3, 2007, `http://epublications.bond.edu.au/ejsie/vol2/iss3/2`.

[4] ***, `Mathematica`, Wolfram Research Inc. (2012).

The construction of an algorithm with optimal complexity

Gârlă Eugeniu

ASEM, Chişinău, Moldova
e-mail: `eugeniugarla@yahoo.com`

The paper analyses a class of nonlinear optimization problems with special restrictions, we propose a concept for solving the auxiliary problem, for which we calculate complexity, we also assesses the maximum number of elementary operations and describe the optimal algorithm for performing numerical calculations.

Reorientation of the transitive graphs

Grigoriu Nicolae

State University of Moldova
e-mail: grigoriunicolae@gmail.com

Transitive graphs have many applications in computer science and many other theoretical and practical fields[1], [4]. Many of the NP-problems that have a transitive graph representation can be solved in polynomial time [3]. In this context it is important to have an optimal solution for reorientation of the transitive graph.

Theorem 1. *[2] If $\overrightarrow{G}$ is a transitive orientation of the undirected graph $G = (X; U)$ that has more then two orientations, then there is a set of arcs $\overrightarrow{E} \neq \overrightarrow{U}$, $\overrightarrow{E} \neq \emptyset$ so the reverse orientation of the arcs in $\overrightarrow{E}$ leads to new orientation of the graph G.*

Definition 1. *The subset of edges of the B-stable subgraph $F = (X; U_F)$ is called internal factor defined by subgraph F.*

Definition 2. *If F is a B-stable subgraph of the transitively orientable graph G, and $x \in X_G \setminus X_F$ is an adjacent vertex to the set X_F, then the set of edges $[x, y], \forall y \in X_F$ is called external factor defined by the subgraph F.*

Theorem 2. *If E_{F_i} is an external factor defined by the B-stable graph F_i, then there is an internal factor I_{F_j} so the following condition is satisfied: $E_{F_i} \subseteq I_{F_j}$, $0 \leq i \leq k-2$, $i+1 \leq j \leq k-1$. Where k is the length of the complete set of the factor graphs of the transitively orientable graph G.*

Bibliography

[1] S. C. Basak, G. Restrepo and J. L. Villaveces, Eds. *Advances in Mathematical Chemistry and Applications* Vol. 1, 2014.

[2] S.Cataranciuc, N.Grigoriu *Algorithmic approach in reorientation of comparability graphs* Vol. 60 Issue 2, p37-46. 10p, 2015.

[3] Chính T Hoàng, *Efficient algorithms for minimum weighted colouring of some classes of perfect graphs* Vol. 55, no. 2, pp. 133-143, 1994.

[4] R. Wisniewski, A. Karatkevich, M. Adamski and D. Kur. *Application of comparability graphs in decomposition of Petri nets* in Human System Interactions (HSI), 2014 7th International Conference on, 2014.

Fractional order integro-differential equations solution by artificial neural networks approach

Ahmad Jafarian, Safa Measoomy Nia

Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran
e-mail: Jafarian5594@yahoo.com, measoomy@yahoo.com

Great care must be taken in considering the fact that neural networks moved in the direction of a systematic world such as applied mathematics and engineering sciences. Such certain movement helped shaping fantastic changes in the numerical solution of complicated cases which are overt in natural phenomena. In the present study, a comprehensive optimization mechanism consisting of a reliable three-layered feed-forward neural network is formed to solve a class of fractional order ordinary integro-differential equations. One point should be kept in mind that the supervised back-propagation type learning algorithm which is based on the gradient descent method, is capable of approximating the mentioned problem on an arbitrary interval to any desired degree of accuracy. Besides, some comparative test problems are given to reveal the flexibility and efficiency of the proposed method.

Keywords: Fractional order integro-differential equation; Artificial neural networks approach; Least mean squares cost function; Supervised back-propagation learning algorithm.

Hyperbolic equations by `Mathematica`

Marian Mureşan

Babeş-Bolyai University, Cluj-Napoca
e-mail: mmarianus24@yahoo.com

The goal of the paper is to see the power of *Mathematica* for studying partial differential equations of hyperbolic type. A given code, even if it was developed by a gifted programmer, hardly can solve any such equation. In our paper we introduce some hyperbolic partial differential equations, study if they are solvable by closed-form formulas, if not we approach them by numerical means, and finally plot the results. Everything is carried out under the umbrella of `Mathematica`.

An online air pollution monitoring system with an integrated early warning mechanism based on hybrid neural networks

Alin Pohoata[1], Daniel Dunea[2], Emil Lungu[1]

Valahia University Targoviste
[1]*Faculty of Science and Arts,* [2]*Faculty of Environmental Engineering*
e-mail: alinpohoata@yahoo.com

One of the major airborne pollutants in urban environments is Particulate Matter (PM) containing inhalable particles that penetrate the thoracic region of the respiratory system determining considerable negative health effects, which aggravates with the lower sizes of particles, exposure duration and people Rs vulnerability(age, medical record, socio-economical status). We developed an

on-line monitoring system for $PM_{2.5}$ (fine particulates) which uses self-designed microstations with an integrated early warning mechanism based on a neural network with a wavelet decomposition preprocessing. Using the Daubechies db3 wavelets as a decomposing preprocessor of hourly averages time series of $PM_{2.5}$ has significantly improved the out of sample forecasted values compared to the soled use of FANN.

A Comparasion of Some Numerical Methods for Semiconductor Device Problem

Pațiuc Vladimir, Sprincean Galina

Moldova State University, Republic of Moldova
e-mail: `patsiuk@mail.ru, sgalina11@rambler.ru`

The considered problem consists in determination of semiconductor diode parameters. The mathematical formulation of the problem is based on Drift-Diffusion model [1]. The model is given by a set of equations for three unknown functions: φ - the electrostatic potential, n, p - the concentrations for electrons and holes, respectively. These functions φ, n, p are to satisfy the following system of nonlinear differential equations:

$$-\nabla \cdot (\varepsilon \nabla \varphi) = q(p - n + N);$$

$$-\nabla \cdot (J_n) = -q(R_{SRH} + R_{AUG});$$

$$\nabla \cdot (J_p) = -q(R_{SRH} + R_{AUG});$$

$$J_n = q\mu_n nE + qD_n \nabla n;$$

$$J_p = q\mu_p pE - qD_p \nabla p;$$

$$E = -\nabla \varphi.$$

Here E is the intensity vector of electric field; ε is the dielectric constant; q is the (positive) electron charge; D_n and D_p are the electrons and holes coefficients of diffusion; N is the so-called doping profile; J_n and J_p are the electrons and holes current densities; R_{SRH}, R_{AUG} represent the Shockley-Hall-Read and Auger recombination rates; μ_n, μ_p are the electrons and holes carrier mobilities.
The problem is solved numerically on the Scharfetter - Gummel discretization [4],[5], by means of BI- Conjugate Gradient [2] and Gauss-Jordan methods [3]. As the equations are strongly nonlinear, then in order to obtain the convergent solution we apply the iterative procedure that consists in gradually increasing of the input voltage with small step. The obtaining solutions are used for equation linearization.

Bibliography

[1] W.H.A. Schilders, E.J.W. ter Maten. *Numerical Methods in Electromagnetics. Special Volume of Handbook of Numerical Analysis, vol. XIII, ELSEVIER, Amsterdam. Guest, ed. 2005. pp.317-443*

[2] Fletcher, R. Watson, G.*Conjugate gradient methods for indefinite systems. Numerical Analysis. Lecture Notes in Mathematics. Springer Berlin / Heidelberg. Alistair, ed. 1976. pp. 73 – 89*

[3] Lipschutz, Seymour, Lipson, Mark. Schaum @ Ys Outlines: Linear Algebra. Tata McGraw-hill edition. Delhi 2001. pp. 69-80.

[4] Frensley, W.R. *Scharfetter-Gummel discretization scheme for Drift -Diffusion ecuation. University of Texas at Dallas. 2004. pp.1-3*

[5] Pinakpani. Nayak. *1D Drift Diffusion simulator for modeling pn - junction Diode, Arizona State University, 2008, pp.11-13*

Mathematical Model of Adsorption in Wastewater Treatment

H.A. Simonyan, M.Z.Petrosyan, G.H.Torosyan

Department of Biochem&Environmental Technologies, National Polytechnic University of Armenia
e-mail: hayksim1@gmail.com

It's had been presented methods of sewage treatment, a mathematical model of the selected process. The purpose of modeling is to determine the optimum process conditions, managing them on basis of a mathematical model and transfer the results to an object.
A complete mathematical model includes a description of the relationships between the main process variables in steady-state regimes (static model) and in time from one mode to another (dynamic model).
The solution of the model under consideration consists in selecting the appropriate equations for describing the operating mode, checking the restrictions on the input and output parameters, calculating the objective function. Variable input parameters include granulometric composition and sorbent layer height. The adsorption capacity of the adsorbents used, the quality and composition of the initial wastewater, the specified productivity and the degree of purification are the unvariable input parameters of the process.
Since the regeneration of the zeolite in the process is not envisaged, then in solving the mathematical model we were satisfied with the selection of the corresponding equations for describing the operating mode.
Keywords: mathematical model, adsorption, wastewater, kinetics.

The stressed-strain state of a rotating ring

Nataliia V. Snizhko

Zaporizhzhya National Technical University, Ukraine
e-mail: snizhko.nataliia@gmail.com

In the paper the boundary plane elasticity problem for the ring is solved using Muskhelishvili complex potentials method. The centrifugal forces of inertia are taken into account. The mutual rotation angle of the boundary points is obtained by analyzing the displacement field.
It is shown that the centrifugal inertial forces have no effect on the mutual rotation angle. Agreement with the results known for the static case is received.

Real, Complex, Functional and Numerical Analysis

Hardy inequalities and Hamiltonian operators: some applications to PDEs

Cristian Cazacu

Department of Mathematics, University of Bucharest, Romania, "Simion Stoilow" Institute of Mathematics of the Romanian Academy, Romania
e-mail: cristian.cazacu@fmi.unibuc.ro

We consider singular operators of the form $P-\lambda V$, $\lambda \geq 0$, where P is a second order differential operator (typically the Laplacian or the magnetic Laplacian) and V denotes (typically) a positive Hardy weight with quadratic singularities. Due to Hardy inequalities, for such operators we discuss various properties such as positivity, criticality, optimality, spectral properties, etc. Then we emphasize how such operators apply to study the well-posedness and the asymptotic behavior of some evolution PDEs with singular coefficients.

A comparison between Akima and Hermite type cubic spline with minimal quadratic oscillation in average

Larisa Cheregi

Technical University of Cluj-Napoca
e-mail: cheregi.larisa@yahoo.com

Over the past decade, theory of splines is being widely used in computerized graphics and based on it, this article presents a comparison between Akima and Hermite type cubic spline with minimal quadratic oscillation, using an example from diabetology, a glycemic profile with experimental data from Municipal Hospital of Oradea, and a design construct of aerodynamic profiles to determine which function is the most efficient for increasing speed.

Stancu-type operators on a triangle with one curved side

Larisa Cheregi

Technical University of Cluj-Napoca
e-mail: cheregi.larisa@yahoo.com

There are constructed some Stancu-type operators defined on a triangle with one curved side, their product and boolean sum operators and their properties, their orders of accuracy and the remainders of the corresponding approximation formulas are studied.

On Birkhoff weak integrability in Banach spaces setting

Anca Croitoru[1], Alina Gavriluţ[1] and Alina Iosif[2]

[1] *Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania*
[2] *Department of Computer Science, Information Technology, Mathematics and Physics, Romania*
e-mail: croitoru@uaic.ro, gavrilut@uaic.ro, emilia.iosif@upg-ploiesti.ro

In this talk, different problems concerning atoms/non-atomicity related to Gould and Birkhoff integrals are treated in the non-additive set-valued case.

On semi-compact operators

Omer Gok

Yildiz Technical University,Mathematics Department, Istanbul, Turkey
e-mail: gok@yildiz.edu.tr

Let X be a Banach space and E be a Banach lattice. An operator $T : X \to E$ is called a semi-compact operator if every $\epsilon > 0$ there is an $0 \leq x$ in E such that $T(ball(X)) \subseteq [-x, x] + \epsilon(ball(E))$. In this talk, we investigate semi-compactness of an operator and its adjoint.

Bibliography

[1] C.D. Aliprantis, O.Burkinshaw, Positive Operators, Academic Press, Orlando, 1985.

On properties of the positive solutions of a class of nonlinear evolution equations in an abstract Lebesgue space

Cecil P. Grünfeld

"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of ROMANIAN ACADEMY
e-mail: grunfeld@venus.nipne.ro

We extend previous known results on the existence theory of global-in-time solutions to the Cauchy problem for a class of nonlinear evolution equation in an abstract Lebesgue space, arising from the collisional kinetic theory of fluids. Our results are obtained in the framework of equations satisfying an abstract generalization of Povzner's moment - closure inequality for the Boltzmann equation. The analysis relies on the control of the nonlinearity by means of intrinsic a priori estimates extending, in abstract form, conservation/dissipation properties of various kinetic equations.

On Characterization of Bessel System

Al-Jourany Khalid Hadi Hameed

Deparetment of Functions and Approximations Theory.Faculty of Mathematics and Mechanics, Saratov State University,Russia.
e-mail: khalidaljourany@gmail.com

The new notion of affine system of Walsh type was introduced , studied and proved results about orthogonalizing and completion with preservation of structure of affine system in[1] .In [2] , we study an affine system of Walsh type generated by a periodic function in connection with multishifte in Hilbert space as well as we gave a necessary sufficient conditions on an affine system of Walsh type to be complete sequence in the space $L^2[0,1]$,finally we showed that an affine system is a fundamental sequence.In this work , we will give a necessary and sufficiently conditions on the function φ an affine system of Walsh type $\{\varphi_n\}_{n\geq 0}$ to be Bessel system in the space $L^2(0,1)$.
Definition(1): Let H be a Hilbert space,and $W_0, W_1 : H \to H$ isometric operators operating in space H .Let's say that the two of isometrics W_0 and W_1 defines the structure of multishifts , if there is a vector $e \in H$ such that :

$$W_{\alpha_1} \dots W_{\alpha_{k-1}} e, \alpha_v \in \{0,1\}, 0 \leq v \leq k-1, k \geq 0$$

Forms an orthonormal basis of the space H.
The concepts of multishift introduced and studied in the works [3]-[5].
Suppose that , the function $\varphi(s)$, $s \in$ Re , (where Re is a real number space) , satisfied the condition :

$$\varphi(s) \in L^2[0,1], \int_0^1 \varphi(s)ds = 0, \varphi(s+1) = \varphi(s)$$

and let $L_0^2 = L_0^2(0,1)$ be a space such functions (where, L_0^2 is the space of square - integral and having a zero integral), as well as , we denote a linear operators in this space as :

$$W_0\varphi(s) = \varphi(2s), W_1\varphi(s) = r(s)\varphi(2s) \tag{1}$$

Where $r(s)$ is the periodic function : Haar-Rademacher-Walsh.
For any $n \in N$, using the binary representation , $n = \sum_{v=0}^{k-1} \alpha_v 2^v + 2^k$ we set :

$$\varphi_n(s) = \varphi_\alpha(s) = \varphi_{kj}(s) = W^n\varphi(s) = W^\alpha\varphi(s) = W_{\alpha_1} \dots W_{\alpha_k}\varphi(s)$$

Where , $k = 0,1,...; j = 0,1,...,2^{k-1}, \alpha = (\alpha_1, \dots, \alpha_k) \in \Omega, \Omega = \bigcup_{k=0}^{\infty} \{0,1\}^k$.Be sides , we set $\varphi_0(s) \equiv 1$,

$$W_{\alpha_1} \dots W_{\alpha_k}$$

Denote the product of the operators : the operator W_{α_k} acts first , W_{α_1} acts last , and the empty product is set the equal to the identity operator I .For any function $\varphi \in L_0^2$, we have :
$\varphi_\alpha(s) = W^\alpha\varphi(s) = W_{\alpha_0} \dots W_{\alpha_{k-1}}\varphi(s) = \varphi(2^k s) r^{\alpha_{k-1}}(2^{k-1}s) \dots r^{\alpha_0}(s) = \varphi(2^k s) \prod_{v=0}^{k-1} r_v^{\alpha_v}(s)$
Where , $r_k(s) = r(2^k s), k = 0,1,...$ is Rademacher system .
Definition(2):The system $\{\varphi_n\}_{n\geq 0} = \{W^\alpha\varphi\}$ is the affine system of Walsh type of the function φ without the constant $\varphi_0(s) \equiv 1$.
If the generating function select $\omega(s) = r(s)$, then the system $\{\omega_n\}_{n=0}^{\infty}$ will the classical system of Walsh - Paley system .Walsh functions (without constant $\omega_0(s) \equiv 1$):

$$\omega_n(s) = \omega_\alpha(s) = W^\alpha \omega(s) = W_{\alpha_0} \dots W_{\alpha_{k-1}} \omega(s) = r_k(s) \prod_{v=0}^{k-1} r_v^{\alpha_v}(s)$$

Forms an orthonormal basis of the space $H = L_0^2(0,1)$, therefore according to the definition(1) operators :$W_0\varphi(s) = \varphi(2s), W_1\varphi(s) = r(s)\varphi(2s)$, define the structure of multishift , [2].
Theorem(1):Let $\varphi \in L^2(0,1)$, supp$\varphi \subset [0,1]$, $\int_0^1 \varphi(s)ds = 0$. If the inequality :

$$\sum_{k=0}^{\infty} \left(\sum_{j=0}^{2^k-1} |(\varphi, \omega_{kj})|^2 \right)^{1/2} = c \prec \infty$$

Then , the affine system of Walsh type $\{\varphi_n\}_{n\geq 0}$ is Bessel system with Bessel constant $B = max\{1, c\}^2$.

Bibliography

[1] Terekhin, P.V.: *Affine Systems of Walsh type . Orthogonalization and Completion* . Izv Saratov University(N.S).Ser.Math.,Mech.,Inform. **14**(4), (2014), 395-400.(in Russian).

[2] Khalid, H.H., Mironov V.A., and Terekhin P.V.: *Affine System of Walsh type . Completeness and Minimality.* Izv Saratov University(N.S).Ser.Math.,Mech.,Inform. , **16**(3), (2016), 247-256.(in Russian).DOI:10.18500/18169791-2016-16-3-247-256.

[3] Terekhin, P.V.: *On representation properties of a system of contractions and shift of functions on an interval* . Izv Tul'sk.Gos. Univ.,Ser.Math.,Mech.,Inform. **4**(1), (1998), 136-138.(in Russian).

[4] Terekhin, P.V.: *On the multiplicative structure of the centeralizer of a multishift on a Hilbert space* . Mathem.,Mech.: Collection of Scientific Papers , Saratov,Saratov University. **2**, (Press2000), 119-122.(in Russian).

[5] Terekhin, P.V.: *Multishift in Hilbert spaces* . Funct . Anal.Appl., **39**(1), (2005), 57-67.DOI:10.1007/s10688-005-0017-5.

Inequalities related to some types of entropies

Nicuşor Minculete

Transilvania University of Braşov, Romania
e-mail: minculeten@yahoo.com

The aim of this paper is to discuss new results concerning some types of entropies. We study some generalized entropy based on Tsallis and Rényi entropies and study mathematical properties by the use of scalar inequalities to develop the theory of entropies.

Keywords: Tsallis's entropy, Rényi's entropy, Shannon's entropy
2010 MSC: Primary 46C05; Secondary 26D15, 26D10

Bibliography

[1] Furuichi, S., *Information theoretical properties of Tsallis entropies*, J. Math. Phys., Vol. 47, 2006.

[2] Furuichi, S., Minculete, N., Mitroi, C., *Some inequalities on generalized entropies*, J. Inequal. Appl. 2012:226, 2012.

[3] Rnyi, A., *On measures of entropy and information.* In: Proc. 4th Berkeley Symp., Mathematical and Statistical Probability, vol. 1, 1961, 547-561.

[4] Sun, L.-H., Li, G.-X., Ficek, Z. *Continuous variables approach to entanglement creation and processing*, Appl. Math. Inf. Sci. 4, 2010, 315-339.

[5] Tsallis, C., *Possible generalization of Bolzmann-Gibbs statistics*, J. Stat. Phys., 52 (1988), 479487.

Analytic and geometric consequences of Orlicz-Poincaré inequalities on metric measure spaces

Marcelina Mocanu

"Vasile Alecsandri" University of Bacău, Romania
e-mail: mmocanu@ub.ro

In Theorem 5.1 from the paper *Geometric characterizations of the $p-$Poincaré inequality in the metric setting, Publ. Mat. 60 (2016),* E. Durand-Cartagena, J. A. Jaramillo and N. Shanmugalingam obtained in the setting of $Q-$Ahlfors regular metric measure spaces two characterizations of $p-$Poincaré inequalities for $p > Q$, one in terms of the (local) Hölder continuity of Newtonian functions in $N^{1,p}$ and the other in terms of the p-modulus of quasiconvex curves connecting pairs of points in the space. We partially extend this result to the case of Orlicz-Poincaré inequalities. We characterize a weak Orlicz-Poincaré inequality through the (local) Hölder continuity of locally integrable functions possesing upper gradients in the corresponding Orlicz space L^{Φ}, then we establish connections between some estimates of the $\Phi-$modulus and the above-mentioned Hölder continuity property.

Dynamical Approximation Entropies in Operator Algebras

George Popescu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: grgpop@gmail.com

The paper studies the approximation entropies introduced by D. Voiculescu in 1995. We will consider only approximation by subalgebras the $C^{\star}$-algebra case. Let M be a $C^{\star}$-algebra and $\alpha : M \to M$ an endomorphism. Then let $\mathcal{P}f(M)$ be the collection of all finite subsets of M and $\mathcal{F}(M)$ the collection of all finite dimensional $C^{\star}$-subalgebras of M. We define the δ-rank of $\omega \in \mathcal{P}f(M)$ as: $r(\omega, \delta) = \inf\{\text{rank}(A) \mid A \in \mathcal{F}(M)\ ,\ \omega \subset_{\delta} A\}$, where $\omega \subset_{\delta} A$ means $\forall x \in \omega$, $\exists a \in A$ so that $\|x - a\| < \delta$.
The pattern of defining dynamical approximation entropies is the following:

- $h(\alpha, \omega, \delta) = \limsup\limits_{n\to\infty} \frac{1}{n} \log r(\bigcup\limits_{k=0}^{n-1} \alpha^k(\omega), \delta)$
- $h(\alpha, \omega) = \sup\limits_{\delta>0} h(\alpha, \omega, \delta)$
- $h(\alpha) = \sup\limits_{\omega\in\mathcal{P}f(M)} h(\alpha, \omega),$

h is called dynamical approximation entropy of the endomorphism α.
Such entropies describe the dynamic of orbits generated by an endomorphism. We show these

entropies are invariant to strong equivalence of endomorphisms, unitary equivalence and approximative unitarily equivalence. We consider the "reduced" entropies, and show that they are also invariant to these equivalences (under suitable conditions).
We investigate the case when these entropies are zero in the AF $C^\star$-algebra case. From a physical point of view meaning the "evolution" is stationary or with "slow growth".
Then also investigate the case when such entropies are strictly positive, meaning they have "exponential growth".

Vector Equilibrium Problem

Cristina Stamate

"Octav Mayer" Institute of Mathematics Iasi, Romania

e-mail: cstamate@ymail.com

We present existence and properties for solutions of some vector equilibrium problems in the case of empty interior cone.

Finite difference scheme for solving one-dimensional fractional diffusion equation

Vitalie Ţîcău

Alecu Russo Balti State University, Balti, Republic of Moldova

e-mail: VitalieSTicau@gmail.com

Fractional differential equations (FDE) have drawn increasing attention because they have applications in various fields of science and engineering. For example, they can describe many physical and chemical processes, biological systems, etc.
The main physical purpose for investigating the diffusion equations of fractional order is to describe the phenomena of anomalous diffusion in transport processes through complex and/or disordered systems including fractal media, and fractional kinetic equations have proved to the particularly useful in the context of anomalous slow diffusion.
There have been several numerical methods proposed for solving the space and/or time FDE up to now. Lynch developed two numerical schemes, one explicit and one semi-implicit, for solving the transport problem with anomalous diffusion modeled by a partial differential equation of fractional order [1, 2].
The main purpose of this paper is to solve the fractional diffusion problem using the compact difference scheme and to offer a stability and convergence analysis. Previous methods for fractional subdiffusion problems have been limited to second-order accuracy in space. The compact scheme is a high-order method and the coefficient matrix of the linear system of equations of the unknown variables is tridiagonal and can be easily solved through the Thomas algorithm [3].
This compact implicit difference scheme has the advantage of high accuracy with the coefficient matrix still being a tridiagonal one, therefore, the linear system of equations are easy to solve.
We have proved that the method is unconditionally stable for $0<\gamma<1$. We have also shown that the method has accuracy of four in the spatial grid size and one in the fractional time step. The conclusions are verified by some numerical experiments.

Bibliography

[1] Secrieru I. Application of the decomposition principle for plane fractional diffusion equation. Proceedings of the Conf. "Modelare matematică, Optimizare si Tehnologii Informaţionale". ATIC. Chişinău, Evrica,2014. Republica Moldova. 201 @ S205.

[2] Secrieru I., Ticau V. Weighted approximate scheme for fractional order diffusion equation. Buletinul Institutul Politehnic din Iaşi, vol. LVII(LXI), fascicol, secţa MATEMATICA, MECANICA TEORETICA, FIZICA, Ed. Politehnium. Iaşi, România, 2011.

[3] G.I. Marchuk. Mathematical modeling in problem to protection the environment. Moscow: Nauka, 1982 (in Russian).

The Arens triadjoint of an almost orthosymmetric bilinear map

Ruşen Yilmaz

Recep Tayyip Erdoğan University, Rize, Turkey
e-mail: `rusen.yilmaz@erdogan.edu.tr`

We introduce a new concept of bilinear map on a vector lattice and prove that if A, B are vector lattices and a bilinear map $T : A \times A \to B$ is a positive almost orthosymmetric bilinear map, then so is the bilinear map $T^{***} : (A')'_n \times (A')'_n \to (B')'_n$. In addition, the relations between the concept of almost orthosymmetric bilinear map and the concepts of bi-orthomorphism and orthosymmetric bilinear map are given.

Probability Theory, Mathematical Statistics, Operations Research

Population growth combined with wide offspring distributions can increase fixation rate and reduce genetic diversity

Idan Alter, Yoram Louzoun

Dept. of Mathematics, Bar-Ilan University, Israel
e-mail: alteridan@gmail.com, louzouny@math.biu.ac.il

Genetic diversity grows with the population size in most neutral evolution models. Empirical evidence of large populations with limited diversity have been proposed to be due either to genetic bottlenecks or to selection. An alternative explanation is that the limited diversity is a result of rare reproduction events. Indeed, recent estimates of the offspring number distribution highlight the role of large reproduction events. We here show that in a large class of models containing such rare events, genetic diversity decreases as the population size increases, in neutral evolution models.
For many realistic offspring number distributions, the contribution of rare events to the dynamics grows with the population size. In the context of genetic diversity, these rare events induce a decrease in the time to the most recent common ancestor and in the genetic heterogeneity as the population grows. This phenomenon may explain the observed rapid fixation of genes in large populations, in the absence of observable selection or bottlenecks.

Generalized divergence measures for Markov chains

Vlad Stefan Barbu[1], Alex Karagrigoriou[2], Vasile Preda[3]

[1] *Université de Rouen–Normandie, Laboratoire de Mathématiques Raphaël Salem, France,*
[2] *University of the Aegean, Samos, Greece,*
[3] *University of Bucharest and ISMMA Gh. Mihoc–C. Iacob, Bucharest, Romania*
e-mail: barbu@univ-rouen.fr, alex.karagrigoriou@aegean.gr, vasilepreda0@gmail.com

Divergence measures are of great importance in statistical inference as well as their limiting versions, known as divergence rates. In this presentation we focus on generalized divergence measures for Markov chains. We consider generalizations of Alpha divergence measure (Amari and Nagaoka, 2000) and Beta divergence measures (Basu et. al, 1998) and investigate their limiting behaviour. We also study the corresponding weighted generalized divergence measures and the associated rates (Belis and Guiasu, 1968; Guiasu, 1971; Kapur, 1994). Special attention is paid to the generalized form of the popular Cressie and Read power divergence class of measures. Illustrative 2-state and 3-state Markov chain examples are furnished and analysed.
Acknowledgements. The research work of Vlad Stefan Barbu was partially supported by the project *FEDER–XTerM: Complex Systems, Territorial Intelligence and Mobility*, University of Rouen Normandy, within the Large Scale Research Networks from the Region of Normandy, France.

Bibliography

[1] S. Amari and H. Nagaoka, *Methods of Information Geometry*, Oxford University Press, New York, 2000.

[2] V. S. Barbu, A. Karagrigoriou, V. Preda, Entropy and divergence rates for Markov chains: I. The Alpha-Gamma and Beta-Gamma case, to appear in Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, 4, 1-8, 2017.

[3] V. S. Barbu, A. Karagrigoriou, V. Preda, Entropy and divergence rates for Markov chains: II. The weighted case, to appear in Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, 1, 1-8, 2018.

[4] V. S. Barbu, A. Karagrigoriou, V. Preda, Entropy and divergence rates for Markov chains: III. The Cressie and Read case and applications, to appear in Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, 2, 1-8, 2018.

[5] A. Basu, I.R. Harris, N.L. Hjort and M.C. Jones, Robust and efficient estimation by minimising a density power divergence, *Biometrika*, **85**, 549-559, 1998.

[6] M. Beliş and S. Guiaşu, A quantitative-qualitative measure of information in cybernetic systems, *IEEE Transactions on Information Theory*, **14** (4), 593-594, 1968.

[7] N. Cressie and T.R.C. Read, Multinomial goodness-of-fit tests, *J. R. Statist. Soc*, **5**, 440-454, 1984.

[8] S. Guiaşu, Weighted entropy, *Reports on Mathematical Physics*, **2** (3), 165-179, 1971.

[9] J.N. Kapur, *Measures of Information and Their Applications*, Wiley, New Delhi, 1994.

Robust estimator of conditional tail expectation in the case of heavy-tailed losses

Lala Bouali Dalal and Benatia Fateh

Laboratory of Applied Mathematics, Mohamed Khider University of Biskra, 07000, Algeria
e-mail: lalabouali.dalal@gmail.com

The risk mesure is an important and useful tool in actuarial and financial risk assessment, one of the most applicable mesure is the conditional tail expectation (CTE), see Denuit et al (2006). CTE is the average amount of loss, given that the loss exceeds a specified quantity. That is, this risk measure gives information on the distribution of the random loss variable beyond the α order quantile and thus on the thickness of the distribution tail. As a result, CTE provides a measure of capital required because of exposure to loss and is therefore used to measure risk. It is therefore not surprising that CTE continues to receive increased attention in the actuarial and financial literature, where we also find its many extensions and generalizations.
The new contribution in our work is to derive a robust estimator of the conditional tail expectation in the case of heavy tailed losses, it's an improvement of the work of Necir, A. et al (2010).
We use the so-called t-Hill tail index estimator proposed by Fabián (2001), instead of Hill's estimator, this estimator allows us to improve the robustness of the conditional tail expectation in the case of heavy-tailed losses.
The asymptotic normality of the proposed estimator is established under appropriate conditions and its performance evaluated through simulated data sets.
The determination of this new estimator, proving its robustness and establishing the asymptotic normality, they are the objectives to which one aspires.

A Parallel System with Arrival and Maximum Repair Time of the Server

Chitaranjan Sharma

Government Girls College Dhar M.P. India
e-mail: drcsharma3@gmail.com

An attempt has been made to discuss a parallel system of two identical units in which some arrival time is given to the server for carrying out repair activities. The server repairs the unit at its failure up to a maximum repair time. If repair of the unit by the server is not possible in the time, it is replaced by new one with some replacement time. The failure time of the unit and the time to which unit undergoes for replacement are exponentially distributed while the distributions of arrival time of the server, repair and replacement time of the unit are taken as arbitrary with different probability density function. The semi-Markov process and regenerative point technique are used to derive the expressions for various reliability measures of the system model. The graphical study of the results obtained for a particular case has also been made.

On the Strong Convergence for Negatively Orthant Dependent Random Variables

Zahra Shokooh Ghazani

Department of Mathematics, Central Tehran Branch, Islamic Azad university,Tehran, Iran
e-mail: zah.shokooh_ghazani@iauctb.ac.ir

In this paper, we present some results on the complete convergence for arrays of negatively orthant dependent (NOD, in short) random variables by using the Rosenthal-type inequality, Kolmogorov exponential inequality and the truncation method. the result is an improvement of complete moment convergence and the Marcinkiewicz-Zygmund type strong law of large numbers for NOD random variables.

Bibliography

[1] A. T. Shen, Y.Zhang,A.Volodin,Applications of the Rosenthal-type inequality for negatively superadditive dependent random variables, Metrika 78, (2015), pp. 295-311.

[2] X.J.Wang, S.H.Hu, Complete convergence and complete moment convergence for martingale difference sequence, Acta Mathematica sinica, English series 30 (2014),pp. 119-132.

[3] A.Bozorgnia, R.F. Paterson,R.L. Taylor, limit theorems for dependent random variables, World Congress of Nonlinear Analysis 92 Berlin, Walter de Grunter(1996) 1639-1650.

Bimatrix perfect information game

Hâncu Boris

Moldova State University, Chişinau, Republic of Moldova
e-mail: boris.hancu@gmail.com

Consider the bimatrix game $\Gamma = \langle I, J, A, B\rangle$ in complete and $\left(1 \overset{\inf}{\leftrightarrows} 2\right)$ – perfect information over the sets of pure strategies. So we can use the set of informational extended strategies $\mathbf{I} = \left\{\mathbf{i}^{\alpha} = \left(i_1^{\alpha}, i_2^{\alpha}, ..., i_j^{\alpha}, ...i_m^{\alpha}\right), \alpha = \overline{1, n^m}\right\}$ of the player 1 and $\mathbf{J} = \left\{\mathbf{j}^{\beta} = \left(j_1^{\beta}, j_2^{\beta}, ..., j_i^{\beta}, ...j_n^{\beta}\right), \beta = \overline{1, m^n}\right\}$ of the player 2 that mean the following: if the player 2 will choose the column $j \in J$ then the player 1 will choose the line $i_j^{\alpha} \in I$ and if the player 1 will choose the line $i \in I$ then the player 2 will choose the the column $j_i^{\beta} \in J$. The players do not know the informational extended strategies of each others. For these type of game it is very difficult to construct utility matrices: for all strategy profile $\left(\mathbf{i}^{\alpha}, \mathbf{j}^{\beta}\right)$ which element of the matrix A and B should be considered as a payoff value of the player 1 and 2 ? Thus, in order to solve games in informational extended strategies, we propose to use the following methodology. Using the informational extended strategies we construct the following normal form of the incomplete and imperfect information game $\widetilde{\mathbf{\Gamma}} = \left\langle \{1,2\}, I, J, \left\{AB(\alpha,\beta) = \left\| \left(a_{ij}^{\alpha\beta}, b_{ij}^{\alpha\beta}\right) \right\|_{i\in I}^{j\in J} \right\}_{\alpha=\overline{1,n^m}}^{\beta=\overline{1,m^n}} \right\rangle$. For the game $\widetilde{\mathbf{\Gamma}}$ we construct the associated Bayesian game $\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}, \widetilde{\mathbf{J}}, \mathcal{A}, \mathcal{B}\right\rangle$,where $\widetilde{\mathbf{I}} = \bigcup\limits_{\alpha\in\Delta_1} \widetilde{\mathbf{I}}(\alpha)$, $\widetilde{\mathbf{J}} = \bigcup\limits_{\beta\in\Delta_2} J(\alpha)$. Here $\widetilde{\mathbf{I}}(\alpha)$ is the set of all pure strategy of the $\alpha-$type player 1 and $\widetilde{\mathbf{J}}(\beta)$ the set of all pure strategy of $\beta-$type player 2. Payoff matrices of the $\alpha-$type player 1 and $\beta-$type player 2 are respectively $\mathbf{A}(\alpha) = \left\|\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$ and $\mathbf{B}(\beta) = \left\|\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$, where $\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum\limits_{\beta\in\Delta_2} p(\beta/\alpha) a_{ij}^{\alpha\beta}$, $\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum\limits_{\alpha\in\Delta_1} q(\alpha|\beta) b_{ij}^{\alpha\beta}$. The matrices $\mathcal{A}$ and $\mathcal{B}$ are the "big matrices", which consist of the submatrices type $\mathbf{A}(\alpha)$ and $\mathbf{B}(\beta)$ respectively. It is true the following theorem

Theorem. *The strategy profile* $\left(\widetilde{\mathbf{i}}^*, \widetilde{\mathbf{j}}^*\right)$ *is a Bayes-Nash equilibrium in the game* Γ_{Bayes} *if and only if it is a Nash equilibrium for the subgame* $sub\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}(\alpha), \widetilde{\mathbf{J}}(\beta), \mathbf{A}(\alpha), \mathbf{B}(\beta)\right\rangle$.

Algorithms for determining nontrivial convex covers of graphs

Buzatu Radu

Moldova State University
e-mail: radubuzatu@gmail.com

Nontrivial convex p-cover and nontrivial convex p-partition of a graph were defined in [1]. The problems of determining whether a graph has a nontrivial convex p-cover or a nontrivial convex p-partition are NP-complete for a fixed $p \geq 2$ [1]. Further, there is a specific interest in studying these problems for some classes of graphs. Particularly, nontrivial convex p-covers of trees are studied in [2] and [3].

Let G be a graph with n vertices and m edges.

Theorem 1. *It can be decided in time $O(n^3)$ whether a tree G, $n \geq 6$, has a nontrivial convex p-partition for a fixed p, $2 \leq p \leq \lfloor \frac{n}{3} \rfloor$.*

Also, a recursive algorithm is proposed to establish if a tree has a nontrivial convex p-cover for a fixed $p \geq 2$ [3].
Obviously, there exist graphs for which there are no nontrivial convex covers or nontrivial convex partitions or both. It is NP-complete to decide whether a graph can be partitioned into nontrivial convex sets [3].

Theorem 2. *It can be decided in time $O(n^4 m)$ whether a graph G can be covered by nontrivial convex sets.*

Bibliography

[1] R. Buzatu, S. Cataranciuc. *Convex graph covers.* Computer Science Journal of Moldova, 23 no.3 (69), 2015, pp. 251–269.

[2] R. Buzatu, S. Cataranciuc. *Nontrivial convex covers of trees.* Bulletin of Academy of Sciences of Republic of Moldova, Mathematics, no.3 (82), 2016, pp. 72–81.

[3] R. Buzatu, S. Cataranciuc. *On covers of graphs by nontrivial convex sets.* Computer Science Journal of Moldova, 2017, under publication.

An Effective Linearizing Method for the Specific Nonlinear Minimax Optimization Problem

Mio Horai, Takashi.G. Nitta

Ise gakuen High school, Kurosecho, Ise, 516-0018, Japan,
Department of Mathematics, Mie University, Kurimamachiyamachi, Tsu, 514-8507, Japan
e-mail: mio@com.elec.mie-u.ac.jp, nitta@edu.mie-u.ac.jp

In this paper, we propose an effective method for the Specific Nonlinear Minimax Optimization Problem including the nonlinear function h_{kj} satisfied "Inada conditions", which is well-known in economics, communication technology, and engineering, to their problems. To solve the problem, we transform the nonlinear problem to the linearized problem using Jiao's linearized method, and we find the optimal solution by Branch and Bound Algorithm. As the extension of our proposal, we apply our method to the optimization problem that h_{kj} do not satisfy "Inada conditions," for example, sin and cos.
Key words: Nonlinear Minimax Optimization, linearized method, Branch and Bound Algorithm.
2000 AMS subject classifications: 49K35, 90C30.

Cost-Benefit Analysis of a Cold Standby System with Preventive Maintenance Subject to Arrival Time of Server

Ashish Kumar

Department of Mathematics and Statistics,
Manipal University Jaipur, Jaipur-303007, Raj.(India)
e-mail: ashishbarak2020@gmail.com

The main concentration of the present paper is on the cost-benefit analysis of a cold standby system with the concept of priority, preventive maintenance and arrival time of the server. For, this purpose a stochastic model is designed in which one unit is operative initially and other unit is kept as cold standby. A single repair facility is provided which takes some time to arrive at the system to do PM and repair of the components as per the requirement of the system. Server does not do the preventive maintenance if there is no spare unit available for use. All random variables are statistically independent. The unit after repair and preventive maintenance is considered as good as new. All times distributions are exponential except the repair, arrival time of server and preventive maintenance time distributions which are considered as general. The expression for various important measures of system effectiveness is obtained by using semi-Markov process and regenerative point technique. The numerical study of the results for mean time to system failure (MTSF), availability and profit has also been made giving some particular values to various costs and parameters
Key words : Cold Standby System, Preventive Maintenance, Priority, Arrival Time of Server and Cost- Benefit Analysis.

Dynamical systems with final sequence of states and almost asymptotically periodic transition probabilities

Alexandru Lazari

Moldova State University, Republic of Moldova
e-mail: lazarialexandru@mail.md

Let L be a stochastic system with finite set of states V. The system L starts its evolution from the state v with probability $p^*(v)$, $\forall v \in V$ and, at each moment of time $t = 0, 1, 2, \ldots$, it passes from one state $u \in V$ to another state $v \in V$ with probability $p^{(t)}(u, v)$. Additionally, a sequence of states $X = (x_1, x_2, \ldots, x_m) \in V^m$ is given and the system stops transitions as soon as the states $x_1, x_2, \ldots, x_m$ are reached consecutively in given order. The goal is to obtain the expectation of the evolution time of the system L.
The system L represents a dynamical stochastic system with final sequence of states X. The case when the transition probabilities $p^{(t)}(u, v)$, $t = 0, 1, 2, \ldots$, do not depend on given moment of time t, was studied in [1]. Also, a game on that system was investigated in [2] and a polynomial algorithm for determining the game duration expectation was developed.
Next, for each transition $(u, v) \in V^2$, we consider that the sequence $(p^{(t)}(u, v))_{t=0}^{\infty}$ is almost asymptotically periodic, i.e., for a given small accuracy $\varepsilon > 0$, there exists an $\varepsilon - index$ $\tau(u, v)$ and a periodic sequence $(\overline{p}^{(t)}(u, v))_{t=0}^{\infty}$ with least period $\theta(u, v)$, such that $|p^{(t)}(u, v) - \overline{p}^{(t)}(u, v)| < \varepsilon$, $\forall t \geq \tau(u, v)$. So, the matrix sequence $((p^{(t)}(u, v))_{u,v \in V})_{t=0}^{\infty}$ is also almost asymptotically periodic with least period $\theta = LCM(\{\theta(u, v) \mid u, v \in V\})$ and the $\varepsilon - index$ $\tau = \max(\{\tau(u, v) \mid u, v \in V\})$.

We show that the evolution time of L can be approximated with duration of the game Γ, defined on L, with $\tau+\theta$ players, having corresponding strategy $(p^{(t)}(u,v))_{u,v\in V}$, $t=\overline{0,\tau+\theta-1}$. The acting order of the players is represented by one eventually periodic sequence $(\phi_\ell)_{\ell=0}^{\infty}$. This game can be handled similarly with the game investigated in [2], redefining the $\oplus$ operation in the following way: $r\oplus s=\phi(r+s)$, for all integers r and s.

Bibliography

[1] Lazari A., Lozovanu D., Capcelea M. *Deterministic and Stochastic Dynamical Systems: Evolution, Optimization and Discrete Optimal Control (in romanian).* Chisinau, CEP USM, 2015, 310p.

[2] Lazari A. *Stochastic Games on Markov Processes with Final Sequence of States.* Bul. Acad. Stiinte Repub. Mold. Mat., 2017, No. 1(83), 77 V94.

Estimation of psychological personality factors by combining different tests. Bayesian networks

Natalya Litvinenko

Institute of mathematics and mathematical modeling CS MES RK, Kazakhstan, Almaty
e-mail: n.litvinenko@inbox.ru

In the past 30 years, interest in the theory and practice of Bayesian networks has extremely increased in various fields of science. This can be explained by very promising results obtained with the Bayesian network models in various scientific fields: medicine, genetics, sociology and psychology.
Psychological and sociological tests are very popular; the applicability area is huge, from marriage agencies to military units. Psychologists and sociologists are constantly working on improvement of the quality and the informativity of their tests. To provide reliable and accurate conclusions/estimates, running of multiple tests may be required. An open question, which we research here, is how to combine various results of all these tests into one result. For this purpose we use Bayesian network graphs.
We made an attempt to extend Bayesian network methods for applications in psychology and sociology. Particularly, we develop new methods to estimate certain psychological characteristics on a basis of few psychological tests. It helps to describe psychological and social climate in a team. For numerical computations we use the R programming environment.
Obtained results:

- We built the mathematical model with using Bayesian networks. This model explains the principle of constructing a general estimate of the psychological factor based on estimates from several tests.
- We made the corresponding computations, based on this model.
- The obtained estimate corresponds to the subjective assessments of experienced psychologists.

Possible application: to model and to forecast psychological/sociological compatibility of different people. This can be especially important for group of people in isolated locations or for military.

Future: for more complex tasks we will develop a Bayesian networks software toolbox in C# programming language. For large computations, we will use the processing power of NVIDIA GPUs with CUDA.

Exchange rate risk, simulation and statistical analysis

Ludmila Novac

Moldova State University
e-mail: novac-ludmila@yandex.com

The issue of currency risk management for non-financial firms is always important and is usually dealt by their corporate treasuries. Exchange rate risk management is an integral part in every firm's decision about foreign currency exposure.
However, the currency changes can affect all of us, whether we are actively trading in the foreign exchange market, shopping online for goods from another country, making investments in different currencies or just buying food or other things imported from abroad. Making the analyze of the time series of the daily exchange rates we must deal and follow all the parameters that have influences to the exchange rates. The value of a currency depends on factors that affect the economy, such as trade, inflation, employment, interest rates, growth rate and other. In this research paper we use some statistical methods in order to analyze the exchange rate fluctuations and to make some prediction.

On Non-Homogeneous Continuous Time Markov Chains and Divergences With Applications in Banking Industry

George-Andrei Padureanu

University of Bucharest
e-mail: george.padureanu@gmail.com

A common subject in banking industry, related to credit risk modelling, is that of multi-year Probabity of Default Calibration term structures. This is done by means of Non-Homogeneous Continuous Time Markov Chains, under Basel Committee on Banking Supervision Framework.
The paper uses an entropic measure called divergence of conditional distributions in order to verify if term structures of default probabilities can be satisfactorily modelled by a Markov chain technique.
A simulation study allows the identification empirical distributions of these divergences.

Brownian motion with barriers

Oana Rachieru

Transilvania University of Brasov, Romania
e-mail: oana.rachieru@unitbv.ro

In this talk we present the case of a Brownian motion on $x > 0$ with an absorbing barrier, and the Brownian motion on $x > 0$ but with a reflecting barrier at the origin.

A methodology for calculation exchange rates using neural networks and fuzzy logic

Mutali Sezen

"Ovidius" University of Constanta
e-mail: m.sezenn@yahoo.com

Due to the high volatility and complexity of the exchange markets, the exchange rate prediction is a very thought-provoking application of time series forecasting. It can be approached by several methods, among which we restrict to the soft computing ones. More precisely, in this paper we propose a methodology for calculating the USD/RON exchange rate using a method that combines artificial neural networks and fuzzy logic, resulting in an Adaptive Neuro-Fuzzy Inference System (ANFIS). Such an ANFIS generalizes early studies which used only neural networks for forecasting. In the first step of the ANFIS (called system identification), the significant technical and economic indexes used as inputs are identified, the fuzzy if-then rules are generated and the parameters of the model are selected, while in the second step (called fuzzy reasoning), the output is produced from the rule base.

New Ratio Estimators Using Stratified Random Sampling and Stratified Ranked Set Sampling

Monika Saini

Department of Mathematics and Statistics, Manipal University Jaipur, Jaipur-303007, Raj.(India),
e-mail: drmnksaini4@gmail.com

The aim of this paper to proposes ratio estimators for the population mean by using auxiliary information efficiently under stratified random sampling (SRS) and stratified ranked set sampling (SRSS). We obtain the bias and mean square error (MSE) for the proposed estimators and show that the proposed estimator under SRSS is more efficient than the estimator under SRS. The results have been illustrated numerically through simulation study.
Key words : Finite Population Stratified Random Sampling Stratified Ranked Set Sampling, Auxiliary Variable Ratio Estimator Efficiency.

The "Bottleneck" Fractional Multi-Objective Transportation Problem with Fuzzy Type of Parameters

Tkacenko Alexandra

Department of Applied Mathematics, Moldova State University, A. Mateevici str., 60, Chisinau, MD–2009, Moldova
e-mail: alexandratkacenko@gmail.com

In this paper we propose to develop a new approach to solving the multi-objective fractional transportation problem with the same denominators, which are functions of time. Additionally, we include separately this function as a time "bottleneck" type criterion. We study the case when all

of the model's parameters aren't of deterministic type, but of fuzzy one.[1],[2]. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^k x_{ij}}{\max_{ij} \tilde{t}_{ij} \,|x_{ij} > 0\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij} \tilde{t}_{ij} \,|x_{ij} > 0\,\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} x_{ij} = a_i, j = 1, 2, \ldots, n; \tag{3}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, r. \tag{4}$$

In order to solve the model (1)-(4) we proposed an iterative algorithm. It generates the crowds efficient model solutions for different types of approaches to the time required for transport from optimistic to pessimistic, using for this purpose the possible ranges of variation thereof. The algorithm was tested on several examples and was found to be quite effective.

Bibliography

[1] P. Surapati, R. Tapan Kumar, *Multiobjective transportation model with fuzzy parameters: priority based fuzzy goal programming approach,* Journal of Transportation Systems Engineering and Information Technology, V.8 (3), 2008, p.[40-48].

[2] A. Tkacenko, *The multi-criteria fractional transportation problem with fuzzy "bottleneck" criterion,* Journal of Economic Computation and Economic Cybernetics Studies and Research, ISI Thomson Reuter Serv., ISSUE 3/2016, V..50, Bucuresti, Romania, pp. 117-134.

From Matemathical Statistics to Operational Research: techniques and methods used in public policies evaluation

Dorina Ţicu

"Alexandru Ioan Cuza" University, Faculty of Philosophy and Social and Political Sciences, Departament of Political Sciences, International Relations and European Studies
e-mail: ticudorina@yahoo.com

The present article aims to identify and present in an initial phase some features of the evaluation used in the field of public policies such as: the impact evaluation, the cost-benefit evaluation, ex-ante evaluation, post-hoc evaluation, etc. On the other hand, this analysis aims to identify which matemathical statistics methods and techniques can be activated in order to correlate the field of the operational research of the public administration, of the public policies with the types of the evaluation previously identified in order to respect the principle of the internal validity and of the external validity to reach valid results.
Keywords: statistics, multilevel government, public policies, internal validity, external validity

Games of Matching Embeddings and Strategic Equilibria

Valeriu Ungureanu

Moldova State University
e-mail: v.a.ungureanu@gmail.com

The game of Matching Embeddings is stated as an extension and a generalization of such well known games as Matching Pennies and Rock–Paper–Scissors. Various variants of the game of Matching Embeddings are considered both as multi-objective and infinite strategy games. Formally, the payoff matrix of the game of Matching Embeddings is defined similar with the definition of the payoff matrix in the games of Matching Pennies and Rock–Paper–Scissors. As the game of Matching Embeddings is appropriate geometricaly to problems of set nesting/embedding [2], it may be solved by similar special algorithms [3, 4] and approaches [1] when the objective functions are linear and strategy sets are convex polyhedral sets. As a consequence of mentioned affinities, the properties of the game may be stated and proved, as well the approaches to find strategic equilibria may be developed.

Bibliography

[1] GAINDRIC, C., AND UNGUREANU, V. *Systems of linear inequalities with right-hand parametric parts and some optimization problems,* Optimization Methods and Their Applications, Book of Abstracts of the International School and Seminar, Irkutsk, September 10–19, 1989, pp. 53–54.

[2] LOZOVANU, D., AND UNGUREANU, V. *On the nesting of convex polyhedral sets defined by systems of linear inequalities,* Kibernetika (Kiev), 1989, No. 1, pp. 104–107, 135.

[3] LOZOVANU, D., AND UNGUREANU, V. *An algorithm for testing the embeddability of a k-dimensional unit cube in an orthogonal projection of a polyhedral set defined by a system of linear inequalities,* Mat. Issled., No. 110, Mat. Model. Optim., 1989, pp. 67–78, 162 (in Russian).

[4] UNGUREANU, V. *NP-hardness of the problem of testing the embeddability of a k-dimensional unit cube in an orthogonal projection of a convex polyhedral set,* Mat. Issled., No. 110, Mat. Model. Optim., 1989, pp. 120–128, 16 (in Russian).

Some possible generalizations of comonotonicity

Gheorghiță Zbăganu

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania
e-mail: gheorghitazbaganu@yahoo.com

If X is a random vector with n components we say that X is comonotonic if all it can be written as $X = f(U)$ where f is a monotonous function from R to the n-dimensional space. Or, in terms of probability distributions, F is a comonotonic distribution if its support is carried by an increasing path in the n-dimensional space.
It is obvious that if we have a selection of volume N from a comonotonic distribution, all the N points are on the graph of a monotonous curve.
We intend to define an index of comonotonicity for any distribution F.

Algebra, Logic, Geometry (with applications)

Upper bounds for the size of binary codes

Fatemeh Alinaghipour, Bahman Ahmadi

Department of Mathematics, Shiraz University, Shiraz, Iran
e-mail: bahman.ahmadi@gmail.com

Any subset C of the group $\mathbb{Z}_q^n$ is called a *q-ary* code of length n. For any element $c \in C$, the *weight* of c, i.e. the number of non-zero entries of c, is denoted by $\mathrm{wt}(c)$. The *Hamming distance* of any two elements $c_1, c_2 \in C$ is defined to be $\mathrm{dist}(c_1, c_2) = \mathrm{wt}(c_1 - c_2)$. Also the minimum hamming distance of a code C is defined to be the largest integer d such that for any $c_1, c_2 \in C$, we have $\mathrm{dist}(c_1, c_2) \geq d$. The maximum size of any q-ary code of length n with minimum Hamming distance d is denoted by $A_q(n, d)$. It this talk, we will provide an overview of a technique which employs representation theory of finite groups as well as some graph theoretical techniques to obtain some upper bounds for $A_2(n, d)$.

Some notes on the maximum nullity of graphs

Fatemeh Alinaghipour, Bahman Ahmadi

Department of Mathematics, Shiraz University, Shiraz, Iran
e-mail: bahman.ahmadi@gmail.com

For a given simple graph G with n vertices, the *maximum nullity* (respectively, *positive semidefinite maximum nullity*) of G, denoted by $\mathrm{M}(G)$ (respectively, $\mathrm{M}_+(\mathrm{G})$) is defined to be the maximum nullity of all the real symmetric (respectively, positive semidefinite) $n \times n$ matrices whose (i, j) entries (for $i \neq j$) are nonzero if and only if the vertices v_i and v_j are adjacent in G. The problem of determining and/or approximating $\mathrm{M}(G)$ ($\mathrm{M}_+(G)$) are important deep problems which have attracted considerable attentions in the recent years. In this talk we will describe some of our results concerning the bounds of these parameters. In particular, we will introduce the graph theoretical *zero-forcing* parameters and discuss how they can bound maximum nullity parameters. As well, we will provide some of our results towards proving the well-known Delta Conjectures of graphs, which state that $\mathrm{M}(G)$ ($\mathrm{M}_+(G)$) are bounded bellow by the minimum degree of the vertices of the graph.

On special bundles and superstrings

Cristian Anghel

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Cristian.Anghel@imar.ro

We intend to prezent the class of stable omalous bundles, a construction of them and their applications in superstrings.

On Almost Contact Metric Structures of Cosymplectic Type

Mihail Banaru, Galina Banaru

Smolensk State University
e-mail: mihail.banaru@yahoo.com

1. As it is known, the almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\, \xi,\, \eta,\, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\cdot\rangle$ is the Riemannian metric [5], [6]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N. As examples of almost contact metric structures we can consider the cosymplectic structure, the nearly cosymplectic structure, the Sasaki structure and the Kenmotsu structure.
The cosymplectic structure is characterized by the following conditions:

$$\nabla\eta = 0,\ \nabla\Phi = 0,$$

where ∇ is the Levi–Civita connection of the metric. It was proved that the manifold, admitting a cosymplectic structure, is locally equivalent to the product $M \times R$, where M is a Kählerian manifold [6].
An almost contact metric structure $\{\Phi,\, \xi,\, \eta,\, g\}$ is called nearly cosymplectic, if the following condition is fulfilled [6], [8]:

$$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = 0,\ X, Y \in \aleph(N).$$

We note that the nearly cosymplectic structures have many remarkable properties and play an important role in contact geometry. We mark out a number of articles by H. Endo on the geometry of nearly cosymplectic manifolds as well as the fundamental research by E.V. Kusova on this subject [8].
It is known that if $(N,\, \{\Phi,\, \xi,\, \eta,\, g\})$ is an almost contact metric manifold, then an almost Hermitian structure is induced on the product $N \times R$ [5], [6]. If this almost Hermitian structure is integrable, then the input almost contact metric structure is called normal. A normal contact metric structure is called Sasakian [6]. On the other hand, we can characterize the Sasakian structure by the following condition:

$$\nabla_X(\Phi)Y = \langle X, Y\rangle\,\xi - \eta(Y)X,\ X, Y \in \aleph(N).$$

For example, Sasakian structures are induced on totally umbilical hypersurfaces of Kählerian manifolds [6], [10]. As it is well known, the Sasakian structures have also many important properties. In 1972 K. Kenmotsu has introduced a class of almost contact metric structures, defined by the condition

$$\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\,\xi - \eta(Y)\Phi X,\ X, Y \in \aleph(N).$$

The Kenmotsu manifolds are normal and integrable, but they are not contact manifolds, consequently, they can not be Sasakian. We mark out that the fundamental monograph by Gh. Pitiş [9] contains a detailed description of Kenmotsu manifolds and their generalizations and a set of

important results on this subject.

2. Let us consider the first groups of Cartan structural equations of the most important almost contact metric structures [6], [8], [10]. The first group of structural equations of the cosymplectic, nearly cosymplectic, Kenmotsu and Sasakian structures are the (1), (2), (3) and (4), respectively:

$$\begin{gathered} d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta; \\ d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta; \\ d\omega = 0; \end{gathered} \tag{1}$$

$$\begin{gathered} d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + H^{\alpha\beta\gamma}\omega_\beta \wedge \omega_\gamma + H^{\alpha\beta}\omega_\beta \wedge \omega, \\ d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + H_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + H_{\alpha\beta}\omega^\beta \wedge \omega, \\ d\omega = -\frac{2}{3}G_{\alpha\beta}\omega^\alpha \wedge \omega^\beta - \frac{2}{3}G^{\alpha\beta}\omega_\alpha \wedge \omega_\beta; \end{gathered} \tag{2}$$

$$\begin{gathered} d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + \omega \wedge \omega^\alpha, \\ d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + \omega \wedge \omega_\alpha, \\ d\omega = 0; \end{gathered} \tag{3}$$

$$\begin{gathered} d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta - i\omega \wedge \omega^\alpha, \\ d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + i\omega \wedge \omega_\alpha, \\ d\omega = -2i\omega^\alpha \wedge \omega_\alpha. \end{gathered} \tag{4}$$

In [7], V.F. Kirichenko and I.V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type. As they have established, the condition

$$d\omega = 0$$

is necessary and sufficient for an almost contact metric structure to be of cosymplectic type. V.F. Kirichenko and I.V. Uskorev have also proved that the structure of cosymplectic type is invariant under canonical conformal transformations [7]. We recall also that a conformal transformation of an almost contact metric structure $\{\Phi, \xi, \eta, g\}$ on the manifold N is a transition to the almost contact metric structure $\{\widetilde{\Phi}, \widetilde{\xi}, \widetilde{\eta}, \widetilde{g}\}$, where $\widetilde{\Phi} = \Phi$, $\widetilde{\xi} = e^f\xi$, $\widetilde{\eta} = e^{-f}\eta$ and $\widetilde{g} = e^{-2f}g$. Here f is a smooth function on the manifold N [6].

Evidently, a trivial example of structure of cosymplectic type is the cosymplectic structure, because it has well-known Cartan structural equations (1). Another important example of the almost contact metric structure of cosymplectic type is the Kenmotsu structure with the Cartan structural equations (3). On the other hand, it is easy to see that the nearly cosymplectic and Sasakian structures are not of cosymplectic type.

3. We consider almost contact metric structures induced on oriented hypersurfaces of six-dimensional sphere with the canonical almost nearly Kahlerian structure [2], [4]. For 2-hypersurfaces (i.e. for hypersurfaces with type number 2) we obtain the following result:

Theorem A. *The Cartan structural equations of the almost contact metric structure on an oriented 2-hypersurface of the nearly Kählerian six-sphere are the following:*

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + B^{\alpha\beta\gamma}\omega_\beta \wedge \omega_\gamma + \left(-\sqrt{2}\widetilde{B}^{3\alpha\beta} - \frac{1}{\sqrt{2}}\widetilde{B}^{\alpha\beta 3} + i\sigma^{\alpha\beta}\right)\omega_\beta \wedge \omega;$$

$$d\omega_\alpha = -\omega_\alpha^\beta \wedge \omega_\beta + B_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + \left(-\sqrt{2}\widetilde{B}_{3\alpha\beta} - \frac{1}{\sqrt{2}}\widetilde{B}_{\alpha\beta 3} - i\sigma_{\alpha\beta}\right)\omega^\beta \wedge \omega; \tag{5}$$

$$d\omega = 0.$$

Here the systems of functions $\{B^{abc}\}$ and $\{B_{abc}\}$ are the components of the Kirichenko tensors [1] of the sphere S^6 and σ is the second fundamental form of the immersion of the 2-hypersurface into S^6; $a, b, c = 1, \ldots, 6$; $\alpha, \beta = 1, 2$. The structural equations (5) perfectly correspond to the structure of cosymplectic type, but this almost contact metric structure is not cosymplectic or Kenmotsu. So, we have proved the following result.

Theorem B. *Hypersurfaces with type number two in a nearly Kählerian six-sphere admit non-cosymplectic and non-Kenmotsu almost contact metric structures of cosymplectic type.*

4. At the end, we remark that Theorem B generalize the result from [3], where it has been proved that 2-hypersurfaces in an arbitrary Kählerian manifold also admit non-cosymplectic and non-Kenmotsu almost contact metric structures of cosymplectic type. On the other hand, in [2] and [4], it was proved that almost contact metric structure induced on an oriented 0- or 1-hypersurface of the nearly Kählerian six-sphere S^6 is necessarily nearly cosymplectic, i.e. it is not of cosymplectic type.

Bibliography

[1] A. Abu-Saleem and M. Banaru, Some applications of Kirichenko tensors, Analele Univ. Oradea, Fasc. Mat., 17(2), (2010), 201–208.

[2] A. Abu-Saleem and M. Banaru, On almost contact metric hypersurfaces of nearly Kählerian 6-sphere, Malaysian Journal of Mathematical Sciences, 8(1), (2014), 35–46.

[3] M. Banaru, On almost contact metric 2-hypersurfaces in Kählerian manifolds, Bulletin of the Transilvania University of Braşov. Series III. Mathematics, Informatics, Physics, 9(58) (1), (2016), 1–10.

[4] M. Banaru and G. Banaru, A note on almost contact metric hypersurfaces of nearly Kählerian 6-sphere, Bulletin of the Transilvania University of Braşov. Series III. Mathematics, Informatics, Physics, 8(57) (2), (2015), 21–28.

[5] D.E. Blair, Riemannian geometry of contact and symplectic manifolds, Progress in Math., Birkhauser, Boston–Basel–Berlin, (2002).

[6] V.F. Kirichenko, Differential-geometric structures on manifolds, Pechatnui Dom, Odessa, (2013) (in Russian).

[7] V.F. Kirichenko and I.V. Uskorev, Invariants of conformal transformations of almost contact metric structures, Mathematical Notes, 84 (5), (2008), 783–794.

[8] E.V. Kusova, On geometry of nearly cosymplectic structures (PhD thesis), Moscow State Pedagogical University, (2013) (in Russian).

[9] Gh. Pitis, Geometry of Kenmotsu manifolds, Publ. House of Transilvania University Braşov, (2007).

[10] L.V. Stepanova, Contact geometry of hypersurfaces in quasi-Kählerian manifolds (PhD thesis), Moscow State Pedagogical University, (1995) (in Russian).

Varietes in the category of localy convex spaces

Dumitru Botnaru

Tiraspol State University
e-mail: dumitru.botnaru@gmail.com

In the category $\mathcal{C}_2\mathcal{V}$ of the local convex topological vectorial Hausdorff spaces they are known reflective subcategories closed in relation to the factor spaces: Π - the subcategory of complete spaces with a weak topology; $\mathcal{S}$ - the subcategory of spaces with weak topology; $\mathcal{N}$ ($u\mathcal{N}$ and $\mathcal{S}h$) of nuclear (ultranuclear and Schwartz) spaces etc. In the category $\mathcal{C}_2\mathcal{V}$ there are reflective subcategories closed in relation to the various classes of factorobjects.
Definition 1. Let $\mathcal{B}$ a class of epimorfisms: $\mathcal{B} \subset \mathcal{E}pi$. The reflective subcategory $\mathcal{R}$ is called the $\mathcal{B}$-variete if she is closed in relation to the $\mathcal{B}$-factorobjects.
Definition 2. The class of morfisms $\mathcal{B}$ is called stable to the right, if from $f^{'}b = b^{'}f$ is a puschout square and $b \in \mathcal{B}$, it results also $b^{'} \in \mathcal{B}$.
If $\mathcal{M}$ is a class of monomorphisms, and $\mathcal{A}$ a subcategory, then we denoted by $\mathrm{S}_{\mathcal{M}}(\mathcal{A})$ the full subcategory of all $\mathcal{M}$-subobjects of objects from $\mathcal{A}$.
We will examine the reflective subcategories closed in relation to the epimorphisms $\mathcal{E}pi$. Their class is denoted by $\mathbb{V}(\mathcal{E}pi)$.
THEOREM 1. 1. *The subcategory* Π *of complete spaces with a weak topology is a* $\mathcal{E}pi$*-variete.*
2. *Let* $(\mathcal{E}, \mathcal{M})$ *be a factorization structure with the class of injections stable to the right. Then* $\mathrm{S}_{\mathcal{M}}(\Pi)$ *is a* $\mathcal{E}pi$*-variete.*
3. $\mathcal{S} = \mathrm{S}_{\mathcal{M}_p}(\Pi) \in \mathbb{V}(\mathcal{E}pi)$.
In the category $\mathcal{C}_2\mathcal{V}$ the pair $(\mathcal{E}_u, \mathcal{M}_p)$=(the class of surjective morphisms, the class of topological embedding) form a factorization structure.
A subcategory $\mathcal{L}$ is called c-reflective, if $\mathcal{S} \subset \mathcal{L}$ and the reflector functor $l : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{L}$ is exactly to the left. The class of c-reflective subcategories we denoted by $\mathbb{R}_c$. If $\mathcal{L} \in \mathbb{R}_c$ then the class $\varepsilon\mathcal{L} = \{e \in \mathcal{E}pi |\ l(e) \in \mathcal{I}so\}$ is complete both left and right, and $((\mathcal{M}_p \circ (\varepsilon\mathcal{L}))^{\rceil}, \mathcal{M}_p \circ (\varepsilon\mathcal{L}))$ is a factorization structure with class of injections $\mathcal{M}_p \circ (\varepsilon\mathcal{L})$ stable of left (see[1]). We mention that $\mathcal{S}$, $u\mathcal{N}$, $\mathcal{S}h \in \mathbb{R}_c$.
THEOREM 2. *Let* $\mathcal{L} \in \mathbb{R}_c$ *be, and* $\mathcal{M} = \mathcal{M}_p \circ (\varepsilon\mathcal{L})$. *Then*
1. $\mathrm{S}_{\mathcal{M}}(\Pi) \in \mathbb{V}(\mathcal{E}pi)$. 2. $\mathrm{S}_{\mathcal{M}}(\Pi) = \mathrm{S}_{\varepsilon\mathcal{L}}(\mathcal{S})$.

Bibliography

[1] D. Botnaru, *Structures bicatégorielles complémentaires.* ROMAI Journal, V.5, N 2, 2009, p.5-27.

The Reflector Functors and Lattice $\mathbb{L}(\mathcal{R})$

Botnaru Dumitru, Cerbu Olga

State University from Tiraspol, State University Moldova
e-mail: dumitru.botnaru@gmail.com, olga.cerbu@gmail.com

It is demonstrated that in the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector Hausdorff spaces [4] any reflector functor preserves the classes of factorisation structures (see [3]).
The factorisation structure $(\mathcal{E}_p, \mathcal{M}_u)$ =(the class of precise epimorphisms, the class of universal

monomorphisms) was described in the lattice [2]. In the category $\mathcal{C}_2\mathcal{V}$, a monomorphism $m : X \longrightarrow Y$ is universal then and only when every continuous functional defined on X extends through m. In the class $\mathbb{R}$ of not zero reflective subcategories of the category $\mathcal{C}_2\mathcal{V}$ we introduce the order $\mathcal{R}_1 \leq \mathcal{R}_2$ if $\mathcal{R}_1 \subset \mathcal{R}_2$. In the class of right factorisation structures we introduce the order $(\mathcal{P}_1, \mathcal{I}_1) \leq (\mathcal{P}_2, \mathcal{I}_2)$ if $\mathcal{P}_1 \subset \mathcal{P}_2$. Let Π be the subcategory of complete spaces whith weak topology and $\pi\colon \mathcal{C}_2\mathcal{V} \to \Pi$ – the reflector functor. The subcategory Π is the smallest element in the lattice $\mathbb{R}$.
Let $\mathcal{R} \in \mathbb{R}$. For any object X of the category $\mathcal{C}_2\mathcal{V}$, either $r^X\colon X \to rX$ and $\pi^X\colon X \to \pi X$ where $\mathcal{R}$ and Π-his replique. Because $\Pi \subset \mathcal{R}$, we have $\pi^X = v^X \cdot r^X$, for a morphism v^X. Note $\mathcal{U} = \mathcal{U}(\mathcal{R}) = \{r^X \mid X \in |\mathcal{C}_2\mathcal{V}|\}$, $\mathcal{V} = \mathcal{V}(\mathcal{R}) = \{v^X \mid X \in |\mathcal{C}_2\mathcal{V}|\}$. We have the following factorisation structures $(\mathcal{P}'', \mathcal{I}'') = (\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R})) = (\mathcal{V}^{\urcorner}, \mathcal{V}^{\urcorner\llcorner})$, $(\mathcal{P}', \mathcal{I}') = (\mathcal{P}'(\mathcal{R}), \mathcal{I}'(\mathcal{R})) = (\mathcal{U}^{\llcorner\urcorner}, \mathcal{U}^{\llcorner})$ (see [1]). For $\mathcal{R} \in \mathbb{R}$ note through $\mathbb{L}(\mathcal{R})$ the class of the factorisation structures $(\mathcal{E}, \mathcal{M})$ for which $\mathcal{P}'(\mathcal{R}) \subset \mathcal{E} \subset \mathcal{P}''(\mathcal{R})$ and $\mathbb{L}_u(\mathcal{R}) = \{(\mathcal{E}, \mathcal{M}) \in \mathbb{L}(\mathcal{R}) \mid \mathcal{M} \subset \mathcal{M}_u\}$, where $\mathcal{M}_u$ is the class of universal monomorphisms (see [2]).
Theorem. Let $\mathcal{R} \in \mathbb{R}$. Then the following statements are true:
1. $\mathbb{L}_u(\mathcal{R})$ is a lattice with the smallest element $(\mathcal{P}'_u, \mathcal{I}'_u) = ((\mathcal{E}_p \cup \mathcal{U}(\mathcal{R}))^{\llcorner\urcorner}, (\mathcal{E}_p \cup \mathcal{U}(\mathcal{R}))^{\llcorner})$ and the biggest element $(\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R}))$.
2. Let $(\mathcal{E}, \mathcal{M}) \in \mathbb{L}_u(\mathcal{R})$. Then the reflector functor $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$ preserves both classes $\mathcal{E}$ and class $\mathcal{M}$:
$r(\mathcal{E}) \subset \mathcal{E}$ and $r(\mathcal{M}) \subset \mathcal{M}$.
3. $f \in \mathcal{P}''(\mathcal{R}) \Longleftrightarrow r(f) \in \mathcal{P}''(\mathcal{R})$.

Bibliography

[1] Botnaru D., *Structures bicatégorielles complémentaires.* ROMAI J., v.5, Nr.2, 2009, p.5-27.

[2] Botnaru D., Gysin V.B., *Stable monomorphisms in the category of separating locally convex spaces*, Bulletin Acad. Sciences of Moldova, 1973, Nr.1, p.3-7 (in Russian).

[3] Kennison J. F., *Reflective functors in general topology*, TAMS, 1965, v.118, p.309-315.

[4] Robertson A. P., Robertson W. J., *Topological vector spaces*, Cambridge, England, 1964.

Paramedial groupoids and abelian groups

Liubomir Chiriac, Natalia Bobeica

Tiraspol State University
e-mail: `llchiriac@gmail.com, nbobeica1978@gmail.com`

A groupoid $(G, \cdot)$ is said to be *paramedial*, if the following conditions holds: $xy \cdot zt = ty \cdot zx$, for all $x, y, z, t \in G$ [1, 2].
A groupoid $(G, \cdot)$ is called *medial* if $xy \cdot zt = xz \cdot yt$, for any $x, y, z, t \in G$. The relationship between mediality and associativity was studied in [3].
Theorem 1. *Let G be a paramedial groupoid satisfying the following conditions:*
1. For two any elements $x, y \in G$, there are elements $e, f \in G$ such that $ex = x = xf$ and $ey = y = yf$;
2. For each element $x \in Q$ and each element $e \in G$ such that $ex = x$, there is an element x'_e such that $x'_e x = e$.
Then G is an abelian group.
An element α of the groupoid G is said to be a center associative element, if and only if one has

$x \cdot \alpha y = x\alpha \cdot y$ for all $x, y \in G$ [4].

Theorem 2. *Let G be a paramedial groupoid. If in G there is some center associative element α such that for every $b \in G$, the equations $ax = b$ and $ya = b$ are soluble in G, then the groupoid G is a commutative monoid.*

Theorem 3. *Let G be a paramedial groupoid. If in G there is some center associative element α such that for every $b \in G$, the equations $ax = b$, $ya = b$ and $bz = a$ are soluble in G, then the groupoid G is an abelian group.*

Bibliography

[1] Belousov V.D., Foundations of the theory of quasigroups and loops, Moscow, Nauka, 1967, 223 pp.

[2] Bobeica N., Chiriac L. On topological AG-groupoids and paramedial quasigroups wiht multiple identities. Romai Journal, vol.6, nr.1, 2010, p. 5-14.

[3] Morgado Jose, Entropic groupoids and abelian groups, Gazeta de Matematica, N 103, 01.07.66, p.8-10, Instituto de Fisica e Matematica, Universidade Federal de Pernambuco, Brasil.

[4] Garrison G.N., Quasigroups, Ann. of Math., 41, 1940, p. 474-487.

On Malcev's hyperalgebras

Ina Ciobanu

Alecu Russo Balti State University, Republic of Moldova
e-mail: viorelina@mail.ru

We use the terminology from [1, 3, 5].

Denote by $Com(X)$ the set of all non-empty compact subsets of a space X. A set-valued mapping $\theta : X \longrightarrow Y$ associate with each element x of a space X a non-empty subset $\theta(x)$ of a space Y. Let $\theta : X \longrightarrow Y$ be a set-valued mapping. The mapping θ is upper (lower) semicontinuous if the set $\theta^{-1}(H)$ is closed (open) in X for any closed (open) subset H of Y. The mapping θ is closed (open) if the set $\theta(W)$ is closed (open) in Y for any closed (open) subset W of X.

Let $\{E_n : n \in \mathbb{N} = \{0, 1, 2, 3, \ldots\}\}$ be a sequence of pairwise disjoint topological spaces. The discrete sum $E = \oplus\{E_n : n \in \mathbb{N}\}$ is the continuous signature of universal E-polyalgebras. A structure of an E-hyperalgebra on a non-empty space G is a family $\{e_{nG} : n \in \mathbb{N}\}$, where $e_{0G} : E_n \times G^n \to G$ is a single-valued mapping and $e_{nG} : E_n \times G^n \to G$ is a set-valued mapping for any $n \in \mathbb{N}$, $n \geq 1$. A topological universal hyperalgebra of the signature E or a topological E-hyperalgebra is a family $\{G, e_{nG} : n \in \mathbb{N}\}$, where G is a non-empty space, $e_{0G} : E_n \times G^n \to G$ is a single-valued continuous mapping and $e_{nG} : E_n \times G^n \to G$ is an uper semicontinuous compact-valued mapping for any $n \in \mathbb{N}$, $n \geq 1$.

Follows [4, 6, 8, 2, 7] we introduce the following notions.

A hipergroup is a hyperalgebra G with a unique binary operation $\{\cdot\}$, unary operation $\{^{-1}\}$ and a quasi-identity e for which: (HG1) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in G$; (HG2) $x \in e \cdot x \cap x \cdot e$ for any $x \in G$; (HG3) $x \in y \cdot z$ implies $y \in x \cdot z^{-1}$ and $zy^{-1} \cdot x$ for all $x, y, z \in G$.

A hypergroup G is called a polygroup if: (PG1) e is an identity, i.e. $x \cdot e = e \cdot x = x$ for each $x \in G$; (PG2) $x \cdot x^{-1} = x^{-1} \cdot x = \{e\}$ for each $x \in G$.

If G is a topological space and a hypergroup and the operations $\{\cdot, ^{-1}\}$ are compact-valued and upper semicontinuous, then G is a topological hypergroup. If G is a topological space and a polygroup and the operations $\{\cdot, ^{-1}\}$ are compact-valued and upper semicontinuous, then G is a

topological polygroup. Any topological group is a topological polygroup.
A Mal'cev polyalgebra is a hyperalgebra G with one ternary set-valued operation $m : G^3 \longrightarrow G$ such that $m(y, x, x) = \{y\}$ and $y \in m(x, x, y)$ for all $x, y \in G$. If G is a topological space and m is a compact-valued upper semicontinuous mapping,then G is called a topological Mal'cev polyalgebra.
Theorem 1. *Let (G, m) be a topological Mal'cev polyalgebra. If G is a T_0-space, then G is a T_2-space.*
Theorem 2. *Let (G, m) be a topological Mal'cev polyalgebra. Then any congruence α on G is open. In particular, $T_a(\alpha) = T_q(\alpha)$ and the projection $p_\alpha : G \longrightarrow G/\alpha$ of G onto $(G/\alpha, T_q(\alpha))$ is an open homomorphism.*
Remark 1. *The assertions of Theorems 1 and 2 are not true for topological hypergroups.*
Remark 2. *The assertions of Theorems 1 and 2 are true for topological polygroups and topological homogeneous polyalgebra.*

Bibliography

[1] A.V.Arhangelskii and M.Tkachenco, *Topological Groups and Related Structures,* Amsterdam - Paris. Atlantis Press/World Scientific, 2008.

[2] M.M.Choban. *The structure of locally compact algebras,* Serdica: Bulg. Math. Publ. 18 (1992), 129-137.

[3] M.M.Cioban. *On the theory of topological algebraic systems,* Trudy Moskovskogo Matem. Obshchestva 48 (1985) 106-149. English translation: Trans. Moscow Math. Soc. 48 (1986) 115-159.

[4] P. Corsini and V. Leoreanu, *Applications of Hyperstructures Theory*, Advanced in Mathematics, Kluwer Academic Publishers, 2003.

[5] R. Engelking. *General Topology.* -Warszawa: PWN, 1977.

[6] M. Koskas, *Groupoides, demi-hypergroupes et hypergroupes*, J. Math. Pures Appl. 49:9 (1970) 155-192.

[7] A. I. Mal'cev, *Free topological algebras*, Trans. Moscow Math. Soc. (2) 17 (1961) (Russian original: Izvestia Akad. Nauk SSSR **21** (1957) 171–198).

[8] T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press, Inc, 115, Palm Harber, USA, 1994.

The cardinality of parametric bases in the chain logics

Ion Cucu

Moldova State University
e-mail: cucuion2012@gmail.com

Chain logics are intermediary between classical logic and intutionistic one. A superintuitionistic logic is said to be chain logic if the formula $((p \supset q) \vee (q \supset p))$ is true in it.
Let us consider the pseudo-Boolean algebra

$$Z_m = \langle \{0, \tau_1, \tau_2, \ldots, \tau_{m-2}, 1\}; \Omega \rangle$$

where $0 < \tau_1 < \tau_2 < \ldots < \tau_{m-2} < 1$ and $\Omega = \{\&, \vee, \supset, \neg\}$. The logic of Z_m (LZ_m) denotes the set of valid formulas. The functions $\varphi(x)$ and $\psi(x)$ are not expressible by any formulas on Z_4 :

$\varphi(0) = 0, \varphi(\tau_1) = \varphi(\tau_2) = \tau_2$, $\varphi(1) = 1$ and $\psi(0) = 0, \psi(\tau_1) = 1, \psi(\tau_2) = \tau_2, \psi(1) = 1$.
The function f of the algebra Z_m is called parametrically expressed by means of a system of functions Σ of Z_m if there exist functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via Σ using superpositions, such that the predicate $f(x_1, x_2, \ldots, x_n) = x_{n+1}$ is equivalent to $\exists\, t_1 \exists\, t_2 \ldots \exists\, t_l \big((g_1 = h_1)\& \ \& \ldots \& (g_r = h_r)\big)$ on Z_m. A system of formulas Σ is said to be parametric basis in a logic L if Σ is parametrically complete and parametrically independent in L.
A criterion for parametric completeness has been previously found by the author for all chain logics and it may be stated in the following way.
Theorem 1 *The system of formulas Σ is parametrically complete in LZ_m iff Σ is parametrically complete in LZ_3 and for every relation $\varphi(x) = = y$, $\psi(x) = y$ there is a formula of Σ that does not preserve them on Z_4.*
The next theorem gives the cardinality of parametric basis in the chain logics.
Theorem 2. *The cardinality of any parametric basis in LZ_2 is at most 4, in LZ_3 at most 6, and in any chain logic linked to LZ_4 is at most 7.*
As remark to this theorem, we would like to add that the numbers 4, 6 and 7 given in it may not be decreased.

On left-transitive quasigroups

Natalia Didurik

University of the Academy of Sciences of Moldova
e-mail: natnikkr83@mail.ru

Main concepts and definitions can be found in [1].
A quasigroup $(Q, \cdot)$ is said to be left-transitive if the identity $xy \cdot xz = yz$ holds [2]. The following results are obtained.
Lemma 1. *Any left-transitive quasigroup $(Q, \cdot)$ has a left unit, i.e., there exists an element $f \in Q$ such that $f \cdot x = x$ for all $x \in Q$.*
Lemma 2. *Any left-transitive quasigroup $(Q, \cdot)$ is an LIP-quasigroup.*
Theorem 1. *Any loop, which is an isotope of left-transitive quasigroup $(Q, \cdot)$, is a group.*
Lemma 3. *Any left-transitive quasigroup $(Q, \cdot)$ is isotopic to an abelian group if and only if translation R_f is an automorphism of the quasigroup $(Q, \cdot)$.*
Lemma 4. *Any left-transitive quasigroup $(Q, \cdot)$ is a left F-quasigroup if and only if translation R_f is an automorphism of the quasigroup $(Q, \cdot)$.*
Lemma 5. *Any left-transitive quasigroup $(Q, \cdot)$ is a left Bol quasigroup.*
Lemma 6. *Any quasi-automorphism γ of left-transitive quasigroup $(Q, \cdot)$ has the form*

$$\gamma = R_k R_f \gamma_0,$$

where k is a fixed element of the set Q, f is a left unit of the quasigroup $(Q, \cdot)$, γ_0 is an automorphism of $(Q, \cdot)$.
Theorem 2. *Any autotopy (α, β, γ) of left-transitive quasigroup $(Q, \cdot)$ has the form*

$$(\alpha, \beta, \gamma) = (R_f L_k, R_d, L_k R_d) R_f \theta,$$

where f is a left unit of the quasigroup $(Q, \cdot)$, k, d are the fixed elements of the set Q, θ is an automorphism of $(Q, \cdot)$.

Bibliography

[1] V. D. Belousov. *Foundations of the theory of quasigroups and loops.* Izdat. "Nauka", Moscow, 1967, 223 p.

[2] I. A. Florja. The relation of left-transitive quasigroups with Bol quasigroups. *Nets and quasigroups. Mat. Issled.* Vyp. 39 (1976), 203-215.

On definition of CI-quasigroup

N.N. Didurik[1], V.A. Shcherbacov[2]

[1] *University of the Academy of Sciences of Moldova*
[2] *Institute of Mathematics and Computer Science*
Academy of Sciences of Moldova
e-mail: `victor.scerbacov@math.md`

Necessary definitions can be found in [2, 5, 6]. **Definition 1**. *Loop* $(Q, \cdot)$ *satisfying one of the equivalent identities* $x \cdot yJx = y$, $xy \cdot Jx = y$, *where* J *is a bijection of the set* Q *such that* $x \cdot Jx = 1$, *is called a CI-loop [1].*
Definition 2. *Groupoid* $(Q, \cdot)$ *with the identity* $xy \cdot J_r x = y$, *where* J_r *is a map of the set* Q *into itself, is called a left CI-groupoid [3, 4].*
From the results of Keedwell and Shcherbacov [6, Proposition 3.28] it follows that the left CI-groupoid in which the map J_r is bijective, is a CI-quasigroup. Any finite left CI-groupoid is a quasigroup [4].
Theorem. *Any left CI-groupoid* $(Q, \cdot)$ *is a CI-quasigroup.*

Bibliography

[1] R. Artzy. On loops with a special property. *Proc. Amer. Math. Soc.*, 6:448–453, 1955.

[2] V.D. Belousov. *Foundations of the Theory of Quasigroups and Loops.* Nauka, Moscow, 1967. (in Russian).

[3] V.D. Belousov and B.V. Tsurkan. Crossed-inverse quasigroups (CI-quasigroups). *Izv. Vyssh. Uchebn. Zaved. Mat.*, 82(3):21–27, 1969. (in Russian).

[4] V. Izbash and N. Labo. Crossed-inverse-property groupoids. *Bul. Acad. Ştiinţe Repub. Mold. Mat.*, (2):101–106, 2007.

[5] H.O. Pflugfelder. *Quasigroups and Loops: Introduction.* Heldermann Verlag, Berlin, 1990.

[6] Victor Shcherbacov. *Elements of Quasigroup Theory and Applications.* CRC Press, Boca Raton, 2017.

Some results on the orthogonal $n-$ groupoids

Vladimir Izbash

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova
e-mail: `vladimir.izbas@math.md, izbas@mail.md`

A $n-$*groupoid* is a pair (Q, A), where Q is a set and "A" a $n-$ary operation on Q. We usually write the result of the operation A on the systems $(a_i^n) = (a_1, a_2, ..., a_n)$ as $A(a_1^n)$. Number $|Q|$ is called the order of the $n-$groupoid (Q, A). It is clear that $|Q^n| = |Q|^n$.
An $n-$groupoid (Q, A) is an $n-$*quasigroup* if an only if the equation $A(a_1^{i-1}, x, a_{i+1}^n) = b$ has a unique solution in Q for any $a_1^n, b \in Q$, and for all $i \in \{1, 2, ..., n\} = \overline{1, n}$.

Definition 1. *The systems $A_1, A_2, ..., A_n$ of n $n-$groupoids is said to be orthogonal if for any $b_1, b_2, ..., b_n \in Q$ the system of equations $A_i(x_1^n) = b_i$ has a unique solution in Q, for all $(b_1^n) \in Q$ and $i \in \overline{1,n}$. This is equivalent to the fact that the mapping $\theta : Q^n \mapsto Q^n$, $\theta(x_1^n) = (A_1(x_1^n), A_2(x_1^n), ..., A_n(x_1^n))$ is a permutation of Q^n. In this case the ordered systems of $n-$groupoids $A_2, A_3, ..., A_n$ is called an orthogonal complement for (Q, A_1).*

Proposition 1. *If a $n-$groupoid (Q, A_1) has an orthogonal complement than $A_1(\underbrace{Q, Q, ..., Q}_{n}) = Q$, where $A_1(\underbrace{Q, Q, ..., Q}_{n}) = \{A_1(a_1^n) | a_1^n \in Q\}$.*

For $a \in Q$ we denote $S(a, A_1) = \{(x_1^n) \in Q^n | A_1(x_1^n) = a\}$.

Proposition 2. *i) For all $a, b \in Q$, $S(a, A_1) \bigcap S(b, A_1) = \emptyset \Leftrightarrow a \neq b$;*
ii) and $\bigcup\limits_{a \in Q} S(a, A_1) = \underbrace{Q \times Q \times Q \times ... \times Q}_{n}$, that is $\sum\limits_{a \in Q} |S(a, A_1)| = |Q|^n$

Theorem 1. *i) A finite $n-$groupoid (Q, A_1) has an orthogonal complement if and only if $|S(a, A_1)| = |Q|^{n-1}$ for all $a \in Q$;*
ii) If $|Q| = m$, and $|S(a, A_1)| = |Q|^{n-1}$ for all $a \in Q$ then it has exactly $((m^{n-1})!)^m$ orthogonal complements.

Bibliography

[1] V. D. Belousov. *Systems of orthogonal operations.* Mat. Sbornik, **77** , (119) (1968), 38-58.

[2] L. Euler. *Recherches sur une Nouvelle Espece de Quarres Mgiques.* Verh. Genootsch. der Wet. Vlissingen, **9** , (1782), 85-232.

[3] A. Sade. *Groupoides orthogonaux.* Publ. Math. Debrecen, **5** , (1958), 229-240.

Cyclic crystallographic punctual groups of W_p-symmetry

Alexandru Lungu

Moldova State University, Chisinau, Republic of Moldova
e-mail: lungu.al@gmail.com

W_p-symmetry [1,2] is one of the main generalizations of classical symmetry. In this case the transformations of the qualities, attributed to the points, essentially depend on the choice of points. The cristallographic punctual groups $G^{(W_p)}$ of W_p-symmetry are subgroups of left direct wreath product of initial group P with the generating group G, accompanied with a fixed isomorphism $\varphi : G \to AutW$ by the rule $\varphi(g) = \overleftarrow{g}$, where $\overleftarrow{g}: w \mapsto w^g$. (where W is the direct product of isomorphic copies of group P which are indexed by elements of group G).
Any major group of W_p-symmetry with the finite generating group G and initial group P of permutations it is construct in shape of the left standard direct wreath product of group G with initial group P, accompanied with a fixed isomorphism $\varphi : G \to AutW$ by the rule $\varphi(g) = \overleftarrow{g}$, where $\overleftarrow{g}: w \mapsto w^g$. For the W'-semi-major finite groups of W_p-symmetry with the initial group P and generating group G should: a) we to find in group W that non trivial subgroups W', wich verify the conditions $\overleftarrow{g}\ (W')W' = W'$, for each g from group G; b) we construct the semi-direct product

of the group G with W'.
Any W'-semi-minor (respectively, pseudo-minor) finite group of W_p-symmetry with initial group P and generating group G can be derived from G and P by the following steps: 1) we construct the direct product W of isomorphic copies of the group P which are indexed by elements of G; 2) we find in W so non trivial subgroups W' (respectively, the subset W' with unit, which is not a subgroup) wich verify the condition $\overleftarrow{g}\ (W')W' = W'$, for each g from group G; 3) we construct an exact natural left quasi-homomorphism μ with the kernel H of the group G onto the subgroup W' (respectively, onto the subset W' with unit, which is not a subgroup) by the rule $\mu(Hg) = w$; 4) we combine pairwise each g of class Hg with $w = \mu(Hg)$; 5) we introduce into the set of all these pairs the operation $g_i w_i \circ g_j w_j = g_k w_k$, where $g_k = g_i g_j$, $w_k = w_i^{g_j} w_j$ and $w_i^{g_j}(g_s) = w_i(g_j g_s)$.

From the netrivial cyclical crystallographic punctual groups G and group P ($P \cong C_2$), we obtained: 9 major groups, 20 W'-semi-major groups, 10 semi-minor groups and 20 pseudo-minor groups of W_p-symmetry.
This work was partially supported by the project 15.817.02.26F.

Bibliography

[1] V.A. Koptsik and I.N. Kotsev, *To the theory and classification of color symmetry Groups. II. W-symmetry.* (Russian). OIYI Reports, R4-8068, Dubna, 1974.

[2] A.P. Lungu, *Classification of groups of W-symmetry (Russian).* Studies in modern algebra and geometry. p. 79-84, Chisinau:Shtiintsa, 1983.

About commutative Moufang loops with some restrictions for subloops

N. Lupaşco

Tiraspol State University, Moldova
e-mail: nlupashco@gmail.com

A commutative Moufang loops (abbreviated CML) is characterized by the identity $x^2 \cdot yz = xy \cdot xz$ [1]. The multiplication group $\mathfrak{M}(Q)$ of the CML Q is the group generated by all the translation $L(x)$, where $L(x)y = xy$.
In this paper we examine nonassociative commutative Moufang loop Q with some restriction for subloops and subgroups of its multiplicative groups $\mathfrak{M}(Q)$.
We proved that if the multiplication group $\mathfrak{M}$ of infinite commutative Moufang loop Q has such an infinite subgroup $\mathfrak{N}$, than in $\mathfrak{M}$ every abelian subgroup which has with $\mathfrak{N}$ an infinite intersection is a normal subgroup, then the loop Q is associative. Some results of multiplication group of CML Q we proved in [2, 3, 4].
It is also investigated the construction of subgroups of the groups of automorphisms $\mathrm{Aut}(Q)$ and $\mathrm{Aut}(\mathfrak{M})$. It is proved that $\mathrm{Aut}(Q)$ and $\mathrm{Aut}(\mathfrak{M})$ have matricial representation. The results are related to article [5].

Bibliography

[1] Bruck R. H. *A survey of binary systems.* Berlin: Springer Verlag, 1958.

[2] Lupashco N. T. *On commutative Moufang loops with some restrictions for subgroups of its multiplication groups.* Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica, 2006, N 2(51), 55 – 101.

[3] Lupashco N. T. *On commutative Moufang loops with some restrictions for subloops and subgroups of its multiplication groups.* Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica, 2009, N 3(61), 52 – 56.

[4] Lupashko N. T. *On automorphisms of commutative Moufang loops*, Discrete Math. Appl., 21:3 (2011), 309 – 316.

[5] Lupashko N. T. and Sandu N. I. 2012, *Automorphisms of commutative Moufang loops satisfying the minimality condition*, Matematicheskie Zametki, 2012, Vol. 91, No. 3, pp. 407 – 421.

The Commutator Ideals in C^* Crossed Products by Hereditary Subsemigroups

Mamoon Ahmed

Princess Sumaya University for Technology, Jordan.
e-mail: `m.ahmed@psut.edu.jo`

Let (G, G_+) be a lattice-ordered abelian group with positive cone G_+ and $H+$ a hereditary subsemigroup of G_+. In [3] we introduced a closed ideal I_{H_+} of the C^*-subalgebra B_{G_+} of $l^\infty(G_+)$ spanned by the functions $\{1_x : x \in G_+\}$. Then we showed that the crossed product C^*-algebra $B_{(G/H)_+} \times_\beta G_+$ is realized as an induced C^*-algebra $\mathrm{Ind}_{H^\perp}^{\widetilde{G}} \left(B_{(G/H)_+} \times_\tau (G/H)_+\right)$. In this paper, we prove the existence of the following short exact sequence of C^*-algebras

$$0 \to I_{H_+} \times_\alpha G_+ \to B_{(G/H)_+} \times_\alpha G_+ \to \mathrm{Ind}_{H^\perp}^{\widetilde{G}} \left(B_{(G/H)_+} \times_\tau (G/H)_+\right) \to 0$$

We then show that there is an isomorphism ι of $B_{H_+} \times_\alpha H_+$ into $B_{G_+} \times_\alpha G_+$. Which leads to show some interesting non-trivial results on commutator ideals in C^* crossed products by hereditary subsemigroups as they involve a non-trivial extension of previous results in [1] and [2].

Bibliography

[1] S. Adji, *Crossed products of C*-algebras by semigroups of endomorphisms*, Ph.D thesis, The University of Newcastle, 1995.

[2] S. Adji, I. Raeburn and R. Rosjanuardi, *Group extensions and the primitive ideal spaces of Toepltitz algebras*, Glasgow Math. J. 49 (2007), 8192.

[3] M. Ahmed and A. Pryde, *Semigroup crossed products and the induced algebras of lattice-ordered groups*, J. Math. Anal. Appl. 364 (2010) 498507.

Normal high order elements in cyclotomic finite field extensions

R. B. Popovych[1], R. V. Skuratovskii[2]

[1] *Lviv Polytechnic National University, Lviv, Ukraine*
[2] *Institute of Mathematics of NAS of Ukraine, Kiev, Ukraine*
e-mail: rombp07@gmail.com, ruslan@imath.kiev.ua

Let q be a power of a prime number p, and F_q be a finite field with q elements. For any integer m, a normal basis of F_{q^m} over F_q is a basis of the form $\{\alpha, \alpha^q, ..., \alpha^{q^{m-1}}\}$ for some $\alpha \in F_{q^m}$. In this case the element $\alpha \in F_{q^m}$ is called normal over F_q [1,2].
Let $r = 2n + 1$ be a prime number coprime with q. Let q be a primitive root modulo r, that is the multiplicative order of q modulo r equals to $r - 1$. Set $F_q(\theta) = F_{q^{r-1}} = F_q[x]/\Phi_r(x)$, where $\Phi_r(x) = x^{r-1} + ... + x + 1$ is the r-th cyclotomic polynomial and $\theta \equiv x \ (\mathrm{mod}\Phi_r(x))$. It is clear that the equality $\theta^r = 1$ holds. We have the following tower of finite fields: $F_q \subset F_{q^n} \subset F_{q^{2n}}$.
Theorem. *Let b be such element of the field F_q that $2nb \neq 1(\mathrm{mod}p)$. Then the following statements are true*:
(a) *element $\theta + b \in F_{q^{2n}}$ is normal over F_q*;
(b) *element $\theta + \theta^{-1} + 2b \in F_{q^n}$ is normal over F_q.*
Note that for $b = 0$ the order of θ equals only to r. But for $b \neq 0$ the element $\theta + b \in F_{q^{2n}}$ has high order according to [3, Theorem 1 (a), (d)]. Also if $2b = (a^2 + 1)a^{-1}$ and $b \neq 0$, then the element $\theta + \theta^{-1} + 2b = (\theta^{-f} + a)(\theta^f + a)$ has high order according to [3, Theorem 1 (b)].

Bibliography

[1] Lidl R., Niederreiter H., Finite Fields. – Cambridge: Cambridge University Press, 1997.

[2] Mullen G. L., Panario D., Handbook of finite fields. – Boca Raton: CRC Press, 2013.

[3] Popovych R., Elements of high order in finite fields of the form $F_q[x]/\Phi_r(x)$. Finite Fields Appl., 2012, Volume 18, pp. 700-710.

[4] Skuratovskii R. V., Finding normal basis of finite field during deterministic polynomial time. Vol. 25. Visnuk of Kiev's National University. Mechanics and mathematics. pp. 49 - 54, 2011.

[5] R. V. Skuratovskii Structure and minimal generating sets of Sylow 2-subgroups of alternating groups. Source: https://arxiv.org/pdf/1702.05784.pdf, 2017.

Integral formulae for codimension-one foliated Finsler manifolds with (α, β)-metric

Vladimir Rovenski

Mathematical Department, University of Haifa, Israel
e-mail: vrovenski@univ.haifa.ac.il

Integral formulae for foliated manifolds provide obstructions for existence of foliations with given geometric properties and have applications in different areas of geometry and analysis. Our

goal is generalization of Reeb integral formula (that the total mean curvature of the leaves of a codimension-1 foliated closed Riemannian manifold is zero) and its counterpart with the second mean curvature for a codimension-1 foliated closed Finsler manifold with a general (α, β)-metric. All of that is done by a comparison of extrinsic and intrinsic curvatures and second fundamental forms of two Riemannian structures and (the Levi-Civita and Chern) connections, which arise in a natural way from a given (α, β)-metric.

Bibliography

[1] V. Rovenski and P. Walczak, Integral formulae for codimension-one foliated Finsler spaces, Balkan J. Geom. & Appl., 21, No. 1 (2016), 76–102.

[2] V. R. and P. Walczak, Integral formulae for codimension-one foliated Randers spaces, Publ. Math. Debrecen, vol. 91, 2017.

[3] V. R., Integral formulae for a codimension-one foliated Finsler manifold with (α, β)-metric, preprint, ArXiv:1607.00989.

Structure of commutant and centralizer of Sylow 2-subgroups of alternating and symmetric groups, minimal generating sets of Syl_2A_n, its applications in cryptography

R. V. Skuratovskii

Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine
e-mail: `ruslan@imath.kiev.ua`

The aim of this investigation is to research the structure of a commutant and a centralizer of Sylow 2-subgroups from Syl_2A_n and Syl_2S_n and find numbers of minimal generating sets for $Syl_2S_{2^k}$ and Syl_2A_n. Let us denote by $X^{[k]}$ a regular truncated binary rooted tree with number of levels from 0 to k, where $X = \{0, 1\}$. The set $X^n \subset X^*$ is called the n-th level of the tree X^* and $X^0 = \{v_0\}$. The vertex of X^j having the number i we denote by $v_{j,i}$ also we denote by $v_{j,i}X^{[k-j]}$ the subtree of $X^{[k]}$ with a root in $v_{j,i}$. Let $n = 2^{k_0} + 2^{k_1} + ... + 2^{k_m}$, where $0 \leq k_0 < k_1 < ... < k_m$ and $m \geq 0$. Recall that $Syl_2S_n = Syl_2S_{2^{k_0}} \times ... \times Syl_2S_{2^{k_m}}$.

Definition 1. *Let us call the index of an automorphism β on X^l a number of non-trivial v.p. of β on X^l.*

Theorem 1. *The centralizer of $Syl_2S_{2^{k_i}}$ with $k_i > 2$, in Syl_2S_n is isomorphic to $Syl_2S_{2^{k_0}} \times \ldots \times Syl_2S_{2^{k_{i-1}}} \times Syl_2S_{2^{k_{i+1}}} \times \ldots \times Syl_2S_{2^{k_m}} \times Z(Syl_2S_{2^{k_i}})$.*

Theorem 2. *The centralizer of $Syl_2A_{2^{k_i}}$ with $k_i > 2$, in Syl_2A_n is isomorphic $Syl_2A_{2^{k_0}} \times \ldots \times Syl_2A_{2^{k_{i-1}}} \times Syl_2A_{2^{k_{i+1}}} \times \ldots \times Syl_2A_{2^{k_m}} \times Z(Syl_2A_{2^{k_i}})$.*

We will call ***diagonal base*** [1, 2] (S_d) for $Syl_2S_{2^k} \simeq AutX^{[k]}$ a generating set that has the on each level only one non trivial v.p. A number of no trivial v.p. that can be on X^j is odd and equal to 2^{j-1}. Thus, general cardinality of S_d for $Syl_2S_{2^k}$ is 2^{2^k-k-1}.
There is minimum one permutation of type `T` [1] in S_d for $Syl_2A_{2^k}$. It can be chosen in $(2^{n-2})^2$ ways. Thus, general cardinality of S_d for $Syl_2A_{2^k}$ is $2^{2^{k-1}-k-2}(2^{n-2})^2$. The cardinality of all bases for $Syl_2S_{2^k}$ is equal to $2^{k(2^k-k-1)} \cdot (2^k-1)(2^k-2)(2^k-2^2)...(2^k-2^{k-1})$. Hence it can be applied in cryptography [2].

Lemma 1. *A commutator of elements from $Syl_2A_{2^k}$ is an element having all even indexes on X^l, $l < k-1$ and all even indexes on X^{k-1} in intersection with $v_{1,1}X^{[k-1]}$ and with $v_{1,2}X^{[k-1]}$.*

Theorem 3. *Sets of commutators of Sylow 2-subgroups $Syl_2A_{2^k}$, $Syl_2S_{2^k}$ of A_n and S_n are commutants of $Syl_2A_{2^k}$ and $Syl_2S_{2^k}$.*

Bibliography

[1] R. V. Skuratovskii Structure and minimal generating sets of Sylow 2-subgroups of alternating groups. Source: https://arxiv.org/pdf/1702.05784.pdf, 2017.

[2] B. Pawlik, The action of Sylow 2-subgroups of symmetric groups on the set of bases and the problem of isomorphism. Algebra and Discrete Mathematics. (2016), Vol. 21, N. 2, pp. 264-281.

[3] Skuratovskii R. V. Corepresentation of a Sylow p-subgroup of a group S_n. Cybernetics and systems analysis, volume 1, (2009), pp. 27-41.

[4] Y. A. Drozd, R. V. Skuratovskii. Generators and and relations for wreath products. Ukr. Math. J., vol. 60., No. 7, (2008), pp. 1168 V1171.

On Some Semitopological Rings

Boris Țarălungă

Ion Creangă", State Pedagogical University, Republic of Moldova

e-mail: Borisstar@mail.ru

The notion of a semitopological ring was introduced in [1]. In [1] is proved that any locally compact semitopological ring is a topological ring.
A topological space X is said to be pseudocompact if X is a Tychonoff space and every continuous function on X is bounded. A topological space X is said to be countably compact if every countable open cover has a finite subcover. A space X is called a separable space if it contains a finite or countable everywhere dense set.
The tightness $t(x, X)$ of a topological space X at a point $x \in X$ is the smallest cardinal α number such that, whenever $x \in cl_x(Y)$ for some Y of X, there exists a subset Z of Y, with $|Z| \leq \alpha$, such that $x \in cl_x(Z)$. The tightness of space X is $t(x) = sup\{t(x, X) : x \in X\}$. A topological space X is called a k-space if it is Hausdorff and is representable in the form of the image of some locally compact space under the quotient map.
Theorem. *Let R be a semitopological pseudocompact ring and R belong to one of the classes below:*

(a) countably compact spaces;

(b) spaces with countable tightness;

(c) separable spaces;

(d) k-spaces.

Then R is a topological ring.

Bibliography

[1] M. Ursul. Topological Rings Satisfying Compactness Conditions. Kluwer Academic Publisher, Dordrecht/Boston/London, 2002, p. 95.

One remark of about some classes of generalized solvable loops

Vasile I. Ursu

Institute of Matematics Simion Stoilow of the Romanian Academy,
Technical University of Moldova
e-mail: `Vasile.Ursu@imar.ro`

Generally solvable loops arose as a result of an attempt to transfer various definitions of the solvability and nilpotency of loops to loops with infinite rows of sub-loops. The fundamental here was the work of Kurosh and Chernikov [1]. The basic concepts of the theory of loops can be found, for example, in [2].
We give a number of definitions. The system S of the subloop of L is called a *subnormal series* of loop L if it:

(i) contains a single subloop $\{e\}$ of the loop L and contains L;

(ii) linearly ordered by inclusion;

(iii) is closed with respect to reunions and intersections of any set of subloops of S is contained in S;

(iv) any subloop $A \in S$ is a normal subloop of loop $\hat{A} = \cap\{B \in S \mid B \in S, A \subset B\}$.

Factor-loops $\hat{A}/A$ $(A \in S)$ are called the *sections* of the subnormal series S. A subnormal series S is said to be *normal* if all its terms are normal subloops in the loop L. A normal series S is said to be *central* if all its sections are central. A subnormal series S is said to be *solvable* if all its sections are abelian groups. A loop with a solvable subnormal series is called generalized solvable.
The following *Kurosh - Chernikov classes* arise:
RN – this is the class of all loops possessing a solvable subnormal series (not necessarily finite);
RI – this is the class of all loops possessing a solvable normal series;
Z – this is the class of all loops possessing a central series.
An axiomatizable class of algebraic systems, which can be defined by a system of quasi-identities, is called a *quasivariety*. The foundations of the theory of quasivarieties were laid down by A.I. Maltsev [3].
The following is proved
Theorem. *The classes RN, RI, Z are quasivarieties.*

Bibliography

[1] Kurosh A.G. , Chernikov S.N. *Solvable and nilpotent groups.* Uspekhi Matem. Nauk. T. 2., 1947, No. 3, p. 18-50.

[2] Bruck R.H. *A survey of binary systems.* Springers Verlag, 1958.

[3] Maltsev A.I. *Algebraic systems.* Publishing Nauka, Moscow 1972.

Computer Science

A well-posed second-order anisotropic diffusion-based structural inpainting scheme

Tudor Barbu[1], Ionut Munteanu[2]

[1] *Institute of Computer Science of the Romanian Academy*
[2] *Institute of Mathematics "O. Mayer", Romanian Academy*
e-mail: tudor.barbu@iit.academiaromana-is.ro

A nonlinear second-order parabolic partial differential equation (PDE) - based model for image interpolation is proposed in this article. The anisotropic diffusion scheme introduced here is also investigated mathematically, a rigorous mathematical treatment being provided for it.
The interpolation (inpainting, completion) techniques, which reconstruct the missing regions by using the known image information around them, are divided into structure-based and texture-based methods. The structure-based interpolation approaches use PDE and variational models to perform the image completion tasks. Some influential variational inpainting schemes are Mumford-Shah Inpainting and Total Variation (TV) Inpainting models. The second-order PDE-based algorithms also follow the variational principles, while the higher-order PDE interpolation models, such as CDD Inpainting or Cahn-Hillard Inpainting, do not derive from variational schemes, being directly given as evolutionary equations.
The novel second-order PDE inpainting approach proposed here is derived from our past nonlinear diffusion-based restoration models, by introducing an image mask corresponding to the missing or highly deteriorated image zones. The introduced structural completion model is composed of a nonlinear parabolic equation and some boundary conditions. It is based on a positive and monotonically decreasing edge-stopping function that is properly chosen for an effective restoration. A robust mathematical treatment of the well-posedness of this differential model is then performed, the existence and uniqueness of a weak solution of the PDE being investigated. A consistent and fast-converging explicit finite-difference based numerical approximation scheme is then constructed for it.
The successfully performed inpainting experiments and method comparison are also described in this paper. Our anisotropic diffusion approach provides an effective structure-based reconstruction, outperforming many existing interpolation methods, but cannot inpaint properly the missing textures. It also works successfully in noisy conditions, reducing the amount of Gaussian noise.
Keywords *Structural image inpainting, Nonlinear diffusion, Second-order PDE model, Mathematical investigation, Finite difference method, Numerical approximation scheme.*

Structural Pattern Recognition for Titania Nanotubes Arrays in SEM Images

Adrian Ciobanu, Mihaela Luca

Institute of Computer Science, Romanian Academy - Iasi Branch
e-mail: adrian.ciobanu@iit.academiaromana-is.ro

As there is a high interest in titania coated materials for their remarkable properties, we propose a structural approach for computer assisted evaluation of images representing titania nanotube arrays (TNA). These are a special kind of TiO_2 materials with a nanostructure prepared by electrochemical corrosion of a Ti surface. Depending on the selected electrolyte content, discharge

current and duration, this results in highly ordered nanotubes arrays. We worked on top-view high-resolution field-enhanced secondary electron microscopy (FE-SEM) images of TNA samples, acquired with electron beam resolutions of 1.1 to 2.5 nm and magnification ranging between 12 W and 2,600,000 W. The gray level composition of these images is far from ideal, so, thresholding methods folowed by segmentation techniques may fail in some regions of the images. In our approach we started by taking into consideration the basic pixel strucure of a tiny nanotube image, with several dark pixels in the middle, surrounded by a ring of bright pixels. Using MATLAB as testing environment, we applied several image processing techniques in order to detect as accurate as possible the position of each nanotube in a TNA sample image. We apply first a histogram equalization, then we detect all the dark pixels under a certain gray level and eliminate blocks of dark pixels that are too numeorus for a nanotube hole. The remaining dark pixels are very good candidates for nanotube holes and we consolidate them with pixels arround even if they are not so dark. Then we detect bright pixels arround these small bloks of dark pixels. Again we consolidate these rings of bright pixels to be sure that we are capturing all the important pixels as holes, nannotube margins or interstitial spaces. The three types of pixels are then painted in different collors and superimposed on the original image to assess the quality of nanotube detection. We also superimposed on the original image the ideal circles representing detected nanotubes, based on their automatically computed center. The obtained results are good, as there are lots of correctly detected nanotubes. We have also a low percentage of fake detected nanotubes and regions were nanotubes were impossible to detect, while the human eye can somehow see them.

Web Crawler and Its Facilities

Victor Cozlov

Alecu Russo Bălţi State University, Republic of Moldova

e-mail: viteak91@mail.ru

The Internet is a vast sourse of information. We usually navigate through immense number of web pages to find necessary information. Then, we unconsciously get rid of redundant information to highlight the essential one. However, sometimes, we have to do this work repeatedly. Little by little this work becomes annoying and a Web Crawler can be really helpful.

A Web Crawler is a system for the bulk downloading of web pages. The basic web crawling work algorithm is rather simple: given a set of seed Uniform Resource Locators (URLs), a crawler downloads all the web pages addressed by the URLs, extracts the hyperlinks contained in the pages, and one by one downloads the web pages addressed by these hyperlinks [1].

The elaborated Web Crawler works in the noi.md site area. As Crawler permanently remains in recourse, each link for the next processing is verified if it corresponds to the pattern.Every new link for the URL list is checked for uniqueness to exclude the possibility of repetitive downloading. Each piece of news is provided in two languages - Romanian and Russian, so for more complete data each piece of news is downloaded in both languages. We need about 17.2 seconds for one complete cycle, where a page download takes no more than 1 second; for the extraction of text information about 1 second and for about 16 second for links downloading [2].

We use Jsoup library for page downloading and link processing as this has a lot of functions for effective work with data. Despite this fact, we had to write some functions for redundant text elliminating and necessary text finding.

The main purpose of this article is to present our approach in the elaboration of a Web Crawler for visual demonstration of time each process takes and for the creation of text corpora for future processing.

Bibliography

[1] C. Olston, M. Najorg. Web crawling. In Information Retrieval, vol. 4, no. 3, 2010. 74 p.

[2] V. Cozlov, M. Petic. Downloading useful information from Web with the help of a Crawler. In: Proceedings CMSM 2017, Chişinău, 2017. pp. 501-504.

Visualization of algorithmic solutions for route selection, using the example of wine pathways

Oleksandr Fedosieiev[1], Oleksandr Lefterov[2], and Andriy Yaremich[3]

[1] *PRA IT-Pro*
[2] *V.M.Glushkov Institute of Cybernetics NAS of Ukraine*
[3] *National University of "Kyiv-Mohyla Academy"*
e-mail: lefterov.alexander@gmail.com, AF1958@gmail.com, stpmsuit@gmail.com

To represent acceptable routes constructed using the algorithm of the shortest k-vertex path to the user, visualization of calculated routes should be performed within the existing database of "wine pathways". The visualization capabilities of GIS software were used for this purpose. The choice of proposed approach of graphical link with the end-user allows us to obtain acceptable solutions quickly and with minimal operating costs. This takes into account the user's perception of additional useful information visually available on the route map. The GIS engine interacts with the semantic mapping unit (SMU), which also offers recommendations for its use in addition to the standard procedure for displaying information and data on the screen. The user gets additional opportunities by minimizing the amount of input information when forming / editing requests within the Service and reducing the operational tasks size. The user selects the start and end points of the path. The priority and necessary preferences for route calculation are inputted by user. The received data are visualized by Semantic mapping block (SMB). In addition, within the settings of the specified route, which is predefined in the buffer zone, the SMB draws up additional information contextually related to the information received. The main feature of the Service is the route calculation engine, in which the user gets the opportunity to see all the target objects represented in the path that correspond to the given preferences. Further, based on the primary result, the user edits or confirms his request. Moreover, the user can do described actions by simple removal, addition, displacement or marking, the points of interest at the interactive map window. Such a process of pre-determination can be iterative. At each iteration an optimization block is started, the operation of which is described in [1]. At the cyrrent moment, the optimization block uses the Gurobi 6.5.0 open access package for calculations. SMU is built on the basis of open source libraries. As a web GIS platform, Leaflet.js is used.

Bibliography

[1] Fedosieiev O., Lefterov O., Lykhovyd O., Stetsyuk P. Optimization tools for selecting wine routes // Theses of the conference CAIM-2015, Stefan cel Mare University of Suceava, Romania, September 17-20, 2015, pp. 37-38.

Station Cone Algorithm for Linear Programming with Non-Negativ Input Data

Le Thanh Hue

Faculty of Informatics Technology, Hanoi University of Mining and Geology, Hanoi, Vietnam
e-mail: lethanhhue.humg@gmail.com

The paper studies the linear programming problem with non-negative input data and applies the station cone algorithm to find the solution. From the non - negative input data and station cone algorithm's properties, the author has shown how to derive the feasible solution in a simple way. This significanly reduces the computational time, because it skips the feasible solution study phase without having to use the phase I of the simplex algorithm to search. The paper also points out how an interior point is the center of a convex polyhedron in the feasible domain. This is the point in order to connect with the vertex of the station cone which serves to identify the variables in and out of the simplex pivot. Another important part of this paper is the experimental calculation of the station cone algorithm for the above problem class. Tests have shown that the station cone algorithm has fewer iterations than the dual simplex method.

Specialized software application for processing customer reviews for various products using non-numeric statistics

Kuznetsov Eugene[1], Cealbash Oleg[2]

[1]*Institute of Mathematics and Informatics, Academy of Sciences of Moldova*
[2]*Institute of Power Engineering, Academy of Sciences of Moldova*
e-mail: kuznet1964@mail.ru, olegch0@yandex.ru

The enormous abundance of diverse information and the freedom of referring to it, associated with the Internet, is an absolute blessing. Attempts to target the search in such "abundance and diversity" often take a lot of time and search results can be qualified as either "not quite what I need" or "not at all what I need".
One possible way to solve the problem of increasing the "sharpness of blurry arrays" is to create a system of targeted profile search services that can become attributes or organic applications for known search platforms.
There are at least two major problems that need to be resolved when developing such search services:

- a high probability of a customer choosing a product that is functionally inadequate to the user's request (consumer requirements);
- a long time spent on search procedures, including acquaintance and generalization of feedback on the product of interest

In turn, the first problem implies the resolution of two more subtasks:

- a formalized representation of the concept of "customer requirements for goods";
- a development of a mechanism ensuring the compliance of the characteristic parameters of the goods with the requirements of the consumer.

Thus, the content of the first problem is to establish as clearly as possible a clear correlation between the variants of the configuration of the characteristic parameters of the product and its possible target uses. Especially this factor, in our opinion, is capable of ensuring the maximum compliance of the product's functional characteristics with a client request.
The content of the second problem consists in replacing the "manual" search with automatic procedures. In particular, the procedures for collecting and processing feedback submitted on a given set of electronic platforms for a fixed list of selected trade names are subject to automation. In this case, under the selected list, we mean a previously filtered set of products that corresponds to a user request.
We offer a software application that can serve as either an add-on or an organic component of various kinds of "search engines" - yandex.ru, google.com, rumbler.ru, etc. This software application is designed to provide support and assistance to potential buyers for choosing goods in various "electronic stores".
The development of the proposed software application involves at least two aspects. The first is purely technological, and relates to methods of collecting the source information from various sites, accumulating it in a database, and also questions of developing an ergonomic interface. The second aspect relates to methods of processing the collected information.
The innovative component of the proposed work is the development of a concept and rational scenario for processing initial data, using of non-trivial mathematical methods of data processing, for example, methods of non-numeric statistics, filtering by comparing data sets with "characteristic masks", etc. The development and application of numerical methods and algorithms for their implementation within the framework of this software application are important too.

Partial unimodality of independence polynomials of 1-well-covered graphs

Vadim E. Levit[1], Eugen Mandrescu[2]

[1]*Ariel University, Israel,*
[2]*Holon Institute of Technology, Israel*

e-mail: `levitv@ariel.ac.il, eugen_m@hit.ac.il`

A graph is *well-covered* if all its maximal independent sets are of the same cardinality, denoted $\alpha(G)$ (Plummer, 1970). If G is a well-covered graph with at least two vertices, and $G - v$ is well-covered for every vertex v, then G is a 1-*well-covered graph* (Staples, 1979). Let s_k be the number of independent sets of size k in a graph G. The polynomial $I(G;x) = s_0 + s_1x + s_2x^2 + ... + s_\alpha x^\alpha, \alpha = \alpha(G)$, is called the *independence polynomial* of G (Gutman and Harary, 1983). If $s_0 \leq ... \leq s_{k-1} \leq s_k \geq s_{k+1} \geq ... \geq s_\alpha$, for some k, then $I(G;x)$ is *unimodal*. It is known that for every permutation σ of the set $\{1, 2, ..., \alpha\}$ there is a graph G with $\alpha(G) = \alpha$ such that $s_{\sigma(1)} < s_{\sigma(2)} < ... < s_{\sigma(\alpha)}$ [1]. There are well-covered graphs with non-unimodal independence polynomials [3]. However, the independence sequence (s_k) of a well-covered graph satisfies $s_0 \leq s_1 \leq ... \leq s_{\lceil \frac{\alpha}{2} \rceil}$ [3]. The Roller-Coaster Conjecture [3], saying that for every permutation σ of $\{\lceil \frac{\alpha}{2} \rceil, ..., \alpha\}$ there is a well-covered graph G with $\alpha(G) = \alpha$ such that its independence sequence (s_k) satisfies $s_{\sigma(\lceil \frac{\alpha}{2} \rceil)} < s_{\sigma(\lceil \frac{\alpha}{2} \rceil + 1)} < \cdots < s_{\sigma(\alpha)}$, was validated in [2].
In this talk we show that the upper part of independence sequence (s_k) of an 1-well-covered graph is in a non-increasing order. Based on this fact, we propose the following.
Conjecture. For an 1-well-covered graph on n vertices, the unconstrained part of the independence sequence (s_k) is $\left(s_{\lceil \frac{2\alpha}{3} \rceil}, s_{\lceil \frac{2\alpha}{3} \rceil + 1}, ..., s_{\min\left\{\alpha, \lceil \frac{n-1}{3} \rceil\right\}}\right)$.

Bibliography

[1] Y. Alavi, P. J. Malde, A. J. Schwenk, P. Erdös, *The vertex independence sequence of a graph is not constrained*, Congr. Numer. 58 (1987) 15–23.

[2] J. Cutler, L. Pebody, *Maximal-clique partitions and the roller coaster conjecture*, Journal of Combinatorial Theory A **145** (2017) 25–35.

[3] T. S. Michael, W. N. Traves, *Independence sequences of well-covered graphs: non-unimodality and the roller-coaster conjecture*, Graphs & Combin. **19** (2003) 403–411.

Parallel programming models on HPC clusters

Carolina Opinca

Moldova State University
e-mail:

High-performance clusters (HPC) represents a system of computing nodes, interconnected by a high-speed communication environment and are used for computational purposes, in particular in scientific research. Computational clusters allow to reduce the calculation time, in comparison with a single computer, by dividing the job into parallel executing branches that exchange data over the binding network.

There are three models of parallel programming that predominate in the field of scientific and technical calculations: a message transfer model, a shared memory model and a data parallelism model. By type of architecture, the HPC cluster refers to systems with distributed memory by nodes, where each cluster node is a system with a shared memory.

Within the framework of the parallel programming model, there are different approaches, oriented to different architectures of high-performance computing systems and various tools. The aims of this study is to investigate the basic principles of creating parallel programs and to analyse how the efficiency of using a multiprocessor/multicore architecture depends on the development tools used and the methods of parallelization.

The most popular mechanism for organizing computing on a cluster is the implementation of the standard MPI (Message Passing Interface) which allows to send messages between nodes. The fact is that MPI processes communicate through a network and this leads to redundant copying of data across multiple buffers and increased memory consumption. For parallel computations within a single machine with shared memory, threads and distribution of tasks between them are much more suitable. Here the most popular in the world of HPC is the OpenMP standard.

The transfer of messages between processes that belong to the same cluster node is implemented on the basis of the OpenMP standard directives. The transmission of messages between processes that belong to different nodes of the cluster is realized on the basis of the communication functions of the MPI standard. However, the use of two frameworks (MPI and OpenMP) instead of one, does not only add to the complexity of programming, but does not always give the desired performance gain - at least not immediately. It is necessary to distribute the calculations between MPI and OpenMP, and, perhaps, solve the problems specific for each level.

Shannon entropy for imprecise and under-defined or over-defined information

Vasile Pătrașcu

Tarom Information Technology,
224F Bucurestilor Road, Otopeni
e-mail: patrascu.v@gmail.com

Shannon entropy was defined for probability distributions and then its use was expanded to measure the uncertainty of knowledge for systems with complete information. In this article it is proposed to extend the use of Shannon entropy to under-defined or over-defined information systems with a certain degree of imprecision. For this purpose, a two-step normalization procedure is proposed: a translation and a homothetic one. After the presentation, the procedure is used for calculating Shannon's entropy in the case of particular representations of information such as neutrosophic information, bifuzzy information, intuitionistic fuzzy information, imprecise fuzzy information and fuzzy partitions. In the case of neutrosophic information, two variants are possible: the first is the trivalent variant in which the certainty has three prototypes: true, neutral and false; the second is the bivalent variant in which the certainty has two prototypes: true and false. The article also shows that the presented method can also be used for other formulas such as Onicescu information energy, Tsallis entropy or Renyi entropy.

Developing classifiers for emotional messages clustering

Mircea Petic

Alecu Russo Balti State University, Bălţi, RM
Institute of Mathematics and Computer Science of ASM, Chişinău, RM
e-mail: petic.mircea@gmail.com

A characteristic feature of contemporary society is the special role of information networks, where different signals related to disasters that may be produced or already have been produced, may occur promptly.

This research is made within a project whose goal is related to the development of information systems oriented to ensure the security of citizens in extreme situations (natural calamities, technogenic catastrophes, etc.). The main source of information for the means of preventing and mitigating the consequences of social disasters are large volumes of unstructured data accessible to global information networks: mass media, social networks, blogs, and so on.

In order the processing phase be more rapid we elaborated a Crawler-based [1] application service. It search through web news articles, downloads and extracts the text of this news, and stores them in the database. As every news site has its own structure we should take into account its particularity.

Then we used a tool that extracts the most frequent words from the unstructured text data. For every of these words, the context, where it is present, is highlighted and rules of inflection and derivation with a high degree of accuracy are applied to generate more semantically related words, thus enriching the set of classifiers [2]. So, the procedure showed how to reduce the number of susceptible words for classifiers and to optimize the processing time.

Bibliography

[1] Najork M., Wiener J. L.Breadth-first crawling yields high-quality pages. In Proceedings of the Tenth Conference on World Wide Web, Hong Kong, May 2001. Elsevier Science pp. 114 - 118.

[2] Cojocaru Sv., Petic M., Horoş Gr. Tools for Texts Monitoring and Analysis Aimed at the Field of Social Disasters, Catastrophes, and Terrorism. In: Computer Science Journal of Moldova, v.24, n.2 (71), 2016, pp. 157-171.

On technology in creation of Games for distance learning on mobile devices and PCs

Victor Seichiuc

Moldova State University
e-mail: seiciuc@mail.ru, runsmaric@gmail.com

Used in E-learning the video games today plays an important role in the educational process. As a result, demand for games and game-making technologies are very appreciated. The developed and proposed *IT Methodologies* it is a set of Programs and Computer Techniques organized in such a way to allow on your PC or on your mobile device to create the desired Game, using minimal time and IT resources.
The basic stages of Game development are: 1) Writing scripts. 2) Integrating sound objects. 3) Integrate external graphic objects. 4) Compiling and exporting to mobile platforms. 5) Testing the application. 6) Final realization on-line.
The proposed *IT Methodology* can be applied to the development of different types of Games, for example: 1) Arcade, 2) Simulators, 3) Action, 4) Platforms, 5) Speed, 6) Training, 7) E-learning, 8) Gambling, etc. Games are organized on several levels. Each level contains its characteristic *elements* (buttons) that are responsible for the realization of certain concrete *events*. It is called the *purpose event*, the one who achieves the main result in the Game level. Otherwise, the event is called an *attendant event*. Here are some examples:

Action Game *Izolda Ninja Girl*: It's a fun game, developed for mobile platform Google Play. The game contains ten levels of play in which the player passes various obstacles and faces different opponents. In the play levels of the Game the *purpose event* is to destroy the Monsters and to move to the next level.
The *main character* as well as auxiliaries (*Monsters*) possesses elements of *Artificial Intelligence*, allowing them to interact by themselves. It is composed of the following parameters: 1) moving and stopping the movement; 2) recognition to the target object and distance calculation to him; 3) change of animation and behavior; 4) combat mode; 5) regeneration.

Speed Game *Space Runner*: It's a Game of speed and reaction, developed for mobile platform Google Play. The *purpose event* in the *Space Runner* is to jump over obstacles and accumulate *maximum points*. The *main character* of speed Game possesses elements of *Artificial Intelligence* - the variation of the speed of movement according to the distance traveled.

E-learning Game *Test-Game TP.T1*: The *Test-Game TP.T1* is developed for testing the knowledge in the distance learning. The test contains a set of evaluation questions and variants of answers to the course Theory of Probability and Mathematical Statistics, placed on the Moodle platform at Trade Co-operative University of Moldova (TCUM), HYPERLINK

http://www.uccm.md/. This *Test-Game TP.T1* was developed for mobile devices and PC and runs on Windows and / or Mac OS. The mobile version can be accessed on Google Play, PC version - accessed on the site TCUM. The *purpose event* in test levels is to get a *maximum of points*. Elements of *Artificial Intelligence* in *Test-Game TP.T1* are possibilities to finish the Game, depending on the *time spent* on the test.

Conclusion. The presence of *Artificial Intelligence* elements categorizes the described Games as *Intelligent Support Systems*.
IT Resources in Creating the Video-Games. The presented Games were created using three tools: *Construct 2* - the game modeling platform at the Programming level and Visual Design level. *Photoshop* - tools for creating Graphics. *Audacity* - Tools for creating and editing Sounds.

On development of Intelligent Software Tools for solving of Integral Equations

Vladislav Seichiuc, Eleonora Seichiuc, Gheorghe Carmocanu

Trade Co-operative University of Moldova, Moldova State University
e-mail: seiciuc@mail.ru, eseiciuc@mail.ru, gcarmocanu@yahoo.com

Many applied problems, especially those with nonsmooth solutions, are mathematically represented by the different classes of linear integral equations (IE). But, IEs can be exactly solved just in special cases, for example, when their kernels are degenerated. Thus, the elaboration of numerical and approximate methods for solving of IE remains a current problem. At present, these methods are increasingly integrated into Intelligent Software Tools aimed for solving of certain IE classes.
One of such Intelligent Software Tools is Intelligent Support System (ISS) for approximate solving of Fredholm and Volterra IE (ISS_IE) of the second kind (see [1]). The ISS_IE is based on the following received theoretical and applicative results:

- new computing algorithms are developed for spline-collocations and spline-quadratures methods for solving of Fredholm and Volterra IE of the second kind;
- a theoretical substantiation of the developed computing algorithms is obtained in the space of continuous functions and in the Hölder spaces;
- for elaborated algorithms the ISS_IE of the second kind is developed.

The core components of ISS_IE are:

1) the Base of Kernel Prototypes of IE (BPK_IE_COMP) for checking the sufficient conditions of IE compatibility,
2) the Base of Kernel Prototypes of IE (BPK_IE_COL) for solving by spline-collocations method,
3) the IE Solver (IES), accompanied by the convenient interface during the solving of integral equations.

For more efficient use, ISS_IE is undergoing upgrading: 1. The implemention of the degenerated kernel method for exact solving of Fredholm and Volterra IE of the second kind; 2. The development of the approximate method of replacing the kernel with degenerate kernel; 3. The utilization of the Wolfram Mathematica Application in the ISS_IE

Bibliography

[1] 1. E. Seichiuc. *The approximate solving of Integral Equations by the Intelligent Software Tools.* Abstract of PhD thesis in physics and mathematics, MSU, Chisinau, (2008), 27 pp.

Modernized Dijkstra's Algorithm and mathematical model for geographic information system

Ruslan Skuratovskii[1], Evgeny Osadchiy[2]

[1] *Institute of Mathematics of NAS of Ukraine, Kiev, Ukraine*
[2] *NTU "KPI", Kiev, Ukraine*

e-mail: `ruslan@imath.kiev.ua, public@sana.net.ua`

For modeling of a domain of the Earth there are usually used a cartographic projection. The goal of our investigation is method of finding of optimal path between adjacent control points that are in some vertices of graph. The tusk of line orienteering is to find optimal path between adjacent control points in some vertices of graph. The goal of our investigation is to construct such method of finding path through all control points in determed sequence. We can choose optimal path between every pair of control points (c.p.)
For this goal we develop modified Dijkstra's algorithm that has complexity $O(\frac{V^2}{4} + \frac{VlnV}{4})$ instead of usual complexity $O(V^2 + V\log_2 V)$. Also our algorithm permits a parallel realizing. As a result of a parallel implementation complexity of algorithm decrease in $|4(|V_o| - 1)|$ times, where $|V_o|$ is number of c.p. Coefficient of relatively effectiveness of parallel search is

$$\kappa_1 = \frac{V^2 + V\log_2 V}{(V^2 + V\log_2 V) \cdot |4(|V_o| - 1)|} = \frac{1}{|4(|V_o| - 1)|}.$$

We propouse to find an optimal path between two vertices using a opposition search method. This method are based on parallel opposition search of shortest path by Dijkstra's algorithm from two vertices A and B which are c.p. on oriented graph. First vertex with minimal distance to both vertices A and B is enclose vertex of a path from A to B. Model for a sequential search of path was constructed. Let e_{ij} is an edge between vertices v_i and v_j that has length d_{ij}. We consider a resistance r_{ij} of e_{ij} and determine new length of labeled graph edge as $\widetilde{d_{ij}} = r_{ij} d_{ij}$. The model for n competitor of sport orienteering has weighted sum objective function

$$\mathbb{F}\left(v_0, v_n, k\right) = \sum\nolimits_{s=i}^{j} d_{ij} r_{ij} s_{ij}^{-1} \to \min,$$

with inequality restriction $r_{ij} < R(k)$, where $R(k)$ is value of a critical load of k-th competitor of orienteering search, s_{ij} – speed of moving by e_{ij} [1, 2].

Let S – set of all c.p. If we can chose set S, then we use method of mask dynamic of search for solution of the problem

$$\min_{2 \le j \le n} \left(dp\left(\{1, 2, \ldots, n\}, j\right) + m\left[j, i\right]\right).$$

$$dp\left(S, i\right) = \min_{j \in S \setminus \{i\}} \left(dp\left(S \setminus \left[i\right], j\right) + m\left[j, i\right]\right).$$

Where $dp\left(S, i\right)$ is shortest path, which starts in v_1 and going by all vertices from $S \setminus \{v_i\}$, ending up in vertex v_j.

Bibliography

[1] R. V. Skuratovskii Generilized Euler's constant. Mathematical visnuk of NTSh. Vol. 10, 2013, pp. 163-168.

[2] Skuratovskii R. V. Modernized Pohlig-Hellman and Shanks algorithm. Visnuk of KNU. Cybernetics. 2015 y. pp. 56.

High Level Petri Nets in Scenario Analysis Related to the Field of Social Disasters

Titchiev Inga

Institute of Mathematics and Computer Science, Tiraspol State University
e-mail: inga.titchiev@gmail.com

People are facing with different kind of disasters [1, 3], which eventually can lead to new accidents and catastrophes. To analysis and mitigate their consequences, the formalism of High Level Petri nets[2] has been proposed. It will be examine the problem of people evacuation in useful time from the multi-storey buildings. So, the aim of this research is to provide potential solutions to respond in case of disaster.

Modeling large, complex systems can be a hard task. But similar to modular programming, these can be done by construction of complex nets that can be broken into smaller pieces by utilizing the facilities within different tools for creating substitution transitions, a transition which can represent an entire piece of net structure.

Acknowledgments. The research is performed in the frame of the project *Development of a toolkit for modeling strategies to mitigate social disasters caused by catastrophes and terrorism* (17.80013.5007.01/Ua).

Bibliography

[1] I. Titchiev, *Petri nets to model disaster prevention*, Proceedings of the Workshop on Foundations of Informatics, August 24-29, Chisinau, Republic of Moldova, pp. 445-449, 2015.

[2] X. He, T. Murata, *High-Level Petri Nets @ S Extensions, Analysis, and Applications*, Electrical Engineering Handbook (ed. Wai-Kai Chen), Elsevier Academic Press, pp.459-476, 2005.

[3] S.Cojocaru, M.Petic, I.Titchiev, *Adapting Tools for Text Monitoring and for Scenario Analysis Related to the Field of Social Disasters*, The proceedings of The 18th International Conference on Computer Science and Electrical Engineering (ICCSEE 2016), October 6-7, Prague, Czech Republic, pp. 886-892, 2016.

New nonbinary quantum codes

Zlatko Varbanov, Borislav Anchev

Department of Information Technologies, University of Veliko Tarnovo, Bulgaria
e-mail: vtgold@yahoo.com; zl.varbanov@uni-vt.bg

In [3] P.Shor gave a randomized algorithm for factorizing an integer in polynomial time on a quantum computer. Since factorization of large integers is the hard problem which underpins

public-key encryption systems such as RSA, the importance of this result is obvious, and a large amount of research into the possibility of building a quantum computer is going on. The relationship between quantum information and classical information is a subject currently receiving much study.

A quantum analogue of a bit of information is called a *qubit*. It is the state of a system in a 2-dimensional Hilbert space $\mathbb{C}^2$, spanned by e_0 and e_1, where e_0 and e_1 are eigenvectors corresponding to the eigenvalues 0 and 1 of the qubit. An error, like any physical process, is a unitary transformation of the state space. The space of errors to a single qubit is 4-dimensional, and is spanned by the four unitary matrices I (no error, $e_0 \to e_0$, $e_1 \to e_1$), X (bit error, $e_0 \to e_1$, $e_1 \to e_0$), Z (phase error, $e_0 \to e_0$, $e_1 \to$) $- e_1$), and $Y = iXZ$ (bit-phase error).

The setting in which quantum error-correcting codes (QECCs) exist is the quantum state space of n qubits. This space is $\mathbb{C}^{2n}$, and it has a natural decomposition as the tensor product of n copies of $\mathbb{C}^2$, where each copy corresponds to one qubit. A quantum error-correcting code is defined to be a unitary mapping (encoding) of k qubits into a subspace of the quantum state space of n qubits such that if any t of the qubits undergo arbitrary decoherence, not necessarily independently, the resulting n qubits can be used to faithfully reconstruct the original quantum state of the k encoded qubits [2]. A $[[n, k, d]]$ QECC can correct $\lfloor (d-1)/2 \rfloor$ errors, where k is the number of encoded qubits. As it is too difficult to construct good quantum codes in general, we can consider simpler constructions. The problem of finding good QECCs can be transformed into the problem of finding classical linear self-orthogonal codes over the finite field $\mathbb{F}_q$. In our work we use the well-known Calderbank-Shor-Steane (CSS) quantum code construction [2]: if there exists a classical linear $[n, k, d]_q$ code C such that $C^\perp \subset C$ (where $C^\perp$ is the orthogonal code of C) then there exists an $[[n, 2k - n, \geq d]]$ quantum code that is pure to d.

Using the CSS construction method, we construct self-orthogonal codes over $\mathbb{F}_3$ and we obtain new quantum codes with the following parameters: $[[18, 6, 4]]_3$, $[[20, 10, 4]]_3$, $[[24, 0, 9]]_3$, $[[24, 16, 3]]_3$, $[[26, 18, 3]]_3$, $[[27, 15, 4]]_3$, $[[28, 20, 3]]_3$, $[[30, 22, 3]]_3$, and $[[32, 24, 3]]_3$. Also, we construct some self-orthogonal codes over $\mathbb{F}_5$ that meet the bounds for the best known quantum codes for $q = 5$. All computer calculations in this work were performed by Q-Extension [1].

Bibliography

[1] I. Bouyukliev, *What is Q-extension?*, Serdica J. Computing **1**, 2007, 115–130.

[2] Calderbank A.R., Rains E.M., Shor P.W., Sloane N.J.A., Quantum error coorection via codes over $GF(4)$. *IEEE Trans. Inform. Theory. 44* 1998, pp.1369-1387

[3] Shor P.W., Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer, *SIAM J.Computing* 26 (1997), pp.1484-1509.

Education

Transfer issue from the perspective of the Situation Based Approach

Nicoleta Bleandura

Alecu Russo State University of Balti, Faculty of Exact, Economic and Environmental Sciences, Department of Mathematics and Informatics
e-mail: nicoleta.bleandura@gmail.com

The concept of transfer is important in the current Competence/Situation Based Approach (SBA) in education. The article presents the process of the parallel evolution of the transfer notion in the psychological and pedagogical field. The main ideas, but also major conceptual divergences implied in this process are described [1]. From the point of view of the SBA, the basic issue seems to consist of the double dimension of the transfer concept: repetition and displacement [2]. The article analyses major interpretations explaining these two mechanisms. Furthermore, the metaphors of mobilization and contextualization are also reflected, as alternative or additional explanations of the metaphor of transfer [3] [4] [5]. It is studied how the SBA in education calls for transfer within a complex and dynamic process of contextualization, de-contextualization and re-contextualization of knowledge [6].

Bibliography

[1] Chaussecourte, Ph. *Le transfert didactique, un faux ami?* In: Cohen-Azria, C., Sayac, N. Questionner l'implicite. Les méthodes de recherche en didactiques (3). Villeneuve d'Ascq: PUS, 2009. ISBN 978-2-7574-0122-4. pp. 137-145.

[2] Perrenoud, Ph. *Vers des pratiques pédagogiques favorisant le transfert des acquis scolaires hors de l'école.* In: Pdagogie collgiale, 10 (3), 1997, pp. 5-16.

[3] Tardif, J., Meirieu, Ph. *Stratégie pour favoriser le transfert des connaissances.* In: Vie pédagogique, 98, 1996, pp. 4-7.

[4] Jonnaert, Ph. *Actualité du transfert en éducation.* In: Jonnaert, Ph. Une notion tenace. Cahiers pédagogiques, 408, 2002, p. 11-12.

[5] Develay, M. *A propos des savoirs scolaires.* In: VEI Enjeux, 123, 2000, p. 28-37.

[6] Cabac, V. Scutelnic, O. *Formarea competenelor prin procesele de contextualizare - decontextualizare - recontextualizare.* In: Studia Universitatis Moldaviae, 9(69), 2013, pp. 10-14.

Abordări metodologie privind studierea compartimentului "Drumuri minime în graf"

Marina Bostan

Universitatea de Stat din Tiraspol
e-mail: bostanmarina@yahoo.com; marina.bostan13@gmail.com

În mod tradiţional cursul universitar Teoria Grafurilor predat, în mod special, în universităţile cu profil pedagogic, se studiază utilizându-se un număr limitat de instrumente didactice cu implementarea limitată a tehnologiilor informaţionale moderne. Eficienţa procesului de predare-învăţare-evaluare poate să crească continuu, doar introducând şi perfecţionând noi instrumente

didactice, în conexiune strânsă cu implementarea noilor tehnologii informaţionale. În experimentul desfăşurat de noi, privind implementarea tehnologiilor moderne în cursul universitar de Teoria Grafurilor s-a analizat din mai multe puncte de vedere oportunităţile şi avantajele oferite de softurile MAPLE 18, DELPHI, TPASACAL, C/C++ . Procesul de predare a compartimentului "Drumuri minime în graf" se axează atât pe partea teoretică cât şi practică, care este una foarte semnificativă în dezvoltarea şi consolidarea abilităţilor necesare pentru soluţionarea problemelor care ţin de algoritmica grafurilor. Unele dintre cele mai frecvente şi populare metode de a găsi cele mai scurte distanţe sunt algoritmii Dijkstra (pentru găsirea drumului minim de la un vârf fixat până la toate celelalte) şi Roy-Floyd (pentru identificarea celei mai scurte căi între toate perechile de vârfuri). Algoritmul Dijkstra a fost descoperit de Edsger W. Dijkstra, informatician olandez, in 1956 şi publicat 3 ani mai târziu. Algoritmul Roy-Floyd a fost publicat sub forma recunoscută în prezent de către Robert Floyd în anul 1962. în principiu, este asemănător algoritmului publicat de către Bernard Roy în 1959 [1,2]. La expunerea subiectelor din compartimentul "Drumuri minime în graf", tradiţional se utilizează limbajele de programare care, evident, au anumite avantaje, dar în acelaşi timp pot fi substituite cu succes de alte softuri matematice moderne cum ar fi de exemplu: AutoCad, MatLab, Maple, Matematica, etc. Utilizarea softurilor de ultima generaţie imprimă lecţiilor care ţin de algoritmica grafurilor un suflu nou şi tot odată sunt cu mult mai atractive, ilustrative şi în unele situaţii cu mult mai eficiente şi mai practice. în cadrul procesului de predare-învăţăre a teoriei grafurilor, se studiază cele mai importante metode din acest domeniu, cât şi implementarea algoritmilor la soluţionarea unor probleme concrete, modelul matematic a cărora este foarte clar şi exact. Chiar dacă această abordare este corectă din punct de vedere al asimilării materiei propuse, acest fapt nu îndeamnă studenţii, mai ales cei cu pregătirea peste nivelul mediu, la o atitudine creatoare privind elaborarea, obţinerea modelului matematic, prin intermediul căruia ar putea fi rezolvată problema respectivă. În contextul respectiv, pentru a îmbunătăţi situaţia la acest capitol, studenţilor le sunt propuse probleme, care necesită cunoştinţe pentru a elabora, întâi de toate, modele matematice şi abilităţi practice pentru a implementa algoritmul identificat, manual, aplicând limbajul C/C++ şi, în mod special, softul matematic Maple 18.

Bibliography

[1] Bang-Jensen, G. Gutin, *Digraphs Theory, Algorithms and Applications*, Springer-Verlag, 2007.

[2] J.A. Bomdy, U.S.R. Murty, Graph Theory, Springer, 2007.

Un model pedagogic centrat pe implementarea Sistemelor de Management al învăţării în procesul de studiu al Programării Orientate pe Obiecte

Andrei Braicov, Ala Gasnaș

Universitarea de Stat din Tiraspol, Republica Moldova
e-mail: abraicov@gmail.com, alagasnas@gmail.com

Programarea Orienată pe Obiecte (POO) este un concept de bază al Informaticii, o paradigmă care pe parcursul a 20 de ani continuă să inspire spre o evoluţie constantă în Programare. Cursul POO se regăseşte ca parte fundamentală a curriculei de formare profesională iniţială a specialiştilor în domeniul Informaticii.
Cercetările [1-5] arată că studenţii se confruntă cu diferite dificultăţi de studiere a conceptului POO, care ţin de:

- înţelegerea noţiunilor din POO (clasă, obiect, proprietate, metodă, modificator de acces, constructor, destructor etc.) şi relaţiile dintre aceste noţiuni;
- percepţia principiilor POO (abstractizarea, incapsularea, moştenirea, ierarhizarea, modularizarea, polimorfismul, supraîncărcarea) [2] şi aspectele dinamice ale programelor OO [3];
- scrierea algoritmilor organizaţi în cheia POO etc. [4];
- problema "schimbării paradigmei", care impune reînvăţarea ("dezvăţarea" de conceptul vechi şi învăţarea conceptului nou) [5].

Pentru depăşirea acestor dificultăţi sunt necesare modalităţi de studiere a cursului POO, care vor implica tehnologiile didactice activ-participative şi Tehnologiile Informaţionale şi de Cominicaţie (TIC).
Predarea cursurilor din domeniul Informaticii este eficientă atunci când se face într-un mediu prietenos, dinamic şi colaborativ, prin asigurarea unui feedback optim. Sistemele de Management al învăţării sunt "user-friendly" şi permit conectivitatea şi furnizarea diferitor tipuri de resurse educaţionale, precum şi o interacţiune bună (sincronă şi asincronă) a studentului cu profesorul şi colegii.
Autorii propun un model pedagogic de studiere a cursului POO prin utilizarea SMî, care este capabil să asigure:

plurivalenţă: pentru dezvoltarea competenţelor POO, cum ar fi, analiza unei problemele din perspectiva orientată pe obiecte, iniţial este necesar de a dezvolta competenţa de descriere a principiilor POO, iar acest lucru se obţine prin abordarea complexă a modulelor didactice care sunt centrate pe nevoile şi posibilităţile individuale ale studenţilor, adaptate intereselor cognitive, ritmului de lucru şi stilului personal de învăţare;

continuitate: valorificarea mecanismului de funcţionare al modelului pedagogic propus asigură legătura dintre competenţele iniţiale, necesare pentru învăţarea POO (prerechizitele) şi competenţele formate la finele studiilor. Ultimile devin prerechizite pentru alte cursuri de studiere a altor limbaje orientate-obiect etc;

colaborativitate: modelul favorizează instruirea în bază de probleme şi proiecte, iar lucrul în comun stimulează studenţii să-şi modeleze cunoştinţele, să asiste şi să ateste dezvoltarea abilităţilor colegilor, să aprecieze contribuţia fiecărui membru al echipei;

contextualitate: ea asigurată de confruntarea cu sarcini din viaţa reală, care sunt larg folosite în activităţile practice ale cursului POO. Multitudinea variată de sarcini condiţionează realizarea transferului cunoştinţelor învăţate.

Modelul propus posedă un şir de avantaje, printre care economicitatea, creativitatea, tehnologizarea şi originalitatea.

Bibliography

[1] Fjuk A., Karahasanovic A., Kaasboll J.*Comprehensive Object Oriented Learning: the Learner's Perspective.* In: Informing Science Press, Santa Rosa, CA, 2006. 226 p.

[2] Fleury A.E. *Encapsulation and reuse as viewed by java students.* In: ACM SIGCSE Bulletin, 2001, nr. 33(1), p. 189-193.

[3] Ragonis N., Ben-Ari M. *A Long-Term Investigation of the Comprehension of OOP Concepts by Novices.* In: Computer Science Education, 2005, nr. 5(3), p. 203-221.

[4] Schulte C., Niere J. *Thinking in Object Structures: Teaching Modeling in Secondary Schools.* In: The materials of the 16th European Conference on Object-Oriented Programming, 6th Workshop on Pedagogies and Tools for the Teaching and Learning of Object Oriented Concepts, Malaga, Spain, 2002.

[5] Luker P.A. *There's more to oop than syntax!* In: ACM SIGCSE Bulletin, 1994, nr. 26(1), p. 56-60.

Dezvoltarea competenţelor digitale în formarea iniţială a profesorilor de filologie

Andrei Braicov, Tatiana Veveriţa

Universitatea de Stat din Tiraspol, Republica Moldova
e-mail: abraicov@gmail.com, veveritat@gmail.com

Dezvoltarea fulminantă a societăţii informaţionale impune reorganizarea formării profesionale a specialiştilor, inclusiv a cadrelor didactice de filologie.
Optimizarea procesului de formare iniţială a profesorilor de filologie trebuie să valorifice dezvoltarea Tehnologiilor Informaţionale şi de Comunicaţie, inclusiv a softurilor specializate.
Un şir de documente europene şi naţionale [de ex., 1 - 4] stipulează standarde de competenţă digitală pentru cadrele didactice, cu referire la: comunicarea digitală; gestionarea informaţiei; crearea de conţinuturi digitale educaţionale; implementarea aplicaţiilor de management instituţional; gestionarea conţinuturilor educaţionale; utilizarea echipamentelor digitale în educaţie; respectarea normelor etice şi legale în spaţiul digital.
Pornind de la acestea şi de la competenţele profesionale ale profesorilor de filologie con-cluzionăm următoarele competenţe digitale pentru cadrele didactice de filologie, care respectă o structură descrisă de Livia Armanu în [5].

a. Competenţe ştiinţifice, metodice şi psihopedagogice:

- să caute şi să selecteze materiale pentru procesul didactic în diferite formate (text, grafică, audio, video);
- să extragă şi să prelucreze informaţii de diferite tipuri (textuală, grafică, audio, video) utilizând procesoare de text, editoare tabelare, aplicaţii video, audio, etc.
- să sistematizeze informaţia în fişiere şi directoare atăt în medii locale, cât şi externe;
- să identifice, să utilizeze şi să promoveze echipamentele digitale în activitatea de instruire;
- să cunoască şi să poată folosi aplicaţiile specializate;
- să utilizeze instrumente de securitate cibernetică;
- să aplice tehnici de căutare a resurselor digitale;
- să identifice şi să rezolve avertizările de securitate a datelor;
- să utilizeze instrumente care asigură confidenţialitatea datelor cu caracter personal;
- să îmbunătăţească şi să creeze resurse digitale proprii;
- să folosească dicţionare şi enciclopedii electronice.

b. Competenţe manageriale:

- să utilizeze şi să promoveze sistemele de management şcolar în planificare, comunicare şi evaluare;
- să utilizeze sistemele de gestiune a clasei digitale;
- să utilizeze şi să integreze resursele educaţionale în sistemele de gestiune a conţinuturilor educaţionale;
- să se implice şi să încurajeze elevii în utilizarea şi crearea resurselor digitale;
- să promoveze utilizarea resurselor şi instrumentelor TIC.

c. Competenţe de evaluare:

- să analizeze calitatea resurselor digitale şi să propună îmbunătăţirea resurselor digitale;
- să aplice criterii de evaluare a validităţii informaţiei;
- să deţină tehnici de analiză şi control în realizarea sarcinilor, în luarea deciziilor.

d. Competenţe decizionale:

- să identifice problemele tehnice în utilizarea dispozitivelor digitale şi să contribuie la soluţionarea lor.
- să explice şi să promoveze respectarea drepturilor de autor asupra resurselor digitale;
- să respecte şi să promoveze legislaţia în domeniul protecţiei copiilor în mediul digital;
- să utilizeze şi să promoveze instrumentele anti-plagiat.

e. Competenţe de relaţionare:

- să informeze şi să promoveze utilizarea resurselor digitale în rândul elevilor, colegilor, părinţilor;
- să organizeze activităţi de instruire asupra drepturilor de autor şi instrumentelor anti-plagiat;
- să informeze elevii, părinţii, colegii despre agresiunea cibernetică şi să întreprindă măsuri de prevenire;

f. Competenţe de formare:

- să participe la cursuri de formare contună;
- să consilieze şi să motiveze colegii în dezvoltarea profesională continuă;
- să îndrume cadrele didactice pentru a contribui la îmbunătăţirea procesului didactic.

Autorii propun un şir de tehnologii didactice cu utilizarea TIC pentru formarea competenţelor digitale necesare viitorilor profesori de filologie.

Bibliography

[1] UNESCO. ICT in Education, *Performance Indicators on ICT Use in Education project.* URL. `http://www.unescobkk.org/education/ict/ict-in-education-project/monitoring-andmeasuring-change`

[2] ***, *Cadrul Strategic ET 2020.* `http://ec.europa.eu/education/policy/strategic-framework_ro`

[3] ***, *Strategia "Educaţia 2020"*. Hotărârea Guvernului Republicii Moldova nr. 944 din 14.11.2014. în: Monitorul Oficial al Republicii Moldova, nr. 345-351 din 21.11.2014, art. nr. 1014.

[4] ***, *Standarde de competenţe digitale pentru cadrele didactice din învăţământul general*. http://edu.gov.md/sites/default/files/cnc4_finalcompetente_digitale_-profesori_22iulie2015_1.pdf

[5] Armanu Livia. *Competenţele cadrului didactic.* Revista iTeach. https://iteach.ro/experientedidactice/competentele-cadrului-didactic.

Personal learning paths for E-learning

Olesea Caftanatov

Institute of Mathematics and Computer Science Academy of Sciences of Moldova
e-mail: caftanatov@gmail.com

The aim of our work is to figure out how pupils like to learn and design personal learning paths that will help them to study such school subject like geometry.

Digital culture are increasingly applied in e-learning, it also contributes to improve the educational process by adapting to the student's interests, capabilities and knowledge. Nowadays there are many different kinds of educational software for students based on adaptive learning, personalized learning or even personal learning pathway.Learning pathwayis described as the chosen route, taken by a learner through a rage of e-learning activities, which allows them to build knowledge progressivelyi [1].

For students personal learning paths are the best solution, because they can more effectively acquire and retain knowledge and skills that will help them in real world. However, what about the elementary school children? They definitely do not knowwhat the best is for them. In this case, one of solution can be designing an intelligent system that will analyze the behavior and preferences of pupils and afterward will automatically recommend a personal learning pathway.

This paper presents a study on the methodology of design of personal learning paths for e learning. This project consists of three parts: content model, learner model and instructional model. The content model includes the subject matter that pupils need to learn based on schools programs for each subject. In this part are designed and elaborated course material based on different learning styles like: short video lessons, presentations, quizzes, worksheets, games or even learning by investigation. For example, the main subject that pupils should learn is the types of angles. For this purpose, the application asks pupils to makes some photo with angles that they will find in real world on their way home from school or when they play outside. This way makes pupils motivated, because they can show their work to their friends and discuss what new they discovered. Afterwards, they need to apply their images, so the teacher could check their work.

The purpose of the second model is to find out how pupils like to learn, what the best learning style is for them. One of the methods to get the answers is the psycho-pedagogical questionnaire for identifying the learning style. The purpose of the third model is to design the adaptive testing. This part should answer on two main problems: what pupil's ability is at real time, and what should system recommend next. Adaptive testing methodology will select questions with a specific level of difficulty based on the previous pupils answers. If the pupil's answers are incorrect then adaptive testing engine will give another similar question to check if they do not understand the subject, or they were absent-minded and gave the wrong answer. In case, they gave the correct answer, the system will also first check if they didn't guessed the answer and after will gave questions with a higher difficulty grade.

Keywords: Adaptive learning, personal learning pathways, adaptive testing.

Bibliography

[1] ***, https://en.wikipedia.org/wiki/Learning_pathway

Abordări didactice în procesul de studiere a ştiinţelor biologice prin prisma interdisciplinarităţii Biologie-Matematică

Eugenia Chiriac

Universitatea de Stat Tiraspol
e-mail: eugeniachiriac@gmail.com

Promovarea pluridisciplinarităţii /interdisciplinarităţii/ transdisciplinarităţii în studierea ştiinţelor biologice este o necesitate determinată de creşterea vertiginoasă a volumului de informaţii ştiinţifice, acumulările cognitive imense din diverse ramuri ale ştiinţelor moderne şi complexitatea problemelor biologice cu care se confruntă societatea la etapa actuală. Pluridisciplinaritatea (multidisciplinaritatea) ţine de situaţia în care un subiect, o temă din ştiinţele biologice este examinată, analizată din perspectiva mai multor discipline, acestea din urmă menţinându-şi intactă structura şi păstrând independenţa unele în raport cu celelalte. Interdisciplinaritatea, din punct de vedere ştiinţifico-metodic, reprezintă o formă de cooperare între discipline, adaptată principilor didactice şi care implementează diferite instrumente şi metode de cercetare ştiinţifică, caracteristice diverselor domenii de studii în scopul analizei şi înţelegerii complexe a fenomenului respectiv [1]. În acest context, caracterul interdisciplinar al procesului de predare-învăţare a biologiei de cele mai dese ori nu este reflectat în manualele existente, programele de studii şi nici în abordările prelegerilor ori lecţiilor de laborator. În abordarea interdisciplinară, ca regulă, sunt ignorate frontierele stricte ale disciplinelor, căutându-se teme comune a diferitelor obiecte de studiu, în cazul nostru, intre ştiinţele biologice şi alte ştiinţe, care pot duce la realizarea obiectivelor de învăţare de grad mai avansat, a competenţelor transversale, considerate cruciale pentru soluţionarea problemelor biologice complexe, etc. O formă avansată a interdisciplinarităţii este transdisciplinaritatea, care presupune integrarea conceptelor, metodologiilor şi limbajelor care tind să devină universale în abordarea problemelor din ştiinţele biologice (matematizarea, utilizarea ciberneticii şi informaticii, modelarea, etc.).

În continuare vom examina în mod special interdisciplinaritatea biologie-matematică. Astfel, împărţirea unui segment în scopuri pur geometrice de către Euclid din Alexandria, în jurul anului 300 î. Cr. a avut şi continuă să aibă consecinţe în diferite domenii: de la botanică până la structura galaxiilor, de la matematică la arte, furnizând un sentiment de uimire şi o valoare exactă egală cu 1,61803..., număr care nu se sfârşeşte şi nu se repetă nici odată, şi care se numeşte Secţiunea de Aur, sau numărul phi. Anume Secţiunea de Aur are o capacitate de a apărea acolo unde o aştepţi mai puţin [2]. Aceasta se poate confirma prin mai multe exemple biologice cum ar fi: forma spirală a cochiliei unui melc. Cum anume melcul îşi construieşte cochilia este, în general, o problemă de chimie şi genetică. Matematica în acest caz, permite: contabilitatea moleculelor care participă în reacţiile chimice care au loc; descrie structura anatomică a moleculelor folosite în cochilii; calculează rezistenţa şi rigiditatea materialului cochiliei în comparaţie cu fragilitatea şi pliabilitatea corpului melcului etc. Este interesant faptul, că molusca îţi construieşte cochilia în spirală în aşa fel, ca să-şi ocupe complet spaţiul fizic. Odată cu creşterea în dimensiuni, molusca construieşte cămăruţe din ce în ce mai mari, sigilându-le pe cele mai mici, rămase nefolosite. Fiecare creştere în lungime a cochiliei este însoţită de o creştere proporţională a razei sale, aşa încât forma rămâne neschimbată [3]. Creşterea cochiliilor într-o spirală logaritmică, care urmează un model guvernat de Secţiunea de Aur a rămas esential neschimbată timp de milioane de ani, prezentând şi astăzi, un interes şi admiraţie în cercetare.

Bibliography

[1] E. Chiriac, L. Chiriac, *Applying Mathematical Models through the Prism of Integrating Factor in the Study of Biological Process*, The 19th Edition of the Annual Conference on Applied and Industrial Mathematics - CAIM Iasi, September 22-25, 2011, p.77

[2] I. Stewart, *Numerele naturii.* Humanitas, Bucureşti, 2010

[3] M. Livio, *Secţiunea de aur. Povestea lui phi, cel mai uimitor număr*, Bucureşti, 2005

Reflecţii didactice asupra sistemului de pregătire a profesorilor de informatică

Liubomir Chiriac, Angela Globa

Universitatea de Stat din Tiraspol (cu sediul la Chişinău), Republica Moldova
e-mail: llchiriac@gmail.com, angelagloba@gmail.com

Slaba pregătire a viitorilor profesori de informatică, în special, la unul din compartimentele fundamentale care ţine de această disciplină Algoritmizare şi programare are mai multe cauze şi motive. Să examinăm succint situaţia la acest capitol.
Informatica conţine două noţiuni fundamentale algoritm şi algoritmizare. Prin algoritmizare, în sens îngust, vom înţelege ştiinţa despre elaborare şi analiza algoritmilor. În sens mai larg, prin alogoritmizare vom înţelege elaborarea programelor (programare). Din aceste considerente studierea, din punct de vedere metodologic, a procesului de elaborare a algoritmilor şi programelor repezintă un obiectiv central al informaticii şi didacticii informaticii.
Este necesar de punctat că studierea informaticii în instituţiile pedagogice, la etapa iniţială, ne referim la anii 80-90, din considerente obiective, era redusă, de cele mai dese ori, la algoritmizare (elaborarea şi analiza algoritmilor, construirea programelor etc.). În pofida lipsei echipamentului electronic performant elevii şi studenţii erau pregătiţi la un nivel sufficient de avanasat la compartimentul algoritmizare şi programare. În prezent, pregătirea atât a elevilor cât şi a viitorilor profesori de informatică, la compartimentul respectiv, este cu mult sub aşteptări. Vom scoate în evidenţă doar următoarele cauze:

1. Numărul de ore prevăzut penru studierea compartimentului algoritmizare, conform noilor programe, s-a micşorat, comparativ cu cele din anii 90, aproximativ cu 40%.

2. Algoritmizarea, conform unor experţi în domeniu, ţine de cultura algoritmică a fiecărui persoane în parte şi este responsabilitatea personală a fiecăruia de a performa la nivel de excelenţă. Cu regret, numărul de autodidacţi pasionaţi de acest compartiment este foarte limitat.

3. Numărul de ore dedicate studierii altor compartimente care ţin de informatică (tehnologii informaţionale, reţele de calculatoare, internet etc.) este în creştere.

Datorită dezvoltării intensive a informaticii şi tehnologiilor informaţionale, în programele universitare, periodic, sunt prevăzute includerea unor disciplini ce conţin informaţii şi rezultate de ultima oră din domeniu TIC (robotica şcolară, tehnologiile 3D etc). Din aceste considerente mărirea numărului de ore la compartimentul algoritmizare şi programare este puţin probabilă. Astfel, contradicţia dintre nivelul general de pregătire a viitorilor profesori de informatică la compartimentul algoritmizare şi programare, pe de o parte, şi cerinţele şi obiectivele didactice care trebuie realizate, pe de altă parte, devine tot mai pronunţată şi vizibilă.

Care sunt modalităţile de soluţionare a problemei respective? Mai jos vom trasa unele linii directoare care, în opinia noastră, ar putea să influenţeze pozitiv procesul de studiere a informaticii:

1. Studierea elementelor care ţin de metodele şi tehnicile de programare începând cu ciclul de studii gimnaziale şi terminând cu ciclul liceal [1].

2. Studierea conceptelor fundamentale de matematică cu aplicaţii majore în informatică menţinând un echilibru intre dificultatea teoretică a modulelor de matematică şi importanţa practică în informatică.

Astfel, studierea modulului "Elemente de teoria numerelor" ar fi bine să aibă în calitate de aplicaţii "Analiza eficienţei algoritmilor şi timpul de execuţie a algoritmilor". Studierea "Elementelor din teoria grupurilor" ar fi oportun să aibă conexiune cu aplicaţii în criptografie cu chei publice (Criptosistemul RSA, semnături d igitale etc.). Compartimentul "Spaţii vectoriale" să posede în calitate de aplicaţii codurile detectoare şi corectoare de erori, conceptul de cod de lungime variabilă etc. Bineînţeles, această abordare necesită o strânsă cooperare profesională între profesorii de matematică şi informatică: pregătirea comună a programelor de studii la matematică, pregătirea comună a ciclurilor de lecţii etc.

Bibliography

[1] Chiriac L., Globa A. *Studierea informaticii în învăţământul preuniversitar prin prisma metodelor şi tehniciilor moderne de programare.* În: Studia Universitatis. Seria cStiinţe ale educaţiei, Nr. 5(95), 2016. Chişinău: Universitatea de Stat din Moldova, 2016. p.231-241. ISSN:1857-2103.

[2] Globa A. *Aspecte didactice privind implementarea strategiilor de instruire la predarea tehnicilor de programare în alte ţări.* În: Didactica Pro. Revistă de teorie şi practică educaţională a Centrului Educaţional PRO DIDACTICA. Nr.5-6 (93-94), 2015. p.52-58. ISSN:1810-6455.

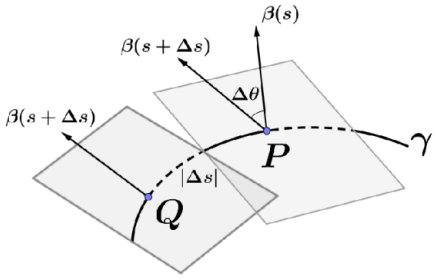
Vizualizarea torsiunii unei curbe prin intermediul softului interactiv GeoGebra

Ina Ciobanu

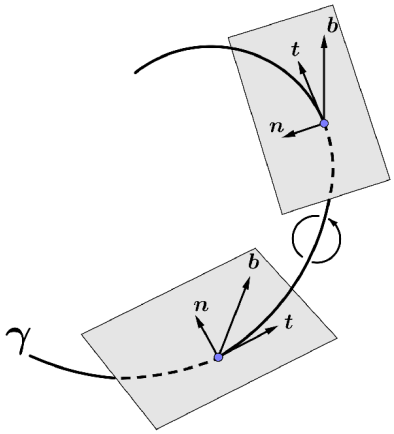
Universitatea de Stat „Alecu Russo" din Bălţi

e-mail: ina.ciobanu@usarb.md

Geometria diferenţială este unul din compartimentele geometriei, studiat şi în procesul pregătirii profesorilor de matematică în instituţiile superioare de învăţământ din RM. Una din noţiunile fundamentale ale geometriei diferenţiale clasice este noţiunea de torsiune a unei curbe.

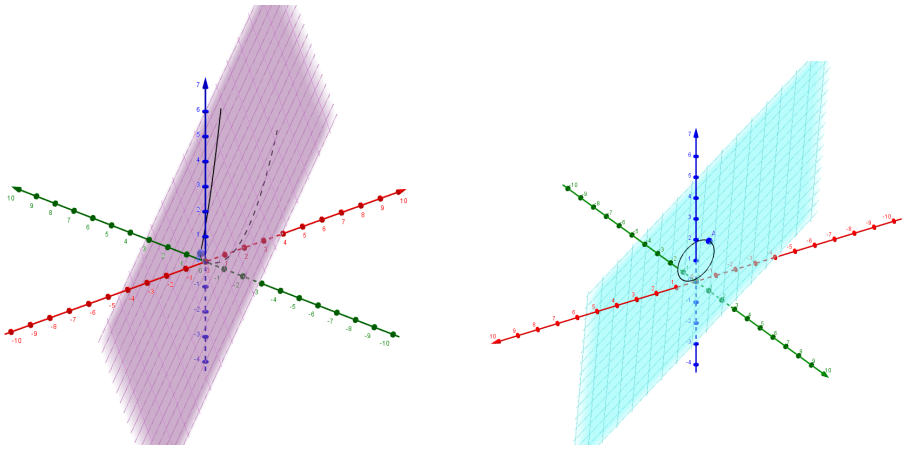


Torsiunea unei curbe.



Semnul torsiunii.

Pentru însuşirea efectivă cursului de geometrie diferenţială studenţii trebuie să aibă un nivel sucient de înalt al imaginaţiei şi gândirii spaţiale. Pentru aceasta structura, consecutivitatea şi tehnologia transmiterii cunoştinţelor matematice necesită efectuarea unor schimbări.
Odată cu apariţia softurilor matematice, trebuie mărit acel volum de informaţie, expunerea căreia poate fi făcută prin intermediul calculatorului. însă, în prezent, se observă disponibilitatea joasă a studenţilor de a aplica calculatorul în activitatea de cercetare în matematică. Unul dintre cele mai efective softuri matematice ce poate contribui atât la unificarea matematicii şi informaticii, cât şi la studierea matematicii clasice, este softul interactiv GeoGebra. în favoarea acestei alegeri, putem aduce mai multe caracteristici ale acestei aplicaţii.
Mai mulţi studenţi preferă să lucreze cu animaţie, elaborând construcţii interactive. Prin aceste modele studenţii pot înţelege mai bine şi noţiunea de torsine a unei curbe.
Denumirea de ,,torsiune" se explică prin faptul că, mărimea sa reprezintă viteza de schimbare a planului osculator în raport cu deplasarea punctului pe curbă. Ne închipuim că schimbarea planului osculator la o deplasare mica a punctului presupune o ,,rasucire" a curbei.



Vizualizarea torsiunii unei curbe.

Cu ajutorul softului interactiv GeoGebra putem vizualiza ,,viteza de schimbare a planului osculator " pentru orice curbă, eficientizând astfel procesul de înţelegere a noţiunilor fundamentale din

geometria diferenţială clasică.

Bibliography

[1] M.M. CIOBAN, I. CIOBANU. *Geometrie diferenţială. Ghid metodic de rezolvare a problemelor.* Bălţi, 2016.

[2] V.F. OCHKOV, E.P. BOGOMOLOVA. *Teaching Mathematics with Mathematical Software.* Journal of Humanistic Mathematics, nr. 5(1) (2015), pp. 265-285.

[3] ***, *Ce este GeoGebra*?. Disponibil la `https://www.geogebra.org/about`

Modeling the n-queens problem using mathematical software

Olga Danilkina[1], Elisabete Alberdi Celaya[2], Judit Munoz-Matute[2]

[1] *University of Dodoma (UDOM), Dodoma, Tanzania*
[2] *University of the Basque Country UPV/EHU, Bilbao, Basque Country (Spain)*
e-mail: `olga.danilkina@gmail.com`

This work places in the subject of Modeling taught in many engineering degrees. In this subject students learn how to model a problem mathematically and how to solve it by means of mathematical programming. At the end of this subject students are required to develop a project in which they have to use different strategies and skills learnt during the subject. In this work, the problem of the n-queens is tackled, which can be developed as a project, as it can be modeled mathematically and solved computationally using different strategies known in the subject. We define the problem, we solve it using different algorithms and we present a comparison of the algorithms. Students having to develop this type of project can use this work as an example of what is expected from them as a final project.
Keywords: n-queens problem, backtracking, recursive algorithms, MATLAB

A proposal to work the digital competence in mathematics in secondary school

Olga Danilkina[1], Elisabete Alberdi Celaya[2], Judit Munoz-Matute[2], Mª Isabel Eguia Ribero[2], Mª Josefa González Gómez[2]

[1] *University of Dodoma (UDOM), Dodoma, Tanzania*
[2] *University of the Basque Country UPV/EHU, Bilbao, Basque Country (Spain)*
e-mail: `olga.danilkina@gmail.com`

As teachers of mathematics one of the challenges that we have is how we can increase students' interest and engagement in classroom. The curriculum of the baccalaureate is presented as the set of competences that students have to acquire when they finish this period. In this sense, the competences can be divided in: general learning competences and fundamental competences. The general learning competences are such as, to live in a responsible way, to know how to communicate, etc. and they are learnt in all the subjects of the baccalaureate. Among the fundamental competences, we can find the digital competence. This paper is focused on this competence, not only because this is a competence to make more attractive the learning of mathematics, but also because it favours the development of some skills such as the analytical ability or the critical thinking ability. In this work how the digital competence can be inserted in the subject Mathematics

II of the baccalaureate is studied. We have analyzed the assessment criteria of the subject, looking especially at the ones related to the digital competence. We have identified the units which assessment criteria are connected to this competence and we have set out different activities using mathematical software for the acquisition of these competences.
Keywords: baccalaureate, digital competence, mathematical software.

Analysis and enhancement of course content using typical problems of the IT industry

Radu Dumbrăveanu, Ghennadi Cabac

Alecu Russo Balti State University, Republic of Moldova
e-mail: rndumbraveanu@gmail.com, gcabac@gmail.com

In this paper we propose a method for enhancing course content by analysing the questions from Stack Overflow site database [1]. Stack Overflow database contains about 14 million questions (and 23 million of answers) from programming field which may vary from simple questions (mostly asked by junior and enthusiast programmers) to very deep questions. The main idea behind the proposed method is to find and analyse duplicated questions, i.e. questions generated by specific problems which already have an answer within a more generic question. We extract the common information from duplicated question and their corresponding generic question and integrate it in the course content in order to enhance the student experience. We also propose different search criteria (for duplicated questions) in order to maximize the extracted information. There is already a growing body of research [2, 3] toward teaching enhancement by means of student's questions within classroom. Our approach is different and does not rely on direct inquiring of students having the following advantages, e.g. collecting questions from students' needs big sample, often it is very hard to collect questions from students since they ask more non-academic than academic questions [2], students do not ask higher cognitive questions while the questions from Stack Overflow are asked by (an answered by) people coming directly from IT industry. We share the same view as [3] by considering the question as "a technology for education" [3]. This point of view is interesting from the perspective of building learning objects [4].

Bibliography

[1] ***, https://data.stackexchange.com/stackoverflow/queries

[2] Commeyras M. (1995). What can we learn from students' questions? Theory Into Practice, 34, 101-106.

[3] Hokanson, Brad. The Technology of the Question: Structure and Use of Questions in Educational Technology. Educational Technology, v55 n6 p14-24 Nov-Dec 2015

[4] Cabac Ghennadi. Aspecte teoretice ale generării conţinuturilor dinamice în instruire The 4th Conference of Mathematical Society of the Republic of Moldova, Comunications in Didactics,

Metoda proiectului şi perfecţionarea competenţelor didactice ale profesorilor de matematică

Natalia Gașițoi, Liubov Zastînceanu

Universitatea de Stat "Alecu Russo" din Bălţi, Republica Moldova
e-mail: gasitoi.natalia@usarb.md, liubaz@mail.ru

Rezultatele formării studenţilor prin programe de studii superioare de master în domeniul Didacticii matematicii presupun cunoştinţe foarte specializate în domeniul matematicii şi a ştiinţelor educaţiei, formarea abilităţilor specializate pentru soluţionarea problemelor didactice şi a competenţelor de gestionare şi transformare a situaţiilor de muncă sau de studiu care sunt complexe, imprevizibile şi necesită abordări strategice noi precum şi asumarea responsabilităţii pentru a contribui la cunoştinţele şi practicile profesionale şi / sau pentru revizuirea performanţei strategice a echipelor [1].

Una din cerinţele faţă de programele de studii superioare de master este prezenţa unei componente de cercetare ştiinţifică, iar planurile de învăţământ prevăd un număr de ore pentru lucrul independent de 2.75 ori mai mare decât pentru orele de contact direct, ceea ce implică selectarea unor metode didactice corespunzătoare. În opinia noastră, metoda proiectului ar putea asigura nivelul respectiv de formare a cunoştinţelor, abilităţilor şi competenţelor didactice.

În contextul cursurilor de didactică a aritmeticii şi algebrei, didactică a analizei matematice, instruirii centrate pe cel ce învaţă în contexte matematice etc., studiate în cadrul programului de studii superioare de master Didactica matematicii oferit de Universitatea de Stat "Alecu Russo" din Bălţi, sunt propuse proiecte atât de grup, cât şi individuale. Proiectele de cercetare didactică propuse pot fi orientate atât spre adaptarea unor conţinuturi matematice, dificil de însuşit de către elevii de liceu şi gimnaziu, cât şi spre valorificarea unor caracteristici psihopedagogice specifice ale instruiţilor (teoria inteligenţelor multiple, instruirea copiilor cu CES, recuperarea lacunelor în însuşirea matematicii). Experienţa realizării unor proiecte de cercetare de grup şi individuale, descrisă în lucrare, demonstrează eficienţa acestei metode didactice pentru perfecţionarea competenţelor didactice ale profesorilor de matematică.

Bibliography

[1] ***, Cadrul European al Calificărilor pentru învăţarea de-a lungul vieţii, https://ec.europa.eu/ploteus/sites/eac-eqf/files/leaflet_ro.pdf

Using BigData Analytics in Universities

Adela Gorea

Alecu Russo Balti State University,
Bălţi, Republic of Moldova
e-mail: adelaosadta@gmail.com

BigData is a term that has been around for some time now but there is still confusion about what it actually constitutes. The concept is continuing to evolve and to be reconsidered, as it remains the driving force behind many ongoing waves of digital transformation, including artificial intelligence, data science and the Internet of Things (IoT) [1].

The amount of machine-generated data is rapidly growing too. Data is generated and shared when

our "smart" home devices communicate with each other or with their home servers.
The amount of data available to us is only going to increase, and analytics technology will become more capable. So, if BigData is capable of all this today just imagine what it will be capable of tomorrow.
Higher Education is a field of application and the possibility of analyzing operation with BigData. Higher education institutions can assess the performance of students, teachers and curricula parallel to similar universities, thus giving a new perspective on the potential for improvement [2].
One of the evaluation modalities the implementation of a range of BigData processing applications i n higher education institutions constitue: student acquisition, student course major Selection, student performance effectiveness, teacher effectiveness, bookstore effectiveness. All the directions of BigData processing applications in higher education institutions listed above will be analyzed in the article.

Bibliography

[1] Rick S., Jennifer E. The Human Face of Big Data, 2012, 224 p. ISBN: 978-1454908272;

[2] Schmarzo B. What Universities Can Learn from Big Data Higher Education Analytics, July 2, 2014, available on: https://infocus.emc.com/william_-schmarzo/what-universities-can-learn-from-big-data-higher-education-analytics.

Rolul mijloacelor tehnice, audio-vizuale, în eficientizarea procesului educaţional

Hoffmann-Bront Viorica Cornelia

Liceul Tehnologic "Ioan Bococi" Oradea
e-mail: vigyorika@yahoo.com

Orice metodă pedagogică rezultă din întâlnirea mai multor factori şi, din acest punct de vedere, educaţia va rămâne mereu o artă: arta de a adapta, la o situaţie precisă, indicaţiile generale date de cărţile de metodologie. (Gaston Mialaret)
Materialul prezentat reprezintă experienţele mele acumulate pe parcursul anilor trecuţi, tot ce am observat în cursul muncii mele de zi cu zi la catedră.Lumea noastră contemporană reprezintă o permanentă şi inedită provocare pentru educaţie.Existenţa fiecărui individ în parte, ca şi a întregii societăţi în ansamblul ei, capătă deci un ritm din ce în ce mai alert, devine tot mai marcată de necesitatea cunoaşterii rapide, complete şi corecte a realităţii înconjurătoare, pentru ca luarea deciziilor să fie făcută ferm, oportun şi competent. Aceasta duce inevitabil, la creşterea volumului de informaţii ce trebuie analizat, la necesitatea stocării şi prelucrării acesteia, deci la necesitatea utilizării calculatorului atât în viaţa de zi cu zi cât şi în procesul instructiv-educativ contemporan. Tehnologiile digitale nu trebuie să reprezinte o simplă adăugare în planul de învăţământ, ele trebuie să fie integrate deplin "în serviciul educaţiei" la toate nivelurile sistemului şcolar actual. Actorii educaţionali trebuie să fie formaţi pentru a face faţă schimbării, incertitudinii şi inovării. Complexitatea crescută a şcolilor şi mediilor de învăţare de astăzi sugerează nevoia realizării într-o nouă manieră a activităţilor educaţionale. De aceea lucrarea prezintă avantajele şi limitele utilizării TIC în procesul de învăţământ şi o analiză asupra eficienţei utilizării TIC în procesul instructiv-educativ.

Abordări didactice privind activităţile integrate din cadrul clasei viitorului

Lilia Mihalache

Universitatea de Stat din Tiraspol (cu sediul în Chişinău)
e-mail: lilia.mihalache@gmail.com

Sunt analizate unele oportunităţi şi avantaje privind Clasa viitorului - Future Classroom Lab (FCL). Se propun unele aspecte didactice vizând proiectarea şi realizarea activităţilor integrate în cadrul FCL. FCL este definită ca un spaţiu de învăţare deschis, unde profesorii şi elevii experimentează diferite scenarii inovatoare, utilizând tehnologia transformaţională: programare (prin Scratch, Kodu, HTML5, JavaScript, AppInventor), robotică (cu BeeBot, Lego WeDo & EV3, LittleBits), medii digitale şi diverse echipamente de producţie (3D-modeling, 3D-printing, VR, AR), Internetul lucrurilor (Arduino, Adafruit Gemma, Kinect, IFTTT).
FCL permite formarea competenţelor secolului 21 şi STE(A)M, totodată dezvoltă gândirea divergentă, adică flexibilitatea mentală, originalitatea, fluenţa şi inventivitatea la elevi.

Limbajul ştiinţific şi cultural al profesorului din domeniul ştiinţelor reale

Dorin Pavel, Maria Pavel

Universitatea de Stat din Tiraspol, cu sediul la Chişinău, RM
e-mail: pruteanupavel@gmail.com

Profesorii din domeniul ştiinţelor pedagogice şi lingvistice sunt formaţi iniţial, în cadrul învăţământului superior, la un nivel înalt din punct de vedere lingvistic. Acest fapt se datorează specificului profesiei pe care o vor exercita: educatori, învăţători la clasele primare, profesori de limbă română etc. Profesorii din domeniul ştiinţelor reale, însă, par a fi văduviţi de o pregătire suficientă sub aspect lingvistic. Mai mult ca atât, elevii care vin la facultate şi optează pentru domeniul ştiinţelor reale, au absolvit licee cu profil real şi, de cele mai multe ori, nu au pus accent deosebit pe pregătirea lingvistică. Pe parcursul studiilor superioare, aceştia vor studia foarte multe discipline reale care vor contribui la formarea unui limbaj ştiinţific înalt. Pe lângă acesta, viitoarele cadre didactice trebuie să posede şi un limbaj al comunicării de o cultură deosebită, din virtutea posturii de model în faţa elevilor sau a studenţilor. La formarea şi dezvoltarea lui, din păcate, pe parcursul studiilor superioare, contribuie doar un singur curs de cultură a comunicării, care se dovedeşte a fi insuficient. În Republica Moldova, în învăţământul superior activează foarte mulţi profesori care au studiat la facultate în limba rusă, sau pe timpul sovietic, când limba română era foarte stâlcită. De aceea, chiar şi astăzi, după 28 de ani de grafie latină şi limbă română la ea acasă, se mai întâlnesc profesori care utilizează cuvintele întroducere în loc de introducere, complect în loc de complet etc. Vocabularul sărac şi cu lacune al unui profesor de ştiinţe reale va influenţa negativ formarea studenţilor, pe de o parte. Pe de altă parte, imaginea profesorului are de suferit în faţa unor studenţi care s-au născut după anii 90 şi vorbesc o limbă română frumoasă. Un alt aspect negativ al limbajului lacunar al cadrelor didactice universitare se reflectă în activitatea ştiinţifică pe care o desfăşoară, prin elaborarea publicaţiilor ştiinţifico-didactice. Aici, deja e vorba de limba scrisă, în care orice greşeală este penalizată de cititor şi creează o imagine nefavorabilă pentru autor. În ajutorul cadrelor didactice astăzi vin tehnologiile informaţionale şi anume editoarele de texte, care conţin instrumente suficiente de ortografie şi gramatică a multor limbi,

inclusiv şi a limbii române. Un cuvânt subliniat cu linie ondulată roşie semnalează o greşeală, care poate fi corectată cu ajutorul unei variante propuse de meniul contextual apărut la efectuarea unui click dreapta peste acesta. Iar o frază subliniată cu linie ondulatorie verde indică către o greşeală sintactică sau semantică. Dicţionarele explicative electronice, de asemenea reprezintă un instrument valoros care contribuie la alegerea cuvintelor sau termenilor ştiinţifici adecvaţi. Specialiştii informaticieni organizează periodic în cadrul Universităţii de Stat din Tiraspol cursuri de formare continuă în domeniul utilizării tehnologiilor informaţionale. Se recomandă ca aceste cursuri, eventual organizate în colaborare cu lingvişti, să includă module ce ţin de utilizarea corectă a limbii române şi evitarea celor mai frecvente greşeli.
La editarea lucrărilor ştiinţifico-didactice, cadrele didactice universitare trebuie să posede abilităţi de tehnoredactare computerizată, care pe lângă limbajul ştiinţific şi cultural întregesc, în era digitală, personalitatea acestora. în lucrarea [1] s-au enumerat cele mai frecvente dificultăţi întâlnite la formatarea unui document, totodată propunându-se şi soluţii de elucidare a lor. Limbajul informaticienilor este unul foarte puternic influenţat de tehnologiile moderne, plin de xenisme (cuvânt străin împrumutat de o limbă, fără a-l fi adaptat la propriul sistem) [2], neologisme, englezisme. Sub acest aspect, informaticienii întâlnesc dificultăţi la formarea şi dezvoltarea limbajului atât ştiinţific cât şi cultural, deoarece există un şir de divergenţe între opiniile specialiştilor cu privire la un termen sau altul din domeniul informaticii. Soluţia problemei ar fi conlucrarea cu filologii, specializaţi în acest domeniu.

Bibliography

[1] Pavel M. *Cultura scrierii electronice sau caligrafia digitală - competenţă esenţială a profesorilor de informatică.* În The Fourth Conference of Mathematical Society of the Republic of Moldova: Communications in Didactics: Proceedings CMSM 4, June 28 July 2, 2017. Chişinău: dedicated to the centenary of Vladimir Andrunachievici (1917-2017) / ed: Mitrofan Cioban, Liubomir Chiriac. Chişinău: Tiraspol State University, 2017. 231 236 p. ISBN 978-9975- 76-203- 8.

[2] ***,`https://dexonline.ro/intrare/xenism/223142`.

Fundamental Principles of Education and Teaching Mathematics

George Popescu

Department of Applied Mathematics, University of Craiova
e-mail: `grgpop@gmail.com`

Traditional education is based on transferring, conveying information, uses the "reward / punishment" pattern, apparently successful, actually it never worked. We may have to reconsider what educations goals are. Education should be addressing to human mind, build an attitude, educate for work, as - observe, analyze, make decisions, take actions, be responsible for consequences of the actions. Education efficiency should not be measured similarly to a business system. Long term and extremely long term goal should replace fast solving problems. Understanding what information could be, the dynamic of information transfer may rewire our brains, change our perception of the surrounding universe.

Aspecte socioeducaţionale ale adaptării elevilor cu potenţial intelectual nevalorificat la exigenţele matematicii de performanţă

Larisa Sali, Marcel Teleucă

Universitatea de Stat din Tiraspol
e-mail: salilarisa@gmail.com

Experienţa din ultimii ani demonstrează că rata elevilor capabili de performanţe înalte la matematică nu este în scădere. însă tot mai mulţi adolescenţi abandonează ideea de a studia aprofundat matematică încă la vârsta 12-13 ani. Acest fenomen este caracteristic pentru sistemul educaţional din Repubica Moldova. Cercetarea realizată se focusează pe identificarea blocajelor de ordin psihosocial care frânează sau împiedică elevul dotat să-şi valorifice propriul potenţial. În opinia noastră este important să se ţină cont de părerile elevului şi a părinţilor pe următoarele dimensiuni: evaluarea succeselor şi a eşecurilor; stabilirea cauzelor eşecurilor, a blocajelor psihologice; identificarea soluţiilor pentru consolidarea forţelor şi capacităţilor; formularea aşteptărilor. Aspecte generale privind adaptarea sociopersonală a adolescenţilor cu performanţe neidentificate au fost analizate de către E. Rodriguez Naveiras ş.a.[1, p. 87]. Investigaţia noastră a cuprins un lot de circa 50 de copii cu vârsta 15-17 ani din municipiul Chişinău şi părinţii acestora. Au fost aplicate chestionare, interviuri semistructurate şi observări. Rezultatele evaluării au permis analiza comparativă a lor în raport cu apartenenţa gender a respondenţilor, vârstă, tipul şcolii. Recomandările formulate sunt utile elevilor capabili de performanţe înalte, părinţilor acestora, vizează cadrele didactice şi actorii responsabili de managementul sistemul educational.

Bibliography

[1] Rodriguez Naveiras E., Borges A., *Sociopersonal adaptationof non-identified high ability teenagers*. În: Programme and the Book of Abstracts, 11-th ECHA Conference "From Giftedness in Childhood to Successful Inteligence in Adulthood", Prague, 2008, p. 87.

Abordări metodice privind utilizarea platformei de învăţare MOODLE în studiul cursului universitar *Analiza matematică*

Ecaterina Rusnac

Universitatea de Stat "Alecu Russo" din Bălţi, Republica Moldova
e-mail: rusnac_c@mail.ru

Secolul XXI este secolul gândirii despre viitor şi a tehnologiilor, care apropie acest viitor în toate ramurile sale de existenţă. Educa ţ ia nu prezintă o excepţie. Profesorii pe parcurs a mai multor ani sunt în căutarea metodelor mai eficiente de pregătire a studenţilor pentru viitor, dar pregătirea adecvată pentru viaţa în secolul XXI necesită o "pedagogie nouă".
Un instrument util şi eficient al aşa numitei "pedagogiei noi" este e-learning-ul – învăţământul prin intermediul calculatorului.
Predarea într-un mediu de învăţare electronic poate contribui la dezvoltarea capacităţilor de a preda, de a învăţa şi cel mai important, de a stabili leăgătura între studenţi şi profesori. Este necesar de ţinut cont că în astfel de pedagogie rolul profesorului şi a studentului un pic se schimbă: profesorul devine producător / manager de resurse de învăţare, tutore şi ghid, care oferă sprijin

studenţilor atunci, când este necesar; iar studentul devine în mare parte responsabil pentru propria învăţare, controlează propriul ritm, profunzimea studiilor etc. Oferirea studenţilor posibilitatea de a se autoevalua şi de a depăşi dificultăţile printr-un studiu individual le încurajeajă şi oferă mai multă încredere în propriile puteri.
Unul din cele mai eficiente instrumente în "pedagogie nouă" este platforma de învăţare MOODLE. Inovaţia pedagogică şi flexibilitatea în timp, oferite de către platforma MOODLE pentru studiului individual şi în grup al studenţilor au o importanţă deosebită, în special pentru studenţi care lucrează sau din anumite motive nu au posibilitatea să frecventeze toate orele. Scopul nu este înlocuirea prezenţei studenţilor în clasă, ci completarea cantitativă a absenţelor.
Din punct de vedere didactic, utilizarea instrumentelor multimedia ale acestei platforme pentru a crea activităţi atractive, face ca învăţarea să devină un proces mai prietenos pentru studenţi, în special studierea unui curs aşa de complicat şi vast ca fiind *Analiza Matematică*.

Eficientizarea realizării activităţilor educative prin intermediul tehnologiilor moderne

Stela Ţîcău

Liceul teoretic "M.Eminescu", Bălţi, Republica Moldova
e-mail:

În contextul societăţii actuale, confruntată cu numeroase şi complexe provocări, cadrele didactice sunt chemate să apeleze cu curaj şi responsabilitate la aplicarea creativă a noilor paradigme în proiectarea şi realizarea activităţilor educative. Aşa se explică orientarea didacticii moderne către dezvoltarea unei metodologii centrate pe elev, pe demersuri interactive ce respectă profilul psihologic de vârstă şi individual al elevului, cerinţele şi ritmul propriu de învăţare[1].
Lumea contemporană reprezintă o permanentă şi inedită provocare pentru educaţie. Existenţa fiecărui individ în parte, ca şi a întregii societăţi în ansamblul ei, capătă deci un ritm din ce în ce mai alert, devine tot mai marcată de necesitatea cunoaşterii rapide, complete şi corecte a realităţii înconjurătoare, pentru ca luarea deciziilor să fie făcută ferm, oportun şi competent.
Problema identificată apare din cauza creşterii volumului de informaţii ce trebuie analizat, la necesitatea stocării şi prelucrării acesteia, deci la necesitatea utilizării calculatorului atât în viaţa de zi cu zi cât şi în procesul instructiv-educativ. Tehnologiile nu trebuie să reprezinte o simplă adăugare în planul de învăţământ, ele trebuie să fie integrate deplin în serviciul educaţiei la toate nivelurile sistemului şcolar. Actorii educaţionali trebuie să fie formaţi pentru a face faţă schimbării, incertitudinii şi inovării. Complexitatea crescută a şcolilor şi mediilor de învăţare de astăzi sugerează nevoia realizării într-o nouă manieră a activităţilor educaţionale[2]. În concluzie, tehnologiile educaţionale moderne urmăresc facilitarea procesului didactic, ajutând profesorii să creeze contexte favorabile învăţării, formării deprinderilor, constituirii unui complex de atitudini care să stimuleze curiozitatea, dorinţa de a cunoaşte mai mult, spiritul competiţional cu sine, plăcerea de a progresa şi de a-şi dezvolta posibilităţi de autoinstruire. Deci, utilizarea tehnologiilor moderne, a softurilor educaţionale reprezintă o necesitate a procesului de instruire, acestea fiind adaptabile la orice vârstă a elevului, înlesnindu-le căile de acces spre cunoaştere.
Combinarea tehnologiilor moderne cu metodele tradiţionale pedagogice reprezintă o schimbare de paradigmă cu implicaţii asupra cunoaşterii în societate în general şi asupra învăţării în special, de aceea disciplina pedagogică trebuie modificată în conformitate cu noul context în care trăim.

Bibliography

[1] Oprea, C. Strategii didactice interactive. Bucureşti: E.D.P., 2006.

[2] Nicu A. Curs de pedagogie. Disponibil pe Internet
dppd.ulbsibiu.ro/ro/cadre_didactice/adriana_nicu/cursuri

Recursive calls in the Pascal programming language

Vitalie Ţîcău

Alecu Russo Balti State University, Balti, Republic of Moldova
e-mail: VitalieSTicau@gmail.com

Applying the diverse materials in class corresponds to the tendency of improving the training process in school. In particular, if the time for the teaching-learning of the study material is limited. In general, computer classes take place according to the manual along with solving some tasks on the computer [1, 2].

The paper addresses the issue of diversifying the teaching-learning materials and is designed to help the teacher in the classroom. In particular, a topic is explored by means practical of solutions, which, at first glance, seems to be difficult the application of recourse to programming.
The way of applying the PASCAL language in subroutines is examined in general and that of recursive calls, in particular. Particular attention is paid to exceptional cases that may occur at a recursive call.
The solution to a set of tasks [3] with the application of recursive and iterative subprograms is presented, which, by means of the presented program codes, can, in a way, facilitate the learning activity of the students: calculating the factorial of a natural number, the sum of the first few natural numbers, the value of the Function-power, generation of the Fibonacci numbers, etc. There are also four examples of the recursive graphics application that may be interesting for students.
It has been found that the recourse is particularly useful in cases when the development of non-recursive algorithms is very complicated: the translation of PASCAL programs into code-machine language, computer graphics, form recognition, and so on. The examples presented are of interest in practice and allow us to understand the benefits of using the recourse.

Bibliography

[1] Gremalschi, A., Mocanu Iu., Spinei I.. Informatica: manual pentru clasa IX. Chişinău: Î.E.P. Ştiinţa, 2011.

[2] Lucanu D. Bazele proiectării programelor şi algoritmilor, volumele I,II, III. Iaşi: ed. Univ. "Al. I. Cuza", 1996.

[3] Atanasiuc A. Culegere de probleme PASCA. Bucureşti: Petrion, 1996.

Conexiuni ale matematicii cu teoria informaţiei, din punct de vedere metodologic şi predare-învăţare (II)

Ion I. Valuță, Evelina Valuță

Universitaea Tehnică a Moldovei, Moldova
e-mail: valutse@mail.md

Pornind din cele considerate în [5] revenim la observaţia lui R. Descartes că ştiinţele exercită influene reciproce în privinţa dezvoltării fiecăreia dintre ele cu, menţionăm, anumite intensităţi în

diferite perioade de dezvoltare ale lor. Matematica, în acest sens nu e excepţie dar, credem, mai mult, este un exemplu concludent (în privinţa perioadelor de dezvoltare a matematicii a se vedea [1], [2]). Astfel, în perioada matematicii practice (Babilon, Egipt) se manifestă influenţa astronomiei (sistemul de numeraţie poziţional în baza şasezeci), al construcţiilor (în particular, al piramidelor - Imhotep), în perioada matematicii teoretice (începând cu cea greacă), a filozofiei şi logicii (Platon, Aristotel), a mecanicei (Arhimede) ş.a.m.d. Există însă, o ştiinţă: *Teoria Informaţiei în cel mai larg sens actual al acestui cuvânt* care datorită sentinţei lui Wiener: *Informaţia este informaţie, nu este nici materie, nici energie* (cu aceste cuvinte se finisează capitolul "Maşinile de calcul şi sistemul nervos" din [3]). prin categoriile sale influenţează orice ramură a ştiinţelor, începând, în genere, cu cele mai primale - ale *cunoaşterii*. Astfel, conform lui N. Wiener, universul se poate vedea ca fiind alcătuit din trei *părţi constituente: materia, energia şi informaţia*. Ultima constituind modalitatea de manifestare a tuturor celor ce există. Tot parcursul istoric este presurat cu diverse puncte cruciale şi cercuri Grothendieck care se datorează modalităţilor de prelucrare a informaţiei. Să ne referim la începuturi. Considerăm că ceia ce este vitalitatea, fiind şi ea eternă, constitue imperiul informaţiei ca atare. în negura vremurilor viaţa pe terra se dezvoltă ajungând la forma lumii animale în care sunt modalităţi avansate de prelucrare a informaţiei ce se realizează prin ceia ce am numi *instinctivitate*. În continuare, la o ramură de umanoizi ia amploare *codificarea sonorică a informaţiei* fapt fixat prin sentinţa biblică *întâi a fost cuvântul*, dezvoltându-se la fiecare individ modalităţi (şi organe) de *emitere, recepţionare şi memorizare* a informaţiei. Ultima o fi exercitat un salt decisive atunci când cei voinici şi-au dat seama că bătrânii *nu pot* dar *ştiu*! Aici, poate pentru prima dată, a apărut ceia ce se numeşte *memorie externă* - memoria bătrânilor pentru tineri. Acest fapt atrage apariţia castei înţelepţilor dar şi al învăţătorilor cu o consecinţă foarte importantă - apariţia grijei faţă de bătrâni (în lumea animală se manifestă doar cea instincivă de pui, de continuitate a speciei) cu toate urmările respective până la apariţia miturilor, zeificărilor, religiilor. Oare nu această explozie infomaţională a adus la apariţia umanităţii ca atare? Din cele ce urmează mai departe menţionăm *scrisul*, o nouă codare a informaţiei, iarăşi memorie externă dar, spre deosebire de cea precedentă, practic, cu caracter veşnic; continuat de apariţia *tiparului*. Din cele ce continue menţionăm, ceva ce-i mai direct din cadrul matematicii, elaborarea, de către Viéte, a *limbajului matematic (algebric, simbolic)*, ca să ne mai oprim (aici) doar numai la cele apărute la mijlocul sec. XIX: a) măsura informaţiei, matematizarea ei şi fondarea teoriei informaţiei ca atare, b) producerea, la momentul cuvenit, a sistemelor electronice de calcul cu posibilităţile lor nemărginite (fie simplu spus) de transmitere, recepţionare, memorizare şi prelucrare a informaţiei, păşind astfel în era civilizaţiei informaţionale. Rolul matematicii în acest domeniu e de ordin fundamental, dar aplicările, în deosebi, în predarea şi matematicii lasă de dorit. Toate acestea ce costituie informatica sec. XXI în modul cuvenit trebuie însuşite de tineri, deci introduse în procesul de învăţământ. Indicaţii în aceste privinţe apar [4], [6] dar problemele de bază sunt de viitor şi se referă în primul rând la pregătirea cadrelor învăţătoreşti şi a manualelor electronice.

Bibliography

[1] Ion Valuţă, Asupra unor probleme de metodologie şi istorie a matematicii, CAIM 2002, a 10-a ediţie a Conferinţei de Matematică Aplicată şi Industrială, Lucrările secţiei a 7-a, p. 7 - 13.

[2] M. Choban, I. Valuă. About Periods of the development of Methodological problems of the Geometrical Sciences. The 17th Conference on Applied and Industrial Mathematics. Constanţa, 2009.

[3] Norbert Wiener, Cybernetics or control and communication in the animal and the machine.1948, ed. a II-a în 1961.

[4] Dan I. Papuc, Universul matematic al civilizaţiei umane. Ed. Marineasa, Timişoara , Ediţia a 2-a, 2010.

[5] I. Valuţă, Conexiuni ale matematicii cu teoria informaţiei, din punct de vedere metodologic şi predare, învăţare. CAIM 2016, Book of Abstracts, Craiova, 2016.

[6] Gh. Căpăţână, D. Lica ş. a., Maple în Matematici, Ed. BREN, Bucureşti, 2005.

Esenţa şi conţinutul conceptului "competenţa profesional-matematică a studenţilor informaticieni"

Teodora Vascan

Universitatea de Stat din Tiraspol, Chişinău, Republica Moldova
e-mail: teodora_vascan@mail.ru

Scopul acestui studiu este analiza interpretările noţiunilor "competenţă profesională" selectând cele mai frecvente puncte de vedere şi de a evidenţia rolul matematicii universitare în pregătirea specialiştilor infomaticieni. Noţiunea de competenţă profesională se foloseşte pentru exprimarea nivelului înalt de calificare şi profesionalism, caracterizând calitatea pregătirii absolventului, potenţialul rezultativităţii activităţii lui de lucru. Competenţa profesională se referă la o caracteristică integrală, care determină capacitatea specialistului de a rezolva sarcini şi probleme profesionale tipice care apar în situaţii reale de muncă profesională, utilizând cunoştinţele, experienţa profesională şi de viaţă, valorile şi tendinţele. În sistemul de niveluri de calificare profesională competenţa este situată între executare şi perfecţiune. Una din componentele principale ale competenţei profesionale este capacitatea de a dobândi în mod independent, cunoştinţe noi, şi de a le aplica la locul de muncă.
Pentru viitorul specialist în domeniul de studiu informatica studierea în universitate a unui set de discipline matematice nu este doar un "divertisment inteligent pentru dezvoltarea generală", ci o necesitate urgentă de dezvoltare a instrumentelor matematice aplicate, ceea ce va determina în mare măsură caracterul de stăpânire a viitoarei activităţi profesionale. Pentru specialistul informatician modern menţinerea "formei matematice", interesul în cele mai recente evoluţii în matematica aplicată trebuie să fie la fel de naturale ca şi interesul pentru progresele tehnologice şi culturale.

Organizarea activităţilor colaborative pentru viitorii profesori de matematică

Liubov Zastînceanu[1], Natalia Gașițoi[2]

[1,2] *Universitatea de Stat "Alecu Russo" din Bălţi, Republica Moldova*
e-mail: liubaz@mail.ru[1], gasitoi.natalia@usarb.md[2]

Activităţile colaborative pentru studenţii instituţiilor de învăţământ superior sunt orientate spre formarea durabilă a competenţelor profesionale şi transversale. Aceste activităţi joacă un rol important în formarea viitorilor profesori de matematică, întrucât ulterior ei vor activa în colective didactice în care vor trebui să colaboreze pentru soluţionarea diverselor probleme profesionale.
Activităţile colaborative de învăţare cer studentului implicare intelectuală şi implicare socială, punându-l în faţa unor situaţii sau întrebări provocatoare, care presupun stabilirea legăturilor dintre nivelul concret şi cel abstract al instruirii, participarea la discuţii, exprimarea propriilor idei atât oral cât şi în scris.
Organizarea instruirii prin proiectarea activităţilor colaborative cere de la cadrul didactic timp şi

experienţă, revederea curriculumului unităţii de curs şi a procesului de evaluare. Dar anume aceste activităţi contribuie esenţial la formarea competenţelor de învăţare prin interacţiune directă, de răspundere individuală, de comunicare interpersonală şi în grup, de monitorizare a activităţii de prelucrare a informaţiei în grup, care sunt necesare pentru o adaptare profesională rapidă.
Activităţile colaborative de instruire pot fi organizate în diverse forme, în timpul orelor de curs / seminar sau în timpul lucrului independent, în regim de contact direct sau la distanţă (de exemplu, prin intermediul activităţilor Chat, Forum, Glosar, Workshop etc. pe platforma de învăţare MOODLE).

Index