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Contents

	Plenary talks	**7**
1	**Partial Differential Equations**	**11**
2	**ODEs; Dynamical Systems**	**15**
3	**Mathematical Modeling**	**29**
4	**Real, Complex, Functional and Numerical Analysis**	**35**
5	**Probability Theory, Mathematical Statistics, Operations Research**	**47**
6	**Algebra, Logic, Geometry (with applications)**	**57**
7	**Computer Science**	**69**
	Index	**73**

Plenary talks

Geometry of distances via fixed point problem

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A b-distance space is a pair (X, d), where X is a non-empty set X and $d : X \times X \longrightarrow \mathbb{R}$ is a real-valued function with the following properties:

i) $|d(x, y)| = |d(x, x)| = |d(y, y)|$ if and only if $x = y$;

ii) there exists a positive number $s \geq 1$ such that $|d(x, y)| \leq s(|d(x, z) + d(z, y)|)$ for all $x, y, z \in X$.

The function d is called a b-distance on X and c is called a b-constant of the b-distance.
A function $\varphi : X \times X \longrightarrow \mathbb{R}$ is called a b-function on X if there exists a positive number $s \geq 1$, called a b-constant, such that $|d(x, y)| \leq s(|d(x, z) + d((z, y)|)$ for all $x, y, z \in X$.
Let (X, d) be a b-distance space. For any point $a \in X$ and any positive number $r > 0$ the set $B(x, r, d) = \{x \in X : d(a, x) < d(a, a) + r\}$ is called the the d-open ball and $B[x, r, d] = \{x \in X : d(a, x) \leq d(a, a) + r\}$ is called the the d-closed ball at the point a. The set $U \subset X$ is called d-open if for any $x \in U$ there exists $r > 0$ such that $B(x, d, r) \subset U$. The family $T(d)$ of all d-open subsets is the topology on X generated by d.
Are natural the following two questions:

Q1. It is true that any d-open ball is a d-open subset?

Q2. It is true that any d-closed ball is a d-closed subset?

The responses are negative for b-metrics.
Let $\varphi : X \times X \longrightarrow \mathbb{R}$ be a b-function on X. A sequence $\{x_n \in X : n \in \mathbb{N}\}$ is called a φ-Cauchy sequence if $lim_{n,m\to\infty}\varphi(x_n, x_m) = 0$. If A and B are non-empty subset of X, then $\varphi(x, A) = inf\{|\varphi(x, y)| : y \in A\}$, $\rho_\varphi(A, B) = sup\{\varphi(x, B) : x \in A\}$ and $h_\varphi(A, B) = sup\{\rho_\varphi(a, B), \rho_\varphi(B, A)\}$ is the analog of the Hausdorff-Pompeiu distance between A and B.
Some results of the following type will be discussed.

Theorem 1. *Let X be a topological space, $f : X \longrightarrow X$ be a continuous mapping, $0 \leq c < 1$, $\varphi : X \times X \longrightarrow \mathbb{R}$ be a b-function on X and the following conditions are fulfilled:*
1. Any φ-Cauchy sequence has a unique limit in the space X.
2. $|\varphi(f(x), f(y))| \leq c \cdot |\varphi(x, y)|$ for all $x, y \in X$.
Then there exists a unique point $x_0 \in X$ such that $f(x_0) = x_0$.

Theorem 2. *Let X be a topological space, $F : X \longrightarrow X$ be a upper semi-continuous set-valued mapping, $0 \leq c < 1$, $\varphi : X \times X \longrightarrow \mathbb{R}$ be a b-function on X and the following conditions are fulfilled:*
1. Any φ-Cauchy sequence has a unique limit in the space X.
2. $\varphi(x, y) = \varphi(y, x)$ for all $x, y \in X$.
3. $h_\varphi(F(x), F(y))| \leq c \cdot |\varphi(x, y)|$ for all $x, y \in X$.
Then there exists a point $x_0 \in X$ such that $x_0 \in cl_X F(x_0)$.

Diffusion Approximation of Near Critical Branching Processes in Random Environment

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joint work with E. Yarovaya[2]

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We consider Bienaymé-Galton-Watson and continuous-time Markov branching processes and prove diffusion approximation results in the near critical case, in fixed and random environment. In one hand, in the fixed environment case, we give new proofs and derive necessary and sufficient conditions for diffusion approximation to get hold of Feller-Jiřina and Jagers theorems. In the other hand, we propose a continuous-time Markov branching process with random environments and obtain diffusion approximation results. An averaging result is also presented. Proofs here are new, where weak convergence in the Skorohod space is proved via singular perturbation technique for convergence of generators and tightness of the distributions of the considered families of stochastic processes.

Fuzzy Data Analysis. A Fuzzy Model for Uninominal Elections

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We start with an overview of fuzzy logic and fuzziness and its importance in data analysis. A few main issues and results in fuzzy data analysis follow. We then continue with an insight into the proportional uninominal elections model used in Romania at the general elections in 2008, 2012, and 2016. We argue that the problem of elections is a problem to be taken care by scientists. We will touch the Greedy principle and show that the use of Greedy choices may lead to wrong results. We continue with an argument that the election problem itself is a fuzzy logic problem, i.e. a problem of defuzzification of finite fuzzy partitions and we demonstrate a few methods that are applicable to this. After analysing their properties we show that one of these methods seems suitable for more than one reason to be used in this case. We end with a brief overview of other applications of fuzzy logic in natural sciences.

The road to AUTOLAB

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Back in the past, the starting idea was to create a proper infrastructure to be able to build electric vehicles in our University. But the effort couldn't be carried by the University alone and

that was the moment when we set up The Electric Vehicles Research Center in partnership with Targoviste Municipality in 2016.
Following that moment, **The Electric Vehicles Research Center** launched, starting with October 1st, 2018, the **AutoLAB** project, dedicated to students in the Integrated Electrical Systems Engineering in Vehicles MSc program, as well as to bachelor's degree students of the Faculty of Electrical Engineering, Electronics and Information Technology.
Integrated Electrical Systems Engineering in Vehicles is a Master of Science Program that involves the **multidisciplinary** integration of Vehicle Systems Engineering derived from **energy, electric, electronics, communication** and **mechanical systems**.
Students will gain skills across vehicles-specific electrical systems engineering with **practical applications in electric, mechanics, electronics, communication, materials, advanced control** and **systems modelling domains**.

1. Partial Differential Equations

On some particular and exact solutions of the stationary bidimensional Navier-Stokes equation

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The following system of partial differential equations are examined:

$$\begin{cases} u \cdot u_x + v \cdot u_y = a u_{yy}, \\ u_x + v_y = 0, \quad a > 0, \end{cases} \qquad (1)$$
$$u = u(x, y),\ v = v(x, y);\ x, y \in \mathbb{R}.$$

The system (1) describes the process of laminar, stationary flow of a liquid or gas on a plane surface of a plaque, specifically in the layer that is forming this surface. The functions u and v represent the flow of the liquid (gas), the constant $a > 0$ is a determined parameter of the liquid's viscosity (of the gas). Starting from the physical perspective of the problem, one can consider that $x \geq 0$, $y \geq 0$.

Various methods for determining the solutions of non-linear partial differential equations are set forth in [1], [2]. The aim of this paper is to determine different particular and exact solutions of the system (1) and to highlight those which can be represented analytically. The subsequent cases are analyzed:

1) u is not dependent on y; then is obtained the following solutions of the system (1): $u = C$; $v = f(x)$, where C are arbitrary constant, and f are arbitrary function.

2) u is not dependent on x or v is not dependent on y; then is obtained the following solutions of the system (1): $u = C_1 + C_2 e^{Cy}$; $v = aC$, where C, C_1, C_2 are arbitrary constants .

3) v is not dependent on x ; then the solutions of the system (1) have form $v = f(y)$; $u = -f'(y)(x+C_1)+C_2 g(y)$, where C_1, C_2 are arbitrary constants and the functions f, g is determined of the system of equations:

$$\begin{cases} af''' = ff'' - (f')^2\,, \\ ag'' = fg' - f'g. \end{cases}$$

In this case are obtained the particular solutions of the system (1): $v = -6a(C + y)^{-1}$; $u = (C_1 - 6ax)(C + y)^{-2} + C_2(C + y)^{-3}$ and $v = C_1 e^{Cy} + aC$; $u = (C_2 - x)C_1 C e^{Cy}$; where C, C_1, C_2 are arbitrary constants.

4) $v = f(u)$; where f is differentiable function; then the functions $f(u)$ and $u(x, y)$ is determined of the system

$$\begin{cases} au_{yy} = (f - uf')u_y, \\ u_x + f'u_y = 0, \end{cases}$$

In case $f(u) = Cu + C_1$ is obtained the following solutions of the system (1): $u = C_2 + C_3 \exp[C_1(y - Cx)]$; $v = CC_2 + aC_1 + CC_3 \exp[C_1(y - Cx)]$, where C, C_1, C_2, C_3 are arbitrary constants.

5) $v = uf(x) + f_1(x)$; then is obtained following solutions of the system (1): $u = g(x)e^{Cy} + C_1$; $v = aC + C^{-1}g'(x)e^{Cy} - (Cg)^{-1}C_1 g'$, where C, C_1 are arbitrary constants and g is arbitrary differentiable function.

6) $u = x^n f(s)$, $v = x^k g(s)$, $s = yx^m$, where differentiable functions f and g is determined of the system

$$\begin{cases} af'' = f'g - fg', \\ g' = -(2k + 1)f - ksf',\ m = k,\ n = 2k + 1. \end{cases}$$

In this case are obtained following particular solutions of the system (1) where $s = yx^k$:
a) for arbitrary k, $u = -6ax^{2k+1}(C+s)^{-2}$; $v = -6ax^k[(k+1)(C+s)^{-1} - ks(C+y)^{-2}]$, C is arbitrary constant.
b) for $k = -\frac{2}{3}$,

$$u = \frac{6aCC_1e^{-Cs}}{x^{\frac{1}{3}}(1+C_1e^{-Cs})^2}; \quad v = \frac{aC}{x^{\frac{2}{3}}} \cdot \left[\frac{4CC_1s}{(1+C_1e^{-Cs})^2} - \frac{(1-C_1e^{-Cs})}{1+C_1e^{-Cs}}\right]$$

and

$$u = \frac{-6aC}{x^{\frac{1}{3}}\cos^2(Cs+C_1)}; \quad v = \frac{2aC}{x^{\frac{2}{3}}} \cdot \left[\operatorname{tg}(Cs+C_1) - \frac{2Cs}{\cos^2(Cs+C_1)}\right],$$

where constant $C > 0$ and C_1 are arbitrary constant.
c) for $k = 0$, $u = CC_1e^{Cs}$; $v = aC - C_1e^{Cs}$, C, C_1 are arbitrary constants.

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A particular eigenvalue problem

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We study the second order eigenvalue problem

$$-(\mu f_x)_x + k^2\mu f = \frac{1}{\sigma}k^2 f\mu_x, \quad x \in (0, L), \tag{1}$$

where $_x$ denotes the derivative with respect to x, $\mu(x)$ is a variable coefficient, k is a parameter, $f(x)$ are the eigenfunctions which verify some boundary conditions B in $x = 0, x = L$. The eigenvalues σ are positive. When μ is linear and continuous, we get an upper bound of σ which is not depending on k. Moreover, σ tends to zero when L becomes very large.

Let us consider the equation (1) when μ is replaced by a step function μ_S and some boundary conditions B_S exist in the jump points of μ_S. The corresponding eigenfunctions and eigenvalues are f_S, σ_S. We choose μ_S, B_S so that μ_S, f_S be "very close" to μ, f. However, we prove that $\sigma_S \to \infty$ for very large values of the parameter k. Therefore we obtain a strong dependence of the eigenvalues with respect to the variable coefficient μ.

Variable exponent problems on non-smooth domains

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We are concerned with the weak solvability of the elliptic problems involving variable exponents. The study of the variable exponent problems may lead to various applications related to elastic materials, electrorheological fluids, thermorheological fluids, image restoration, mathematical biology, etc. Thus the continuous interest in investigating this type of problems. A relatively new feature consists in treating problems that are cast on a general class of bounded domains, which includes, for example, non-Lipschitz domains.

Acknowledgement. *This talk is based on a joint work with Alejandro Vaslez-Santiago, University of Puerto Rico at Mayagazez*

2. ODEs; Dynamical Systems

Distributed Applications for Real Time Control Systems

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Systems and distributed applications represent the most efficient methods of complex problems solving, which requires considerable resources for data storage and processing. Architecturally, these systems and applications are distributed on multiple levels. At the bottom level, there are a lot of computing devices with minimal resources for data storage and processing, and with network communication resources. Higher levels represent a set of services that include large storage capacity database/knowledge access and algorithms with high data processing complexity [1]. The example of such system is Cloud Computing [2], which is a distributed set of computing services, applications, access to information and data storage.

Description. Conceptually, real-time systems is a class of computational architectures hardware&software, in which correctness of the decision depends not only on the logical results of the calculations but also on the time when these results were delivered [3]. A function $e = (t_n, \Delta t_n)$ can be defined for these systems, which characterizes the speed of convergence and the validity of the decision. An absolutely correctly decision obtained for the time t_n, might be catastrophic being applied at the time $t_n + \Delta t_n$ where Δt_n is the required time for the calculation of the decision.

The purpose of these researches is to optimize the computational time Δt_n of decision by defining an optimal quality criterion Q_{opt}, which ensures the reconfiguration of the logical architecture of the computing system.

To solve the problem, let's consider the computing system, defined by a lot of servers $S = \{S_i, \forall i = \overline{1,N}\}$, where each server offers a set of services $QS = \{QS_{i,j}, \forall i = \overline{1,N}, j = \overline{1,M}\}$. The calculation of the decision can be done by generating a request to access services $AS(t_n) = \{AS_1, AS_2, \ldots, AS_N\}$, that are solved by the set of servers S, and returned in response form $RS(t_n, \Delta t_n) = \{RS_1, RS_2, \ldots, RS_N\}$.

The quality criterion is calculated by the expression:

$$Q_{opt}(t_n + \Delta t_n) = min\{RS_i(t_n, \Delta t_n), \forall i = \overline{1,N}\} \tag{1}$$

The reconfiguration of the logic architecture of the decision computing system can be done based on model:

$$AS(t_n) \overset{Q_{opt}(t_n+\Delta t_n)}{\longrightarrow} AS(t_n + \Delta t_n) \tag{2}$$

where $AS(t_n + \Delta t_n)$ is the optimal configuration of required services to satisfy the quality criterion Q_{opt}.

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Bifurcations study of a bidimensional dynamical system

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The aim of this paper is to study the asymptotic behavior of a bidimensional system which comes from a Lotka Volterra system. We found the equilibrium points and we investigate their stability. We drew the phase portrait and the bifurcation diagrams.

Configuration of the type (3,1,1,1) for a subfamily of cubic systems

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Consider the class $\mathbf{CSL}_7$ of non-degenerate real planar cubic vector fields, which possess two real and two complex distinct infinite singularities and 7 invariant straight lines, including the line at infinity and taking into consideration their multiplicity. For the systems in $\mathbf{CSL}_7$ we construct all possible configurations of invariant straight lines of the type $(3,1,1,1)$ (see [6] for the definition of *configuration of invariant straight lines* and *type of configuration*). Such family systems we will denote by $\mathbf{CSL}^{2r2c\infty}_{(3,1,1,1)}$.
We say that a cubic system belonging to $\mathbf{CSL}_7$ possesses a *configuration of type* $(3,1,1,1)$ if there exists a triplet of parallel lines and three additional straight lines, every set with different slope.
We prove that systems belonging to $\mathbf{CSL}^{2r2c\infty}_{(3,1,1,1)}$ have exactly **41** distinct configurations of invariant straight lines and present corresponding examples for the realization of each one of the detected configurations. We note that all configurations of the straight lines are presented on the Poincaré disc. A classification of all cubic systems possessing the maximum number of invariant straight lines, i.e. 9, taking into consideration the line at infinity and multiplicities of invariant lines, have been made in [5] where the authors detected 23 configurations. The same classification of all cubic systems possessing 8 invariant straight lines, have been made in [1], [2], [3], [4] and 51 distinct configurations have been detected. Here we continue this investigation for system in $\mathbf{CSL}_7$.
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On the solutions of a hyperbolic integro-differential inclusion of fractional order

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We consider the following Darboux problem associated to a fractional hyperbolic integro-differential inclusion

$$D_c^{\alpha,\rho}u(x,y) \in F(x,y,u(x,y),(I_0^{\alpha,\rho}u)(x,y)) \quad a.e.\ (x,y) \in \Pi,$$

$$u(x,0) = \varphi(x), \quad u(0,y) = \psi(y) \quad (x,y) \in \Pi,$$

where $\Pi = [0,T_1] \times [0,T_2]$, $\varphi(.) : [0,T_1] \to \mathbf{R}^n$, $\psi(.) : [0,T_2] \to \mathbf{R}^n$ with $\varphi(0) = \psi(0)$, $F(.,.) : \Pi \times \mathbf{R}^n \to \mathcal{P}(\mathbf{R}^n)$ is a set-valued map, $I_0^{\alpha,\rho}$ is the generalized left-sided mixed integral and $D_c^{\alpha,\rho}$ is the mixed Caputo-Katugampola fractional derivative, $\alpha = (\alpha_1,\alpha_2) \in [0,1) \times [0,1)$ and $\rho = (\rho_1,\rho_2)$, $\rho_1,\rho_2 > 0$.
When F is Lipschitz in the third and fourth variables we obtain the existence of a solution of this problem. Also, we prove the existence of solutions continuously depending on a parameter. This result allows to obtain a continuous selection of the solution set of the problem considered.

Boundary Value Problem Solution Existence For Linear Integro-Differential Equations With Many Delays

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The study of the conditions for the existence of unique solutions of boundary value problems with delay using the contraction mapping principle was carried out in the papers [1, 2, 3]. Boundary value problems for differential and integro-differential equations of neutral type are investigated in

[4, 5] with the use of topological methods.
We consider the following boundary value problem

$$y''(x) = \sum_{i=0}^{n}\Bigg(a_i(x)\,y(x-\tau_i(x)) + b_i(x)\,y'(x-\tau_i(x)) \tag{1}$$

$$+\sum_{j=0}^{1}\int_a^b K_{ij}(x,s)\,y^{(j)}(s-\tau_i(s))\,ds\Bigg) + f(x),$$

$$y^{(j)}(x) = \varphi^{(j)}(x),\ j=0,1,\ x\in[a^*;a],\ y(b)=\gamma, \tag{2}$$

where $\tau_0(x) = 0$ and $\tau_i(x)$, $i = \overline{1,n}$ are continuous nonnegative functions defined on $[a,b]$, $\varphi(x)$ is a continuously differentiable function given on $[a^*;a]$, $\gamma \in R$,

$$a^* = \min_{0\le i<n}\left\{\inf_{x\in[a;b]}(x-\tau_i(x))\right\}.$$

In this paper, the coefficient conditions for the existence of a solution of the boundary value problem for linear integro-differential equations with many delays (1)-(2), which are efficient for verification in practice, are investigated.
Remark. *An efficient algorithm for finding an approximate solution of the boundary value problem* (1)-(2) *is the spline approximation method, using the cubic splines with defect 2, which is considered in [6].*

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Approximation schemes of differential-functional equations and theirs application

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The algorithms for the approximation of linear and nonlinear differential-difference equations in various functional spaces were studied in works[1-3].
In this work approximation schemes of differential-difference equations with many delays and their applications are studied.
The initial problem is considered

$$\frac{dx(t)}{dt} = f(t, x(t), x(t-\tau_1), ..., x(t-\tau_p)),\ t \in [t_0, T],\ p \geq 1, \tag{1}$$

$$x(t) = \varphi(t), t\ \in [t_0 - \tau, \tau_0], \tag{2}$$

where $x\ \in R^n, 0 < \tau_1 < ... < \tau_p = \tau, t_0, T\ \in R$. We define the functions $z_j(t) \in R^n,\ j = \overline{1,m},\ n \in N$, as Koshi problem solution for the system of ordinary differential equations [3-4]

$$\frac{dz_0(t)}{dt} = f(t, z_0(t), z_{l_1}(t-\tau_1), ..., z_{l_p}(t-\tau_p)), l_i = [\frac{\tau_i m_i}{\tau}]. \tag{3}$$

$$\frac{dz_j(t)}{dt} = \frac{m}{\tau}(z_{j-1}(t) - z_j(t))\ t \in [l_0, T],\ j = \overline{1,m}, \tag{4}$$

$$z_j(t_0) = \varphi(t_0 - \frac{\tau_i}{m}),\ j = \overline{0,m}. \tag{5}$$

The conditions were defined under which the solution of Koshi problem (3)-(5) approximate the solution for the initial problem (1)-(2) and the following relationship is true

$$\|x(t - \frac{\tau_j}{m}) - z_j(t)\| \to 0,\ j = \overline{0,m},\ t \in [l_1, T],$$

at $m \to \infty$.
The constructive algorithms for making the stability regions of linear systems with many delays were obtained in [5]. In works [3,6] the approximation scheme of non-asymptotic roots of quasipolinomas and the technique of investigating the stability of the solutions for linear differential-difference equations are suggested.

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Hopf and double Hopf bifurcation in a system of coupled oscillators

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Hopf and double Hopf bifurcations are identified in a 4D systems which consists of two coupled oscillators. The normal form for the Hopf-Hopf bifurcation is obtained and analysed. The results are interpreted in terms of the original system and various operating regimes of the system are pointed out.

Integrability conditions for a cubic differential system with one invariant straight line and one invariant cubic

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We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x,y) + p_3(x,y),\ \dot{y} = -x + q_2(x,y) + q_3(x,y), \tag{1}$$

where $p_j(x,y)$, $q_j(x,y) \in \mathbb{R}[x,y]$ are homogeneous polinomials of degree j. The origin $O(0,0)$ is a singular point of a center or a focus type for (1). The problem of the center is still open for cubic systems.

It is known that a singular point $O(0,0)$ is a center for system (1) if and only if it has a holomorphic first integral of the form $F(x,y) = C$ or a holomorphic integrating factor of the form $\mu = 1 + \sum \mu_j(x,y)$ in some neighborhood of $O(0,0)$.

The conditions under which the cubic system (1) has first integrals of the form $\lambda_1^{\alpha_1}\lambda_2^{\alpha_2}\Phi^{\alpha_3} = C,\ \alpha_j \in \mathbb{C}$, where $l_1 = 0,\ l_2 = 0$ are invariant straight lines and $\Phi = 0$ is an invariant cubic curve, were obtained in [1].

We study the problem of the center for system (1) assuming that the system has two algebraic solutions: one invariant straight line $\Phi_1 \equiv 1 + a_1x + b_1y = 0$ and one irreducible invariant cubic curve $\Phi_2 \equiv x^2+y^2+a_{30}x^3+a_{21}x^2y+a_{12}xy^2+a_{03}y^3 = 0$, where $(a_{30}, a_{21}, a_{12}, a_{03}) \neq 0,\ (a_1, b_1) \neq 0$ and $a_{ij}, a_1, b_1 \in \mathbb{R}$. The problem we consider in this talk is the following: find the subclass of cubic differential systems which has first integrals of the form $\Phi_1^{\alpha_1}\Phi_2^{\alpha_2} = C$, where $\alpha_1, \alpha_2 \in \mathbb{R}$.

We determine the conditions for a singular point $O(0,0)$ to be a center in a cubic system (1) with algebraic solutions $\Phi_1 = 0$ and $\Phi_2 = 0$.

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On the Well-Posedness of the Cauchy Problem for a Nonlinear Evolution Equation in an Ordered Banach Space

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In this work, we consider an evolution equation in an ordered Banach space, abstracting common properties of a class of nonlinear collisional kinetic models. Our goal is to complete the outcome of recent investigations on the existence of unique global-in-time solutions to the Cauchy problem of the aforementioned equation with additional results on the properties of the solutions (continuous dependence with respect to initial data, long time behavior).

The absolute stabilization and the optimal control of nonlinear dynamical systems for special cases. Flight control systems in the case of rolling oscillations

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In a previous paper [] it was presented the automatic regulation methods for the absolute stability (a.r.a.s) of the nonlinear dynamical systems that may have applications on the stabilization of the pitch and rolling oscillations for aircraft or rockets. Two methods for the absolute stability were specified there: a) the A.I. Lurie method with the effective determination of the Liapunov function; b) the frequency method of the Romanian researcher V.M. Popov who has used the transfer function in the critical cases. The authors developed a new sufficient criterion for (a.r.a.s.), with efficient technique of calculus. This paper is intended to be the continuation of the above mentioned one, i.e. there are obtained the analytic - numerical solutions and the conditions for the regulator parameters, the goal being to get the absolute stability for the airplane autopilot route in case of rolling oscillations. Finally the authors prove practically the theorem Kalman - Yakubovich - Popov for the equivalence of these methods (Th. K-Y-P). In the last section of the paper it is presented the optimal control for the flight system in the case of rolling oscillations. The optimization is made using the maximum principle of Pontreaguine; the authors solve the problem of minimum time. It is determined the command function and the optimal trajectory for this system.

Keywords:nonlinear systems, automatic control system, absolute stability, optimal control, Pontreaguine principle, rolling perturbation flight.
2010 MSC : 34H05, 49K35, 93C15, 93C73, 93D10.

Invariant stability conditions of unperturbed motion governed by critical differential system $s(1,2,3)$ on a singular invariant manifold

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It was shown in [1-2] that the cubic differential system $s(1,2,3)$, in the case of one zero root of the characteristic equation, by centro-affine transformations can be brought to the form

$$\begin{aligned} \frac{dx}{d\tau} &= gx^2 + 2hxy + ky^2 + px^3 + 3qx^2y + 3rxy^2 + sy^3, \\ \frac{dy}{d\tau} &= fy + lx^2 + 2mxy + ny^2 + tx^3 + 3ux^2y + 3vxy^2 + wy^3, \end{aligned} \tag{1}$$

where $f \neq 0$ and the coefficients and phase variables in 1 take values from the field of real numbers $\mathbb{R}$.
Suppose that $l \neq 0$. In [3], all stability conditions of unperturbed motion, expressed by the coefficients of the system 1 were provided. All these conditions were expressed by centro-affine invariants and comitants of the system $s(1,2,3)$ on a nonsingular invariant manifold [4,5].
Next assume that $f \neq 0$ and $l = 0$. Then constructing the Lyapunov series [1] for system 1, we obtain:
Lemma 1. *The stability of the unperturbed motion described by system* (1) *with* $f < 0$ *and* $l = 0$, *is characterized by one of the following 13 possible cases:*

I. $g \neq 0$, *then the unperturbed motion is unstable;*

II. $g = 0,\ p > 0$, *then the unperturbed motion is unstable;*

III. $g = 0,\ p < 0$, *then the unperturbed motion is stable;*

IV. $g = p = 0,\ ht \neq 0$, *then the unperturbed motion is unstable;*

V. $g = p = h = 0,\ qt > 0$, *then the unperturbed motion is unstable;*

VI. $g = p = h = 0,\ qt < 0$, *then the unperturbed motion is stable;*

VII. $g = p = h = q = 0,\ kt \neq 0$, *then the unperturbed motion is unstable;*

VIII. $g = p = h = q = k = 0,\ t \neq 0,\ r > 0$, *then the unperturbed motion is unstable;*

IX. $g = p = h = q = k = 0,\ t \neq 0,\ r < 0$, *then the unperturbed motion is stable;*

X. $g = p = h = q = k = r = 0,\ st > 0$, *then the unperturbed motion is unstable;*

XI. $g = p = h = q = k = r = 0,\ st < 0$, *then the unperturbed motion is stable;*

XII. $g = p = h = q = k = r = s = 0,\ t \neq 0$, *then the unperturbed motion is stable;*

XIII. $g = p = t = 0$, *then the unperturbed motion is stable.*

In the last two cases, the unperturbed motion belongs to some continuous series of stabilized motion. Moreover, for sufficiently small perturbations, any perturbed motion will asymptotically approach to one of the stabilized motions of the mentioned series.

Remark 1. *The conditions of the Lemma 1 are expressed by centro-affine invariants and comitants of the system $s(1,2,3)$ on a singular invariant manifold.*

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Family of quadratic differential systems with invariant ellipse and a invariant straight line of maximal multiplicity

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In this articles we consider the planar quadratic differential system

$$\dot{x} = P(x,y),\ \dot{y} = Q(x,y), \tag{1}$$

where P and Q are real polynomials such that the maximum of the degree of P and Q is 2 and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to system (1).
An algebraic curve $f(x,y) = 0$ in $\mathbb{C}^2$ with $f(x,y) \in \mathbb{C}[x,y]$ is an invariant algebraic curves of a differential system (1) if the following identity holds

$$\frac{\partial f(x,y)}{\partial x}P(x,y) + \frac{\partial f(x,y)}{\partial y}Q(x,y) \equiv f(x,y)K_f(x,y),\ (x,y) \in \mathbb{R}^2 \tag{2}$$

for some polynomial $K_f(x,y) \in \mathbb{C}[x,y]$ called the cofactor of the curve $f(x,y) = 0$. The invariant algebraic curves $f(x,y) = \alpha x + \beta y + \gamma$ of the order one of system (1) are called invariant straight

lines of the system (1). If m is the greatest natural number such that $(\alpha x + \beta y + \gamma)^k$ divides $det\mathcal{E} = P\mathbb{X}(Q) - Q\mathbb{X}(P)$ then the invariant straight lines $\alpha x + \beta y + \gamma = 0$ has algebraic multiplicity m.
In this work we show that in the class of quadratic differential systems whit a an ellipse and a straight line as invariant algebraic curves the maximal algebraic multiplicity of the straight line is tree.
Theorem. *A planar polynomial differential system of degree 2 having an invariant ellipse and a invariant straight line of maximal multiplicity, after an affine change of coordinates, can be written as*

$$\dot{x} = y(x-1),\ \dot{y} = y^2 + x - 1, \tag{3}$$

where invariant algebraic curves: $f_1(x,y) = x^2 + y^2 - 1, f_{2,3,4}(x,y) = x - 1$ *and first integral:* $(x^2 + y^2 - 1)(x-1)^{-2} = C$.

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Qualitative study of cubic differential systems with invariant straight lines of total multiplicity seven along one direction

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Consider the general cubic differential system, i.e. a differential system

$$\dot{x} = P(x,y), \dot{y} = Q(x,y),$$

where $P, Q \in \mathbb{R}[x,y]$, $\max\{\deg P, \deg Q\} = 3$ and $GCD\,(P,Q) = 1$.
According to [1], it is possible to calculate a Darboux first integral for this system, if this system has sufficiently many invariant straight lines considered with their multiplicities. In the theory of dynamic systems, the investigation of polynomial differential systems with invariant straight lines is done using different types of multiplicities of these invariant straight lines, for example: parallel multiplicity, geometric multiplicity; algebraic multiplicity etc [2]. In this work we will use the notion of algebraic multiplicity of an invariant straight line
In [3] we showed that there are exactly 26 canonical forms of cubic differential systems which possess real invariant straight lines of total multiplicity at least seven (including the invariant straight line at the infinity) along one direction. In this paper we expand these results by including the complex invariant straight lines. And we show that there are 34 canonical forms of cubic systems which have real or complex invariant straight lines of total multiplicity seven along one direction.

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Some types of behavior for a degenerate double Hopf bifurcation

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Starting with a 4D system depending on 2 parameters and possesing the origin as an equlibrium point with two pairs o complex conjugate eigenvalues, we find a new normal form. We used a reduced number of nondegeracy conditions compared to existing literature. For a certain domain of the parameters the local dynamics is analyzed.

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Centers in cubic differential systems with an affine invariant straight line of multiplicity two and the line at infinity of multiplicity three

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We consider the real cubic system of differential equations

$$\begin{cases} \dot{x} = y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3 \equiv p\,(x, y)\,, \\ \dot{y} = -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3) \equiv q\,(x, y)\,, \\ \gcd(p, q){=}1,\, sx^4 + (k + q)x^3y{+}(m + n)x^2y^2{+}(l + p)xy^3 + ry^4 \not\equiv 0. \end{cases} \quad (1)$$

The critical point $(0, 0)$ of system (1) is either a focus or a center, i.e. is monodromic. The problem of distinguishing between a center and a focus is called the *problem of the center* or the *center-focus problem.*
The straight line $\alpha x + \beta y + \gamma = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$ is called *invariant* for (1) if there exists a polynomial $K \in \mathbb{C}[x, y]$ such that the identity $\alpha p(x, y) + \beta q(x, y) \equiv (\alpha x + \beta y + \gamma)K(x, y)$, $(x, y) \in \mathbb{R}^2$ holds.
The homogeneous system associated to the system (1) has the form

$$\begin{cases} \dot{x} = yZ^2{+}(ax^2 + cxy + fy^2)Z + kx^3{+}mx^2y{+}pxy^2{+}ry^3 \equiv P\,(x, y, Z)\,, \\ \dot{y} = -(xZ^2{+}(gx^2 + dxy + by^2)Z + sx^3{+}qx^2y{+}nxy^2 + ly^3) \equiv Q\,(x, y, Z)\,. \end{cases}$$

Denote $\mathbb{X}{=}p\,(x, y)\,\frac{\partial}{\partial x} + q\,(x, y)\,\frac{\partial}{\partial y}$ and $\mathbb{X}_\infty{=}P\,(x, y, Z)\,\frac{\partial}{\partial x} + Q\,(x, y, Z)\,\frac{\partial}{\partial y}$.
We say that the invariant straight line $\alpha x + \beta y + \gamma = 0$ (respectively, the line at infinity $Z = 0$) has *multiplicity* ν (respectively, $\nu + 1$) if ν is the greatest positive integer such that $(\alpha x + \beta y + \gamma)^\nu$ (respectively, Z^ν) divides $E = p \cdot \mathbb{X}(q) - q \cdot \mathbb{X}(p)$ (respectively, $E_\infty = P \cdot \mathbb{X}(Q) - Q \cdot \mathbb{X}(P)$).
Theorem 1. *The cubic systems* (1), *with the line at infinity* $Z = 0$ *of multiplicity three and an affine real invariant straight line* $\alpha x + \beta y + \gamma = 0$ *of multiplicity two, have a center at the origin* $(0, 0)$ *if and only if these systems have an integrating factor of the form* $1/(\alpha x + \beta y + \gamma)^2$.
Theorem 2. *For cubic systems* (1), *with the line at infinity of multiplicity three and an affine real invariant straight line of multiplicity two, the critical point* $(0, 0)$ *is a center if and only if the first Lyapunov quantity vanishes.*

Existence Of Nonnegative Solutions For A Fractional Integro-Differential Equation

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We study the nonlinear fractional integro-differential equation

$$(E) \qquad D^{\alpha}_{0+}u(t) + f(t, u(t), Tu(t), Su(t)) = 0, \;\; t \in (0, 1),$$

with the multi-point boundary conditions

$$(BC) \qquad u(0) = u'(0) = \cdots = u^{(n-2)}(0) = 0, \ \ D_{0+}^{p} u(1) = \sum_{i=1}^{m} a_i D_{0+}^{q} u(\xi_i),$$

where $\alpha \in \mathbb{R}$, $\alpha \in (n-1, n]$, $n \in \mathbb{N}$, $n \geq 3$, a_i, $\xi_i \in \mathbb{R}$ for all $i = 1, \ldots, m$, ($m \in \mathbb{N}$), $0 < \xi_1 < \cdots < \xi_m \leq 1$, p, $q \in \mathbb{R}$, $p \in [1, n-2]$, $q \in [0, p]$, D_{0+}^{α} denotes the Riemann-Liouville derivative of order α, $Tu(t) = \int_0^t K(t,s)u(s)\,ds$, and $Su(t) = \int_0^1 H(t,s)u(s)\,ds$ for all $t \in [0,1]$. Under some assumptions on the function f, we present existence results for the nonnegative solutions of problem $(E) - (BC)$. In the proofs of the main theorems we use the Banach contraction mapping principle and the Krasnosel'skii fixed point theorem for the sum of two operators (see [1]).

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3. Mathematical Modeling

Aspects of the Entanglement Concept

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Keywords:entanglement, quantum information, quantum optics, linear algebra

In the field of quantum optics a certain form of states correlation is called "entanglement". The results of a quantum tomography of a photon source used for quantum cryptography are analyzed. In order to accomplish acceptable experiments it is of practical importance to tell if a given state is entangled or not. The mathematics of entanglement concept is analyzed based on the language of quantum theory. Quantum states, quantum bit, quantum operations, quantum entropy and quantum information are considered. The density matrix formalism is used and the counterintuitive properties of the entangled states are described by means of the projection operators. The closely relation between the entanglement concept and the linear algebra, especially the tensor product mathematical notion is essential for the performed analysis.

Properties of Relative Asymmetric Near-metric Spaces and Applications

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Keywords:asymmetric metric, near-metric, relative metric space, Pearson distance, length structure.

We study the some properties and length structures of not full metric spaces which are with generalized relative distance as component wise M - relative space. Two cases of described properties are given: i) asymmetric relative metric space and ii) near-metric space.
Are described two applications for estimations in engineering practice of physical parameters and probabilities: i) estimation of sensitivity coefficients with relative distance and ii) estimation the variations in Pearson's test procedure by using the Pearson distance.

Mathematical modelling, explicit results and numerical implementation for the cavity resonator

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The research of the electromagnetic field behaviour is proposed here in the case of the cavity rectangular resonator.
The initial mathematical modelling is based on the differential Maxwell equations in the spatial

Cartesian coordinate system. Fulfilling the engineering requirements of the considered process the electromagnetic field vector intensities are assumed to obey the harmonic law regarding the time argument.
Application of the general analytic operator technique [1] to the original Maxwell system reduces it in the equivalent manner to the unified wave PDE (partial differential equation) with respect to all scalar components of the unknown electromagnetic field vector intensities.
The final mathematical simulation of the investigated phenomenon is done by the corresponding boundary value problem in terms of the aforesaid PDE. The given statement is solved explicitly using the integral transform method [2].
The obtained exact formulas give the accurate analytic conclusions permitting to get the proper efficient discretization procedure. The latter leads to the relevant computer implementation whose numerical and graphical results are in conformity as with the physical features, as with the theoretical hypotheses of the suggested present research.
The current results partially were reflected in [3].

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Global sensitivity analysis of a compartmental *within-host* model for dengue virus

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In this study we consider a *within-host* model for dengue virus (DENV) incorporating the dichotomy between mature vs. immature virions. The role of plasmacytoid dendritic cells (pDCs) – important sentinels in an infection, as well as the degree of maturity and infectivity – by adding compartments for infectious and noninfectious DENV – are taken into account. The pDC response is related to various infection parameters such as peak viremia, the time to peak viremia, number of infected cells and numbers of activated immune cells in a primary and secondary infection. The analyzed *within-host* model is compartmental involving target cells, infected cells, free virus and different types of immune cells, and anti-viral compounds. // The aim of this talk is to present several global sensitivity results to explore the parameter sensitivity in the model. Sensitivity heat maps of the model variables, parameter sensitivity spectra and normalised singular spectra are performed.

Modelling of the evapotranspiration processes in wetlands based on remote sensing resources

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The main objective of the study was to estimate the evapotranspiration of the wetland vegetation using a methodology based on the images provided by the remote sensing systems with available ground information for the wetland existing near Poeni village in Glavacioc hydrological basin e.g., Landsat 8, MODIS Terra/Aqua satellites, Proba-V. The methodology uses the synthetic inputs derived from satellite information at 8/16 days intervals: land cover (LULC), leaf area index (for vegetation areas) - LAI, albedo, fAPAR (absorbed photosynthetically active radiation fraction), and meteorological inputs (air pressure, air temperature, air humidity, solar radiation, etc.) provided by ground measurements. The first step involves the partitioning of incident solar radiation into net radiation available for plants and soil. This is done by calculating the vegetation fraction using the LAI parameter. Potential evaporation at the ground level is calculated using the Penman-Monteith method based on the weather parameters and canopy characteristics (height, conductance factor, maximum leaf conductance, albedo, LAI). The transpiration of the plants is then calculated by estimating the conductivity of the wetland canopy, the aerodynamic resistance and the radiation intercepted by the plant species. Thereafter, the transpiration of plants is calculated using an approach based on the Penman-Monteith method. The final purpose of the research is to develop a procedure (statistical downscaling) that can infer high-resolution information from low-resolution variables (i.e., 30 m resolution images) to increase the spatial resolution of the ET estimates.

Transmission dynamics and control mechanisms of vector-borne diseases with active and passive movements between urban and satellite cities

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A metapopulation model which explicitly integrates vector-borne and sexual transmission of an epidemic disease with passive and active movements between an urban city and a satellite city is formulated and analysed, with a view towards investigating how these movements and possible restrictions imposed upon them can affect the spread of the disease.
The sensitivity analysis reveals that the disease is primarily transmitted via the vector-borne mode, rather than via sexual transmission and that sexual transmission by itself may not initiate or sustain an outbreak. Also, increasing the population movements from one city to the other leads to an

increase in the basic reproduction number of the later city but a decrease in the basic reproduction number of the former city. The influence of other significant parameters is also investigated via the analysis of suitable partial rank correlation coefficients. After gauging the effects of mobility, we explore the potential effects of optimal control strategies relying upon several distinct restrictions on population movement.

Shallow Water Model for Flow on Vegetated Landscape

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Water flow on a soil surface with vegetation is a complex phenomenon due to the multitude of factors involved in. In this presentation, we introduce a shallow water model that takes into account the soil topography, the presence of vegetation on the soil surface and the water-soil and water-plant interactions. The model is described by a nonlinear hyperbolic PDE system of equations. We provide a numerical scheme by discretizing the PDE system in space and time. We analyze some physical relevant qualitative properties of the resulting system. Quantitative and qualitative tests on the discrete model are performed.

Lumped parameter model for steady state heat transfer calculation in interconnected electrical systems and fluids

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Applying thermal networks by analogy to electrical circuits, the temperature rise in interconnected electrical systems and fluids can be advantageously calculated with a complete system of equations that considers the physical geometry, heating sources, fluid properties and material characteristics. The model parametrization for steady state calculation is explained in this paper, and comparisons of the calculated results with other methods, analytical and/or finite elements are presented. Based on the proposed lumped parameter model, the heat transfer is calculated here, for example, in case of a) V heating of cooled hollow conductors and b) V heat exchangers of a water heating system.

Multidimensional Scaling As Tool To Analyze The Structure In Datasets : An Application For Understanding Brain Connectivity

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Keywords:multidimensional scaling, classical multidimensional scaling, metric multidimensional scaling, non-metric multidimensional scaling, dimensionality reduction, similarity data, loss function

Stastisticial methods are increasingly relevant for uncovering structure in different datasets. This paper aims to explain one of these methods that is known as Multidimensional Scaling by using simple and intuitive examples. Multidimensional Scaling accepts quantitative, qualitative and mix data. There is a wide variety of methods for obtaining data appropriate for MDS. The most direct way is to ask subjects to give pairwise ratings or to sort stimulus objects according to their similarity, relatedness and association. There are three types of this method : classical MDS, metric MDS and non-metric MDS. For each of them, the purpose is to provide a visual representation of the pattern of proximities among a set of objects in a low number of dimensions.There has been recent interest in the use of non-metric multidimensional scaling (NMDS) for understanding brain connectivity. For the case of a connectivity dataset derived from the primate cortical visual systemgreat caution is needed in interpreting the resulting configuration. The annular configuration that NMDS provides for the two streams view of visual processing, helps us to create a hierarchic view for understanding the brain connectivity .

Solving the non-linear multi-index transportation problem

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In this paper we study the non-linear multi-index transportation problem [1-3]. The aim of this paper is to propose polynomial algorithms that solve the problem. The proposed algorithms solve problems of large sizes in reasonable time, which was proven by the various tests shown in this paper. The algorithms were implemented in Wolfram Language and tested in Wolfram Mathematica.

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4. Real, Complex, Functional and Numerical Analysis

Weakly bounded functions between normed weak linear spaces

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Various problems in some areas, such as computer science, optimization and probabilities, have led to the definition of some spaces that are not linear, but almost linear (i.e. they satisfy only part of the axioms of a linear space), such as spaces with non-negative scalars, weak linear spaces, almost linear spaces, quasilinear spaces, (A) semi-linear spaces, (L) semi-linear spaces. The purpose of our talk is presenting some properties of weakly bounded functions acting between normed weak linear spaces.

On new integral inequalities using mixed conformable fractional integrals

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Keywords:Chebsyev inequality, mixed conformable fractional integral.
2010 MSC : 26A33, 26D10, 26D15.

During the past two decades or so, fractional integral operators have been one of the most important tools in the development of inequalities theory. By this means, a lot generalized intergral inequalities involving various the fractional integral operators have been presented in the literature. Very recently, mixed conformable fractional integral operators has been introduced by T. Abdeljawad and with the help of these operators some new integral inequalities are obtained. The main aim of the paper is to establish some new Chebsyev type fractional integral inequalities by using mixed conformable fractional integral operators.

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Two Open Problems Concerning Quasi-Periodic Eigenproblems

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We are concerned with two open problems related to the presence of spurious eigenvalues when two quasi-periodic eigenproblems are solved by Fourier spectral collocation. In spite of the high accuracy of this method such parasitic eigenvalues can appear.
Our philosophy is to find things more than to prove things, so we use the modern computer technology as an active tool in mathematical research. We make use of some MATLAB codes in order to find out the drift (the numerical instability) of some suspect eigenvalues and the strange behavior of the corresponding eigenvectors in the frequency (spectral) space.
The work is intended to be an experimental mathematical essay.

On the class of AM-compact operators

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Let E be a Banach lattice and X a Banach space. A linear operator $T : E \to X$ is called AM-compact if $T[-x, x]$ is relatively compact for every $x \in E^+$. In this talk, we are interested in the class of AM-compact operators between Banach lattices.We show that the center of the collection of regular AM-compact operators between Banach lattices E, F is an algebra and order isomorphic to the tensor product of the centers of E and F. Also, we inves- tigate the center of the collection of the regular Dunford–Pettis operators between Banach lattices.

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Hadamard Type Inequalities for Co-Ordinated Convex Functions via Fractional Integrals

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Keywords:Coordinated convex functions, coordinates, Riemann-Liouville fractional integral
2010 MSC : 26D15

In this paper, we established some new Hadamard-type integral inequalities for co-ordinated convex functions on the co-ordinates via Riemann-Liouville fractional integrals.
Acknowledgement. *This study was supported by Ağrı İbrahim Çeçen University BAP with project number EF.19.003.*

Some new inequalities via fractional integrals of Caputo-Fabrizio type

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Keywords:Caputo-Fabrizio, Fractional integral, Convexity.

In this article, firstly, Hermite-Hadamard's inequality is generalized via a fractional integral operator associated with Caputo-Fabrizio fractional derivative. Then, a new kernel is obtained and a new theorem valid for convex functions is proved for fractional order integrals.

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Several norm inequalities with applications to p-angular distance

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The main aim of this paper is to establish several new inequalities in an inner product space and in a normed space. Among these we proved some refinements of the Cauchy-Schwarz inequality. We also show a generalization of the inequality due to Maligranda and a generalization of the inequality due to Dehghan. Other estimates which follow from the triangle inequality are also presented in connection with the p-angular distance and the skew p-angular distance

Keywords:Inner product space, Cauchy-Schwarz inequality, normed space **2010 MSC** : Primary 46C05, secondary 26D15, 26D10

Barrlund metrics and hyperbolic type metrics in plane

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The Barrlund metrics $b_{G,p}$, with $1 \leq p \leq \infty$, in a proper subdomain G of $\mathbb{R}^n$, are important examples of $M-$relative distances in the sense of P. Hästo, encountered in relative perturbation theory for matrix eigenproblems. The triangular ratio metric $s_G = b_{G,1}$ is connected to the classical Ptolemy-Alhazen problem from optics. Recently, the study of these metrics turned out to be relevant for Geometric Function Theory, where hyperbolic type metrics play a remarkable role. We discuss explicit formulas for Barrlund metrics in special cases and investigate the dependence of the Barrlund metrics $b_{G,p}$ on the parameter p and on the domain G. We prove lower and upper bounds of Barrlund metrics in terms of various hyperbolic type metrics, as well as distortion results connecting Barrlund metrics to quasiconformal mappings.

Numerical approximation for a nonlocal Allen-Cahn equation supplied with non-homogeneous Neumann boundary conditions. Case 1D

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Keywords:Nonlinear equations; Reaction-diffusion equations; Dinite difference methods; thermodynamics; phase changes.
2010 MSC : 34A34; 35K57; 35Qxx; 65N06; 80Axx.

A first-order implicit difference scheme is proposed in order to solve numerically the nonlocal Allen-Cahn equation (see [1]):

$$\alpha\xi\frac{\partial}{\partial t}\varphi(t,x)=\xi\Delta\varphi(t,x)+\frac{1}{2\xi}\left(\varphi(t,x)-\varphi(t,x)^3\right)+\frac{1}{|\Omega|}\int_{\Omega}\left[\varphi(t,y)-\varphi(t,y)^3\right]dy \tag{1}$$

in $Q=(0,T]\times\Omega$, $T>0$ and $\Omega\subset I\!R_+$, endowed with non-homogeneous Neumann boundary conditions (see [2]):

$$\xi\frac{\partial}{\partial\mathbf{n}}\varphi(t,x)=w(t,x)\quad\text{in } \Sigma=(0,T]\times\Gamma \tag{2}$$

and initial contitions:

$$\varphi(0,x)=\varphi_0(x)\qquad\text{on } \Omega. \tag{3}$$

Numerical experiments are presented and analized in terms of particular values for the physical pharameters (see [3] for more details).

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On The Properties of the $\lambda-$Convex Function and $\lambda-$Convex function on Co-ordinates with Its Inequalities

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Keywords:Convexity, $\lambda-$convex functions, Hadamard Inequality, Jensen convexity.
2010 MSC : 26D15, 25D10

Firstly, we write a short historical background about convex functions. Secondly, we mention some properties of $\lambda-$convex functions with its inequalities. Finally, we extend this study for $\lambda-$convex function on the co-ordinates.

Numerical approximation for a nonlocal reaction-diffusion equation supplied with non-homogeneous Neumann boundary conditions. Case 1D

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We are concerned with a first-order implicit difference scheme to solve numerically a nonlocal reaction-diffusion equation subject to the non-homogeneous Neumann boundary conditions. Numerical experiments are presented and analyzed in terms of physical phenomena, for a particular case of nonlocal reaction-diffusion equation: the Allen-Cahn equation.
Considering the following problem:

$$\begin{cases} p_1 v_t(t,x) = \\ p_2 \left\{ \int\limits_{\Omega} J(x-y)\Big[v(t,y)-v(t,x)\Big]dy + \int\limits_{\partial\Omega} J(x-y)w(t,y)d\gamma \right\} \\ \quad + p_3 f(v(t,x)) + \dfrac{1}{|\Omega|}\int\limits_{\Omega} f(v(t,y))dy & \text{in } Q=(0,T]\times\Omega \\ v(0,x)=v_0(x) & \text{on } \Omega, \end{cases} \tag{1}$$

we are introducing the discrete equation corresponding to (1); consequently, we are formulating a conceptual algorithm to numerically solve the equation and report the numerical experiments.
As a novelty of this work we refer to the numerical scheme introduced in order to approximate the solution to the nonlocal reaction-diffusion problem (1) in presence of the cubic nonlinearity $f(v(t,x)) = v(t,x) - v^3(t,x)$.

To the best of our knowledge, the current work presents for the first time results on the numerical solutions for a nonlocal reaction-diffusion problem stated by (1).

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Terraced Matrices, Rhaly Operators, Statistical Density

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We present a general view upon Rhaly operators on Hilbert spaces, that is operators defined by "terraced matrices" for sequences of complex numbers. There is a strong connection between statistical zero densitity of subsequences of natural numbers and properties of Rhaly operators boundedness, compactness. We also emphasize a singular fact, the value of a Rhaly operator lays actually in the closed subspace generated by the orbit of the right shift, in other words Rhaly operators are weighted power series of the right shift.

Bisingular integral operator with a Cauchy kernel in generalized Hölder spaces

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Let $\omega\left(\delta_1, \delta_2\right)$ be a modulus of continuity; $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the corresponding elementary moduli of continuity [1] satisfying Zygmund-Bari-Stechkin conditions; $\gamma = \gamma_1 \times \gamma_2$ is Cartesian product of closed Lyapunov curves. Let $H_\omega(\gamma)$ denotes the space of continuous functions $x(t, \tau)$ (on γ) whose moduli of continuity satisfy the next conditions:

$$\omega\left(x; \delta_1, \delta_2\right) \leq c_1 \omega\left(\delta_1, \delta_2\right); \tag{1}$$

(2) $$\omega_{1,1}(x;\delta_1,\delta_2) \le c_2\Omega_1(\delta_1)\Omega_2(\delta_2)$$

(where $\omega_{1,1}(x;\delta_1,\delta_2)$ is mixed modulus of continuity of the second order). Introduce a norm in this space in the following way:

$$\|x\|_{H_\omega} = \max_{(t,\tau)\in\gamma} |x(t,\tau)| + \sup_{\delta_1^2+\delta_2^2\neq 0} \frac{\omega(x;\delta_1,\delta_2)}{\omega(\delta_1,\delta_2)} + \sup_{\delta_1^2+\delta_2^2\neq 0} \frac{\omega_{1,1}(x;\delta_1,\delta_2)}{\Omega_1(\delta_1)\Omega_2(\delta_2)}.$$

$H_\omega(\gamma)$ is Banach space for this norm. Singular integrals are considered:

$$(S_1\varphi)(t,\tau) = \frac{1}{\pi i}\oint_{\gamma_1} \frac{\varphi(t_0,\tau)}{t_0-t}\,dt_0, \quad (S_2\varphi)(t,\tau) = \frac{1}{\pi i}\oint_{\gamma_2} \frac{\varphi(t,\tau_0)}{\tau_0-\tau}\,d\tau_0,$$

$$(S_{12}\varphi)(t,\tau) = -\frac{1}{\pi^2}\oint_{\gamma_1}\oint_{\gamma_2} \frac{\varphi(t_0,\tau_0)}{(t_0-t)(\tau_0-\tau)}\,dt_0\,d\tau_0 \quad (= S_1 \otimes S_2)$$

(integrals are understood in the sense of the Cauchy principal value). As is well known, in one-dimentional case singular integral (Cauchy operator) is bounded in generalized Hölder space $H_\omega(\gamma)$ when structural characteristic $\omega(\delta)$ of the space satisfies Zygmund-Bari-Stechkin conditions. But this is not enough in two-dimentional case; bisingular Cauchy integral is unbounded ($S_{12} : H_{\omega^{(1)}} \to H_{\omega^{(2)}}$, $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$). To establish an analogue of the theorem Privalov-Plemelj we need condition (2).

Theorem. Bisingular Cauchy operators S_1, S_2, S_{12} are bounded in generalized Hölder space $H_\omega(\gamma)$.

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Pettis-types integrals for vector multifunctions

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Taking into account that the scalarization is a way for studying the vector problems, the Pettis method provides some types of integrals (as Pettis-Lebesgue, Pettis-Sugeno, Aumann-Pettis-Sugeno integrals) which will be studied in this paper.

Hermite-Hadamard-Fejer type inequalities for p-convex functions via α-generator

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Differentiation and integration are basic operations of calculus and analysis. Indeed, they are infinitesimal versions of substraction and addition operations on numbers. From 1967 till 1970 Michael Grossman and Robert Katz gave definitions of a new kind of derivative and integral, converting the roles of substraction and addition into division and multiplication, respectively. And they established Non-Newtonian Calculus. This calculi is generated by arithmetics, generator, etc. So in this study, firstly we give definition of $\alpha - p$ convex function, and some new theorems for this function class via α-generator. Secondly, some theorems are generalized using Hermite-Hadamard-Fejer inequality for $\alpha - p$ convex functions by this generator. Finally, it is obtained some new corollaries depend on these theorems.

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Some New Inequalities for s-convex functions in the second sense via Hölder-İşcan Integral Inequality

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In this paper, it is established some new developments of Hermite-Hadamard type integral inequalities for s-convex functions in the second sense by using Hölder-İşcan and Improvement Power-Mean integral inequality. Then it is compared with existing theorems in literature.

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5. Probability Theory, Mathematical Statistics, Operations Research

On dynamic modeling in reliability and survival analysis by means of time-varying parameters

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In this work we are interested in a general class of distributions for independent not necessarily identically distributed (inid) random variables, closed under extrema, that includes a number of discrete and continuous distributions like the Geometric, Exponential, Weibull or Pareto. The scale parameter involved in this class of distributions is assumed to be time varying with several possible modeling options proposed. Such a modelling setting is of particular interest in reliability and survival analysis for describing the time to event or failure. The maximum likelihood estimation of the parameters is addressed, and the asymptotic properties of the estimators are discussed. We provide real and simulated examples and we explore the accuracy of the estimating procedure as well as the performance of classical model selection criteria in choosing the correct model among a number of competing models for the time-varying parameters of interest.

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Method and instruments for risk assessment in the audit process of public entities

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At present, internal audit is an important role in the use of evaluating the efficiency, economy and efficiency of the internal control environment, and the recommendations of the external data are oriented towards internal dissemination and processing. The recommendations form fundamental benchmarks for the subsequent decision-making use, because they are granted directly with the purpose of minimizing the thermal, making decisions and creating in the forecast of the unpredictable risks. Internal audit evaluates any area regarding the use and internal use of an institution, placing a special emphasis on the risk issue, the care will be destroyed by the entities. Therefore, making managerial decisions in achieving the planned objectives in the medium or small term is possible to have a large number of influences for personal care can not fully control. That is why it is important, in order to be able to make the decision and to carry out an active activity, to be in the calculation of the production of risks that there may be something that may have adverse effects with respect to. Generally speaking, the carrying out of a study process involves and can make risk, and avoiding risk means minimizing it under acceptable conditions. The risks arising in the system can be influenced by external and internal factors. External risks speak as challenges in accepting or transforming the internal situation depending on the external environment.

Internal risks will be addressed and they can evaluate problems and process functions. The management of the entity tends to optimize the decision making depending on the degree of identified risks, and the internal auditor helps them with the detailed evaluation of the process, which has a major risk and with proposals aimed at improving the process and minimizing the risk.

Currently, the specialized literature in the field of audits proposes to calculate the audit risk by the formula

$$RA = RI \cdot RC \cdot RUN, \tag{1}$$

where RA - audit risk, RI - inherent risk, RC - control risk, RUN - undetected risk.
The role of the audit is to have as little percentage of undetected risks (RUN). The RUN risk is evaluated on the basis of formula (1), by analyzing in detail the process through the risk prism, where the audit risk is not greater than 0.05, and the inherent risks will be identified based on the documents analyzed in the preparation phase of internal audit mission, and the control risk is based on the process analysis according to the national internal control standard (NICS). The more carefully the risks of internal control and the inherent risks will be analyzed and identified, the lower the degree of undetected risks The calculation of the control risk will be based on the algebraic average, such as:

$$\bar{x} = \frac{1}{n}\Sigma_{i=0}^{n} x_i.$$

This method gives a general connotation of the degree of NICS achievement and the degree of risk identified. The inherent risks will be studied depending on the process, objective and will be analyzed from the following criteria:

1 The human resources involved in the press;

2 The financial sources planned and allocated for implementation;

3 Information technologies;

4 The degree of fraud in this process, etc

These criteria help us in analyzing the risk inherent in detecting the type of risk and its degree. The audit, in the process of calculating the inherent risk, uses various statistical, mathematical and economic methods to identify the major degree of each criterion. When summing up these rituals, the ABC method will be analyzed, with the help of which we identify the inherent risk weight of some processes and activities. Today, it is impossible to conceive an audit mission in the fields of economics, management, information technologies, not to use in its process of knowledge methods of quantification, numerical expression, of laws, of interdependencies, of measurement of tendencies in the decision-making process. Moreover, the complexity of the interconnection of the audit activity with the activity areas of the entity, leads the internal audit in the evaluation process to be forced to resort to various modern tools and methods, which guarantee not only the risk assessment and the accuracy of the conclusions, but also efficiency of recommendations.

Parallel data processing for PCB testing

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PCBs (Printed Circuit Board) inspection and testing requires an analysis of some performance criteria for the data acquisition and processing system. These criteria are determined by the physical properties of the processes, which determine the concurrent propagation of the signals in the PCBs [1] and requires their acquisition and parallel processing [2]. In this case it is necessary to apply the methods and models of concurrent processing of multidimensional digital signals [3].
The major issue in the development of PCB testing systems is the spatial and time synchronization of data acquisition and processing [1, 2]. This problem can be solved by developing new discretization and signal acquisition methods and new mathematical models for their processing.
Let‘s consider the PCB board with a set of input signals $\boldsymbol{U}^{\mathrm{In}}$ and a set of output signals $\boldsymbol{U}^{\mathrm{Out}}$, where: $\boldsymbol{U}^{\mathrm{In}} = u_i^{\mathrm{In}}, i = \overline{1,N}$ with N test input points; and $\boldsymbol{U}^{\mathrm{Out}} = u_j^{\mathrm{Out}}, j = \overline{1,K}$ with K measuring test points. The mathematical model for measured test signals calculation according to the input test signals $\boldsymbol{U}^{\mathrm{In}}$ and the electrical parameters $\boldsymbol{Z}_j$ of the PCB is described by the following formula:

$$u_j^{\mathrm{Out}} = g_j\left(\boldsymbol{U}^{\mathrm{In}}, \boldsymbol{Z}_j\right).$$

The functional scheme for the PCB testing system consists of a test signal generator $\boldsymbol{U}^{\mathrm{In}}$ (designed on an FPGA circuit), a device for the injection of $\boldsymbol{U}^{\mathrm{In}}$ signals on the PCB and for obtaining the measured signals $\boldsymbol{U}^{\mathrm{Out}}$ from the PCB. The identification of the measured signals shape $\boldsymbol{U}^{\mathrm{Out}}$ is done by their differentiation $\dfrac{\mathrm{d}u_j^{\mathrm{Out}}}{\mathrm{d}t}$ and further integration $\int\limits_0^T \left(\dot{u}_j^{\mathrm{Out}}\right)\mathrm{d}t$. The differentiation function is performed on a variety of electronic differentiation and fuzzy circuits, the result of which is stored and transmitted to a computer. Parameter testing of the PCB takes place on the PC as a result of the analysis of the influence of the input signals $\boldsymbol{U}^{\mathrm{In}}$ on the output signals $\boldsymbol{U}^{\mathrm{Out}}$:

$$\frac{\partial u_j^{\mathrm{Out}}}{\partial u_i^{\mathrm{In}}}, i = \overline{1,N}, j = \overline{1,K}$$

and the mutual inflection of the output signals $\boldsymbol{U}^{\mathrm{Out}}$:

$$\frac{\partial u_j^{\mathrm{Out}}}{\partial u_l^{\mathrm{Out}}}, l = \overline{1,K}, j = \overline{1,K}.$$

Functional modeling and interaction of parallel data processing system components for PCB testing was performed on the basis of UML diagrams.

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Parallel algorithm to determine the Nash solutions in bimatrix games

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Given a bi-matrix game $\Gamma = \langle I, J, A, B\rangle$ where I – the row index set of the matrix, J – the column index set of the matrix, $||a_{ij}||_{i\in I}^{j\in J}$ and $||b_{ij}||_{i\in I}^{j\in J}$ are the playersâ€™ payoff matrices. We construct the following parallel algorithm to determinate equilibrium profiles.

1) Data parallelization. The MPI process with rank 0 initializes matrices A and B, then distributes, based on the 2D cyclic algorithm, the sub-matrices obtained by each MPI process from the virtual 2-dimensional topology communicator.Thus, each MPI process will only work with sub-matrices obtained as a result of distribution.

2) Each process from the virtual 2-dimensional topology communicator will eliminate, in parallel, from the sub-matrices of the matrices A and B that it possesses based on the step 1), those rows that are dominated in matrix A and those columns that are dominated in matrix B.

3) Determination of the equilibrium strategies for the matrix (A', B'), $A' = \left\|a'_{ij}\right\| \begin{matrix} i \in I' \\ j \in J' \end{matrix}$ and $B' = \left\|b'_{ij}\right\| \begin{matrix} i \in I' \\ j \in J' \end{matrix}$ obtained in step 2). It's clear that $|I'| \le |I|$ and $|J'| \le |J|$. For this using the reduction operations, each process will determine $i^*(j) = \arg\max_{i\in I'} a'_{ij}$ for any $j \in J'$ and $j^*(i) = \arg\max_{j\in J'} b'_{ij}$ for any $i \in I'$.

4) We select those index pairs that are simultaneously selected in both matrix A' and matrix B'. In other words, it is determined $\begin{cases} i^* \equiv i^*(j^*) \\ j^* \equiv j^*(i^*) \end{cases}$ which is actually the intersection of the graphs of the following point to set applications $i^*(\cdot)$ and $j^*(\cdot)$.

5) The equilibrium strategies for the game with the initial matrices A and B are built.

On the Coalitional Rationality and the Egalitarian Nonseparable Contribution

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In earlier works, we introduced the Inverse Problem, relative to the Shapley Value, then the Semivalues. In the explicit representation of the Inverse Set, the solution of the Inverse Problem, we built a family of games, called the almost null family, in which we determined more recently games for which the Shapley Value and the Egalitarian Allocations are coalitional rational. The Egalitarian Nonseparable Contribution, introduced by T.Driessen and Y.Funaki, is another value for cooperative transferable utilities games, showing how to allocate fairly the win of the grand coalition, in case that this has been formed. In the present paper, we solve the similar problem for this new value: given a vector L, representing the Egalitarian Nonseparable Comtribution of a TU game, find out a game in which this value is unchanged, but it is coalitional rational. The new game will belong to the family of almost null games in the Inverse Set, relative to the Shapley Value, and it is proved that the threshold of coalitional rationality will be higher than for the Shapley Value. The needed previous results are shown in the introduction, the second section is devoted to the main results, while in the last section are discussed remarks and connected problems. Some numerical examples are illustratuing the procedure of fimding the new game.

Keywords: Shapley Value, Egalitarian Nonseparable Contribution, Inverse Set, family of almost null games, Coalitional Rationality.

Solving 2D block-cyclic distribution bimatriceal games

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The block-cyclic data layout has been selected for the dense algorithms implemented in DMM parallel systems principally because of its scalability, load balance, and efficient use of computation routines. According to the two-dimensional block cyclic data distribution scheme, an m by n dense matrix is first decomposed into m_A by n_A blocks starting at its upper left corner. These blocks are then uniformly distributed in each dimension of the Process Grid. We consider the bimatrix game in the following strategic form $\Gamma = \langle I, J, A, B\rangle$,where $I = \{1, 2, .., n\}$ is the line index set, $J = \{1, 2, .., m\}$ is the column index set and $A = ||a_{ij}||_{i\in I}^{j\in J}$, $B = ||b_{ij}||_{i\in I}^{j\in J}$ are the payoff matrices of player 1 and player 2, respectively. The game is in complete and imperfect information. Suppose that the process can be referenced by its row and column coordinates, (c, l), within the grid $L \times C$. Denote by $A_{(c,l)} = \|a_{ij}\|_{i=\overline{1,|I_{(l,c)}|}}^{j=\overline{1,|J_{(l,c)}|}}$ and $B_{(c,l)} = \|b_{ij}\|_{i=\overline{1,|I_{(l,c)}|}}^{j=\overline{1,|J_{(l,c)}|}}$ submatrices from matrix A and B which are distributed to $(c, l) \in L \times C$ according to 2D- block cyclic algorithm. So we obtain the set of complete and imperfect subgames $\Gamma_{(c,l)} = \left\langle I_{(c,l)}, J_{(c,l)}, A_{(c,l)}, B_{(c,l)}\right\rangle$. Let $NE[\cdot]$ is the set of Nash equilibrium profiles in the bimatrix game. We can proof the following theorem.

Theorem. *Let* $\left(i^*_{(l,c)}, j^*_{(l,c)}\right) \in NE\left[\Gamma_{(c,l)}\right]$ *and the following conditions are fulfilled:*

1) *for all process* $(\widetilde{l}, c) \in L \times C, \widetilde{l} \neq l$ *for which* $\left(i^*_{(\widetilde{l},c)}, j^*_{(\widetilde{l},c)}\right) \in NE\left[\Gamma_{(c,\widetilde{l})}\right]$, $a_{i^*_{(l,c)}, j^*_{(l,c)}} \geq a_{i^*_{(\widetilde{l},c)}, j^*_{(\widetilde{l},c)}}$;
2) *for all process* $(l, \widetilde{c}) L \times C, \widetilde{c} \neq c$ *for which* $\left(i^*_{(l,\widetilde{c})}, j^*_{(l,\widetilde{c})}\right) \in NE\left[\Gamma_{(\widetilde{c},l)}\right]$, $b_{i^*_{(l,c)}, j^*_{(l,c)}} \geq b_{i^*_{(l,\widetilde{c})}, j^*_{(l,\widetilde{c})}}$.
Then $\left(\varphi\left(i^*_{(l,c)}\right), \psi\left(j^*_{(l,c)}\right)\right) \in NE[\Gamma]$ *where* $\varphi\left(i^*_{(l,c)}\right)$, *respectively* $\psi\left(j^*_{(l,c)}\right)$, *computes the global row, respectively column, index of a distributed matrices* $A_{(c,l)}, B_{(c,l)}$ *entry pointed to by the local index row* $i^*_{(l,c)}$, *respectively* $j^*_{(l,c)}$ *local index column, of the process indicated by* (l, c).

Applications of Discrete time semi-Markov processes in sequences of letters

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Keywords:Discrete time Semi-Markov, word occurrences

Let a discrete-time semi-Markov process $\{Z_\gamma; \gamma \geq 0\}$ with finite space an alphabet Ω. Defining the process $\{U_\gamma; \gamma \geq 0\}$ to be the backward recurrence times of the process $\{Z_\gamma; \gamma \geq 0\}$ we study the Markov process $\{(Z_\gamma, U_\gamma); \gamma \geq 0\}$. As an application we present results concerning word occurrences in a sequence of letters generated by a dicrete time semi Markov process.

Statistical Aspects Of Environmental Employees Registered In Employment Offices On 2017-2019:Considerations And Challenges

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Keywords:binary data analysis, categorical variables, contigency table, odds ratio, jobseeker, unemployed job seekers, employability

The paper performs a statistical analysis of some indicators that characterize the environment of jobseekers registered in the Employment Offices in Albania. The data being analysed relates to the period January 2017 - June 2019. The analysis is performed on the data extracted from the NES database (National Employment Service). A careful description is intended to reveal typical features and to recognize the situation and evolution over the years. Assessment combines a total analysis with a comparative analysis with indicators that affect the employability of the individual.

Mathematical models for Risk Management Options

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An option is a conditional term contract that gives the holder the right to buy or sell a support asset at a predetermined price, called exercise price or strike price, that is valid for any moment, during the period or on the expiration date of the optional contract.

The options, as financial assets, give the possibility of reversibility of the capital investment. That is, an investor who buys options has the possibility as long as the option is valid or even at maturity, either keep this asset or give it away if an opportunity arises advantageous investment. Basically, the options give the possibility of arbitrage between the holding of the underlying assets that they represent and the derivative assets represented by the respective options for speculation or the market risk coverage.

There are two main types of options call and put. As regional types of options we can distinguish: European, American and, so called, exotic options. Exotic options are a category of options contracts that differ from traditional options in their payment structures, expiration dates, and strike prices. The underlying asset or security can vary with exotic options allowing for more investment alternatives.

The only Romanian stock exchange on which options are traded is the Sibiu Monetary and Commodity Exchange, these products being launched in 1998. Options on futures contracts traded in Sibiu offer investors new trading opportunities through which each of them can express and manage their financial interests more securely and broadly.

We analyze the mathematical models for the American and European call and put options, using the binomial model. We apply the general scheme for other exotic types of options and try to show how we can manage the risk for various options.

Optimization by the General Efficiency

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This research work is concerned with the study of the General Efficiency and Optimization. We present the Efficiency and Optimization in their most natural context offered by the Infinite Dimensional Ordered Vector Spaces, following our recent results on these subjects. Implications and Applications in Vector Optimization through of the agency of Isac's Cones and the new link between the General Efficiency and the Strong Optimization by the Full Nuclear Cones are presented. An important extension of our Coincidence Result between the Efficient Points Sets and the Choquet Boundaries is developed . In this way, the Efficiency is connected with Potential Theory by Optimization and conversely. Several pertinent references conclude this investigation.

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The "bottleneck" multi-objective fractional transportation problem with fuzzy criteria

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In this paper is developed an algorithm for solving the multi-criteria fractional transportation with the same bottleneck denominators, additionally, including separately this criterion. In the model all of criteria are of fuzzy type. It generates for each feasible time value the best compromise multi criteria solution. So, finally, we will obtain one finite set of efficient solutions for solving the multi-criteria fractional transportation problem with the same bottleneck denominators, separately including the time "bottleneck" criterion. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} c_{ij}^k \tilde{x}_{ij}}{\max\limits_{ij} \tilde{t}_{ij} \left| x_{ij} > 0 \right\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij} \tilde{t}_{ij} \left| \tilde{x}_{ij} > 0 \right\} \tag{2}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} = a_i,\, i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} \tilde{x}_{ij} = a_i, j = 1, 2, \ldots, n; \tag{3}$$

$$\tilde{x}_{ij} \geq 0,\, i = 1, 2, \ldots, m,\, j = 1, 2, \ldots, n,\, k = 1, 2, \ldots, r. \tag{4}$$

The main idea of the algorithm is based on the use of a probabilistic "optimistic" coefficient for reducing the model (1) - (4) to a deterministic one [2]. It generates for every time possible value the corresponding " best compromise solution " of the first k criteria [1]. The algorithm was tested on several examples and was found to be quite effective.

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On a conjecture concerning the Broken Stick Model

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Let $(X_k)_{k\geq 1}$ be a sequence of iid random variables on the unity interval $I = [0,1]$. Suppose that their distribution has a density f with respect to Lebesgue measure.
Sort ascending the random vector $(X_1, X_2, ..., X_n)$ as $\left(0 = X_{(0,n)} \leq X_{(1,n)} \leq X_{(2,n)} \leq ... \leq X_{(n,n)}\right)$ and consider the lags $Y_{j,n} = X_{(j,n)} - X_{(j-1,n)}, 1 \leq j \leq n$.
The vector $\boldsymbol{Z}_n = (nY_{j,n})_{1\leq j\leq n}$ is a broken stick model.
The problem is: what happens for great n?
Do the empirical distributions of $\boldsymbol{Z}_n$ have a limit? It is known that if X_k are unifromly distributed, the the limit is the exponential distribution $Exp\,(1)$.
We prove that the limit does exist if f is continuous and, more than that the Lorenz curve of this limit, L_f is always below the Lorenz curve of the exponential distribution. That is we prove that $L_f\,(p) \leq p + (1-p)\ln(1-p)$ for $p \in [0,1]$.The meaning is that in a broken stick model the uniform distribution is the most egalitarian at least in the set of all the distributions on $[0,1]$ with continuous density.
We believe that this optimality property of the uniform distribution holds no matter of the density f.

6. Algebra, Logic, Geometry (with applications)

On Duflo's conjecture in algebraic geometry

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joint work with N. Buruiana and D. Cheptea

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We apply the Lie theoretic formalism of Calaque-Caldararu-Tu, to some extension problems of vector bundles to the first infinitesimal neighborhood of a subvariety in the complex projective space.

Kirichenko–Uskorev structures on hypersurfaces of Kählerian manifolds

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Dedicated to Professor Lidia V. Stepanova on her jubilee

1.. Let N be an odd-dimensional smooth manifold, η be a differential 1-form called a contact form, ξ be a vector field called a characteristic vector, Φ be an endomorphism of the module of smooth vector fields on N called a structure endomorphism. In this case the triple $\{\Phi,\, \xi,\, \eta\}$ is called an almost contact structure on the manifold N if the following conditions are fulfilled:

$$1)\,(\xi) = 1,\; 2)\,\Phi(\xi) = 0,\; 3)\,\eta \circ \Phi = 0,\; 4)\,\Phi^2 = -id + \xi \otimes \eta.$$

If in addition there is a Riemannian metric $\langle \cdot, \cdot \rangle$ on the manifold N such that

$$\langle \Phi X, \Phi Y \rangle = \langle X, Y \rangle - \eta\,(X)\,\eta\,(Y)\,, \quad X, Y \in \aleph(N),$$

then the tensor system $\{\Phi,\, \xi,\, \eta,\, g\}$ is called an almost contact metric structure on this manifold [1].
As the most important examples of almost contact metric structures we can consider the cosymplectic structure, the nearly cosymplectic structure and the Kenmotsu structure. The cosymplectic structure is characterized by the following condition:

$$\nabla \eta = 0,\; \nabla \Phi = 0,$$

where ∇ is the Levi-Civita connection of the metric. An almost contact metric structure $\{\Phi,\, \xi,\, \eta,\, g\}$ is called nearly cosymplectic, if the following condition is fulfilled [1]:

$$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = 0,\;\; X, Y \in \aleph(N).$$

In 1972 K. Kenmotsu has introduced a class of almost contact metric structures, defined by the condition

$$\nabla_X(\Phi)Y = \langle \Phi X, Y \rangle\, \xi - \eta(Y)\Phi X,\; X, Y \in \aleph(N).$$

We note that the cosymplectic, nearly cosymplectic and Kenmotsu structures have many remarkable properties and play an important role in contact geometry. We mark out that the fundamental monographs by V.F. Kirichenko [1] and Gh. Pitiş [2] contain a detailed description of above mentioned almost contact metric structures.

2. In [3], V. F. Kirichenko and I. V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type.

V. F. Kirichenko and I. V. Uskorev have also proved that the structure of cosymplectic type is invariant under canonical conformal transformations. We recall also that a conformal transformation of an almost contact metric structure $\{\Phi, \xi, \eta, g\}$ on the manifold N is a transition to the almost contact metric structure $\{\tilde{\Phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g}\}$, where $\tilde{\Phi} = \Phi$, $\tilde{\xi} = e^f \xi$, $\tilde{\eta} = e^{-f}\eta$ and $\tilde{g} = e^{-2f}g$. Here f is a smooth function on the manifold N [3].

Evidently, a trivial example of Kirichenko–Uskorev (KU-) structure is the cosymplectic structure. As a non-trivial example of the almost contact metric KU-structure we can consider the Kenmotsu structure. We also note that the nearly cosymplectic structure is not Kirichenko¡96¿Uskorev.

3. We consider almost contact metric structures induced on oriented hypersurfaces of a Kählerian manifold of dimension at least six [4]. Our main result is the following Theorem.

Theorem. *Kirichenko-Uskorev structures induced on oriented hypersurfaces of a Kählerian manifold of dimension at least six are necessarily cosymplectic.*

Taking into account that the Kenmotsu structure is not cosymplectic, we obtain the following statement.

Corollary. *Kirichenko-Uskorev structures induced on oriented hypersurfaces of a Kählerian manifold of dimension at least six cannot be Kenmotsu structures.*

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On *acm*-structures on hypersurfaces of Hermitian manifolds

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Dedicated to Professor Lidia V. Stepanova on her jubilee

1. It is known that almost contact metric (*acm*-) structures are induced on oriented hypersurfaces of almost Hermian manifolds. In accordance with the opinions of many specialists, namely this fact determines the profound connection between the contact and Hermitian geometries. Almost contact metric structures on hypersurfaces of almost Hermitian manifolds were studied by such remarkable geometers as D. E. Blair, S. Goldberg, S. Ishihara, V. F. Kirichenko, L. V. Stepanova, S. Sasaki, H. Yanamoto and K. Yano.

We remind that an almost contact metric structure on an odd-dimensional manifold N is defined by quardruple of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $\langle\cdot,\cdot\rangle$ is the Riemannian metric [1]. Moreover, the following conditions are fulfilled:

$$\eta(\xi)=1,\ \Phi(\xi)=0,\ \eta\circ\Phi=0,\ \Phi^2=-id+\xi\otimes\eta,$$

$$\langle\Phi X,\Phi Y\rangle=\langle X,Y\rangle-\eta\left(X\right)\eta\left(Y\right),\quad X,Y\in\aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N.

We also remind that the almost Hermitian manifold is an even-dimensional manifold M^{2n} with a Riemannian metric $g=\langle\cdot,\cdot\rangle$ and an almost complex structure J. These objects must satisfy the following condition

$$\langle JX,\ JY\rangle=\langle X,\ Y\rangle\,,\quad X,Y\in\aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on the manifold M^{2n} [1], [2]. An almost Hermitian manifold is called Hermitian if its almost complex structure is integrable [1], [2].

2. Approximately 25 years ago, the first group of Cartan structural equations of an arbitrary *acm*-structure induced on an oriented hypersurface of an almost Hermitian manifold was obtained by L. V. Stepanova [3]. In the present communication, we make more precise these Cartan structural equations for some important cases.

Theorem 1. *The first group of Cartan structural equations of an arbitrary acm-structure induced on an oriented totally umbilical hypersurface of a Hermitian manifold is the following:*

$$\begin{aligned}
d\omega^\alpha &= \omega^\alpha_\beta\wedge\omega^\beta+B^{\alpha\beta}{}_\gamma\omega^\gamma\wedge\omega_\beta+\left(\sqrt{2}\,B^{\alpha n}{}_\beta+i\lambda\delta^\alpha_\beta\right)\omega^\beta\wedge\omega-\\
&\quad-\frac{1}{\sqrt{2}}B^{\alpha\beta}{}_n\omega_\beta\wedge\omega,\\
d\omega_\alpha &= -\omega^\beta_\alpha\wedge\omega_\beta+B_{\alpha\beta}{}^\gamma\omega_\gamma\wedge\omega^\beta+\left(\sqrt{2}\,B_{\alpha n}{}^\beta-i\lambda\delta^\alpha_\beta\right)\omega_\beta\wedge\omega-\\
&\quad-\frac{1}{\sqrt{2}}B_{\alpha\beta}{}^n\omega^\beta\wedge\omega,\\
d\omega &= \left(\sqrt{2}B^{n\alpha}{}_\beta-\sqrt{2}B_{n\beta}{}^\alpha-2i\lambda\delta^\alpha_\beta\right)\omega^\beta\wedge\omega_\alpha+B_{n\beta}{}^n\omega\wedge\omega^\beta+B^{n\beta}{}_n\,\omega\wedge\omega_\beta.
\end{aligned}\tag{1}$$

Theorem 2. *The first group of Cartan structural equations of an arbitrary -structure induced on an oriented totally geodesic hypersurface of a Hermitian manifold is the following:*

$$d\omega^\alpha=\omega^\alpha_\beta\wedge\omega^\beta+B^{\alpha\beta}{}_\gamma\omega^\gamma\wedge\omega_\beta+\sqrt{2}B^{\alpha n}{}_\beta\,\omega^\beta\wedge\omega-\frac{1}{\sqrt{2}}B^{\alpha\beta}{}_n\omega_\beta\wedge\omega;$$

$$d\omega_\alpha=-\omega^\beta_\alpha\wedge\omega_\beta+B_{\alpha\beta}{}^\gamma\omega_\gamma\wedge\omega^\beta+\sqrt{2}B_{\alpha n}{}^\beta\,\omega_\beta\wedge\omega-\frac{1}{\sqrt{2}}B_{\alpha\beta}{}^n\omega^\beta\wedge\omega;$$

$$d\omega=\left(\sqrt{2}B^{n\alpha}{}_\beta-\sqrt{2}B_{n\beta}{}^\alpha\right)\omega^\beta\wedge\omega_\alpha+B_{n\beta}{}^n\omega\wedge\omega^\beta+B^{n\beta}{}_n\omega\wedge\omega_\beta.$$

Here $\left\{B^{ab}{}_c\right\}$ and $\{B_{ab}{}^c\}$ are the components of Kirichenko tensors of the Hermitian structure on the manifold M^{2n} [4]; σ is the second fundamental form of the immersion of the hypersurface N into M^{2n}; $\alpha,\ \beta,\ \gamma=1,\ldots,n-1$.

We remark that the structural equations (1) and (2) are also relevant for *acm*-structures induced on oriented hypersurfaces of Kahlerian, locally conformal Kählerian and special Hermitian manifolds (i.e. of almost Hermitian manifolds of Gray-Hervella classes [2] that are contained in the class of Hermitian manifolds).

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The τ, σ-dual subcategories

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In the category $\mathcal{C}_2\mathcal{V}$ of the local convex topological vectorial Hausdorff spaces are studied the lattice of $\mathcal{S}$-semireflexive subcategories [1], where $\mathcal{S}$ is the subcategory of spaces with weak topology. Among such subcategories we mention the $s\mathcal{R}$ subcategories of semireflexive spaces, the $\mathcal{B}$-$i\mathcal{R}$ of $\mathcal{B}$-inductive semireflexive spaces, $l\Gamma_0$ - of locally complete spaces, Π - the subcategory of complete spaces with a weak topology.
Let $\mathbb{R}^s_f(\varepsilon\mathcal{S})$ be the lattice of $\mathcal{S}$-semireflexive subcategories, $\mathbb{K}(\widetilde{\mathcal{M}})$ - the lattice of non-nulle coreflective subcategories that are included the subcategory $\widetilde{\mathcal{M}}$ of spaces with Mackey topology and $\mathbb{R}(\mathcal{M}_p)$ the lattice of reflective subcategories that are included the subcategory Γ_0 of complete spaces. Regarding the terminology see [3]. With the help of the contravariant functor $d_\tau : \mathcal{C}_2\mathcal{V} \to \mathcal{C}_2\mathcal{V}$, $d_\tau X = X'_\tau$ is defined the application $\delta : \mathbb{R}^s_f(\varepsilon\mathcal{S}) \to \mathbb{K}(\widetilde{\mathcal{M}})$, where for $\mathcal{R} \in \mathbb{R}^s_f(\varepsilon\mathcal{S})$ $\delta(\mathcal{R})$ is the subcategory defined by the objects $\{d_\tau X | X \in |\mathcal{R}|\}$. The application $\gamma : \mathbb{K}(\widetilde{\mathcal{M}}) \to \mathbb{R}^s_f(\varepsilon\mathcal{S})$ it is thus defined: for $\mathcal{T} \in \mathbb{K}(\widetilde{\mathcal{M}})$ $\gamma(\mathcal{T})$ is the subcategory defined by the objects $\{X \in |\mathcal{C}_2\mathcal{V}|, d_\tau X \in |\mathcal{T}|\}$. The applications δ and γ were defined by M. M. Buneacov to apply the subcategories $\mathcal{R}$, $\delta(\mathcal{R})$, $\mathcal{T}$ and $\gamma(\mathcal{T})$ at the demonstration of the exponential law and the closed graph(see [2]).

Theorem. 1. *The applications δ and γ are reciprocal inverse.*

2. *Let $\mathcal{T} \in \mathbb{K}(\widetilde{\mathcal{M}})$ be. $\gamma(\mathcal{T}) \in \mathbb{R}(\mathcal{M}_p)$ iff for any objet $X \in |\mathcal{C}_2\mathcal{V}|$ absolutely convex and weakly compact sets in the spaces X and tX coincide, $t^X : tX \to X$ is the $\mathcal{T}$-coreplique of X.*

3. *$l\Gamma_0$ is the smallest element of the latice $\mathbb{R}^s_f(\varepsilon\mathcal{S}) \cap \mathbb{R}(\mathcal{M}_p)$.*

4. *Restrictions of the applications δ and γ establishes an isomorphism of the latices $\mathbb{K}(\delta(l\Gamma_0), \widetilde{\mathcal{M}})$ and $\mathbb{R}^s_f(\varepsilon\mathcal{S}) \cap \mathbb{R}(\mathcal{M}_p)$, where $\mathbb{K}(\delta(l\Gamma_0), \widetilde{\mathcal{M}}) = \{\mathcal{T} \in \mathbb{K}(\widetilde{\mathcal{M}}) | \delta(l\Gamma_0) \subset \mathcal{T} \subset \widetilde{\mathcal{M}}\}$.*

$$\mathbb{K}(\delta(l\Gamma_0), \widetilde{\mathcal{M}}) \underset{\delta}{\overset{\gamma}{\rightleftarrows}} \mathbb{R}^s_f(\varepsilon\mathcal{S}) \cap \mathbb{R}(\mathcal{M}_p).$$

5. *For $\mathcal{T} \in \mathbb{K}(\widetilde{\mathcal{M}})$ the pairs of subcategories $(\mathcal{T}, \gamma(\mathcal{T}) \cap \mathcal{S})$, $(\mathcal{T}, \mathcal{S}_{\varepsilon\mathcal{S}}(\gamma\mathcal{T}))$, $(\mathcal{S} \cap \mathcal{S}_{\varepsilon\mathcal{S}}(\mathcal{T}), \mathcal{S}_{\varepsilon\mathcal{S}}(\gamma\mathcal{T}))$ and $(\mathcal{S} \cap \mathcal{S}_{\varepsilon\mathcal{S}}(\mathcal{T}), \gamma(\mathcal{T}) \cap \mathcal{S})$ are dual isomorphic.*

Definition. *For $\mathcal{T} \in \mathbb{K}(\widetilde{\mathcal{M}})$ pairs of subcategories that appear in the p.5 of Theorem is caled (τ, σ)-duale.*

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On Topological Subtractive Groupoids with Multiple Identities

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A non-empty set G is said to be a *groupoid* relatively to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.
If the groupoid G is a topological space and the multiplication operation $(a, b) \to a \cdot b$ is continuous, then G is called a topological groupoid.
An element $e \in G$ is called a right identity if $xe = x$ for every $x \in G$.
A groupoid $(G, \cdot)$ is called a *subtractive* groupoid if it satisfies the law $b \cdot (b \cdot a) = a$ and $a \cdot (b \cdot c) = c \cdot (b \cdot a)$ for all $a, b, c \in G$.
Consider a groupoid $(G, +)$. For every two elements a, b from $(G, +)$ we denote:

$$1(a, b, +) = (a, b, +)1 = a + b, \text{ and } n(a, b, +) = a + (n-1)(a, b, +),\ (a, b, +)n = (a, b, +)(n-1) + b$$

for all $n \geq 2$.
If a binary operation $(+)$ is given on a set G, then we shall use the symbols $n(a, b)$ and $(a, b)n$ instead of $n(a, b, +)$ and $(a, b, +)n$.
Let $(G, +)$ be a groupoid and let $n, m \geq 1$. The element e of the groupoid $(G, +)$ is called:
- an (n, m)-zero of G if $e + e = e$ and $n(e, x) = (x, e)m = x$ for every $x \in G$;
- an $(n, \propto)$-zero if $e + e = e$ and $n(e, x) = x$ for every $x \in G$;
- an $(\propto, m)$-zero if $e + e = e$ and $(x, e)m = x$ for every $x \in G$.

Clearly, if $e \in G$ is both an $(n, \propto)$-zero and an $(\propto, m)$-zero, then it is also an (n, m)-zero. If $(G, \cdot)$ is a multiplicative groupoid, then the element e is called an (n, m)-identity.
Let $(G, +)$ be a topological groupoid. A groupoid $(G, \cdot)$ is called a homogeneous isotope of the topological groupoid $(G, +)$ if there exist two topological automorphisms $\varphi, \psi : (G, +) \to (G, +)$ such that $x \cdot y = \varphi(x) + \psi(y)$, for all $x, y \in G$.
For every mapping $f : X \to X$ we denote $f^1(x) = f(x)$ and
$f^{n+1}(x) = f(f^n(x))$ for any $n \geq 1$.
Definition 1. Let $n, m \leq \infty$. A groupoid $(G, \cdot)$ is called an (n, m)-homogeneous isotope of a topological groupoid $(G, +)$ if there exist two topological automorphisms $\varphi, \psi : (G, +) \to (G, +)$ such that:
1. $x \cdot y = \varphi(x) + \psi(y)$ for all $x, y \in G$;
2. $\varphi\psi = \psi\varphi$;
3. If $n < \infty$, then $\varphi^n(x) = x$ for all $x \in G$;
4. If $m < \infty$, then $\psi^m(x) = x$ for all $x \in G$.

A groupoid $(G, \cdot)$ is called an isotope of a topological groupoid $(G, +)$, if there exist two homeomorphisms $\varphi, \psi : (G, +) \to (G, +)$ such that $x \cdot y = \varphi(x) + \psi(y)$ for all $x, y \in G$..

Under the conditions of Definition 1 we shall say that the isotope $(G, \cdot)$ is generated by the homeomorphisms φ, ψ of the topological groupoids $(G, +)$ and write $(G, \cdot) = g(G, +, \varphi, \psi)$.

Some interesting results regarding isotopies of a topological groupoid were obtained in [1,2,3,4]. The notion of (n, m)-identity was introduced in [2].

Theorem 1. *If $(G, +)$ is a subtractive groupoid and e is an $(p, 1)$-zero, then every $(1, n)$-homogeneous isotope $(G, \cdot)$ of the topological groupoid $(G, +)$ is subtractive groupoid with $(np, 1)$-identity e in $(G, \cdot)$ and $a \cdot b + c = (a + c) \cdot b$, for all $a, b, c \in G$ and $n, p \in N$.*

Theorem 2. *If $(G, +)$ is a subtractive groupoid and e is a right zero, then every $(1, 1)$-homogeneous isotope $(G, \cdot)$ of the topological groupoid $(G, +)$ is subtractive groupoid with right identity e in $(G, \cdot)$ and $a \cdot b + c = (a + c) \cdot b$, for all $a, b, c \in G$.*

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On generalization of expressibility and completeness in super-intuitionistic logics

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By a pseudo-Boolean algebra we understand a system $\mathfrak{A} = \langle M; \&, \vee, \supset, \neg \rangle$ which is a lattice with pseudo-complement $\neg$ and relative pseudo-complement $\supset$ over & and $\vee$. Following A. Kuznetsov, the function f of algebra $\mathfrak{A}$ is called *parametrically expressed* via a system of functions Σ of $\mathfrak{A}$, if there exists the functions $g_1, h_1, ..., g_r, h_r$, which are expressed explicitly via Σ using superpositions, such that the predicate $f(x_1...x_n) = x_{n+1}$ is equivalent to $\exists t_1...\exists t_l ((g_1 = h_1) \wedge ... \wedge (g_r = h_r))$ on $\mathfrak{A}$.

We examine the pseudo – Boolean algebras

$$Z_m = \langle \{0,\ \tau_1,\ \tau_2, ...,\ \tau_{m-2},\ 1\}, \Omega \rangle,$$

where $\Omega = \{\&, \vee, \supset, \neg\}$, $0 < \tau_1 < \tau_2 < ... < \tau_{m-2} < 1$ and LZ_m denotes the set of valid formulas, i.e. the logic of Z_m.

Evidently the algebras $Z_2 = \langle \{0, 1\}; \Omega \rangle$; $Z_3 = \langle \{0,\ \tau_1,\ 1\}; \Omega \rangle$ is a subalgebra of $Z_4 = \langle \{0,\ \tau_1,\ \tau_2,\ 1\}; \Omega \rangle$; $Z_5 = \langle \{0,\ \rho, \sigma, \tau_1,\ 1\}; \Omega \rangle$ and

$Z_6 = \langle \{0, \rho, \sigma, \tau_1, \tau_2, 1\}; \Omega \rangle$, where $0 < \rho < \tau_1 < \tau_2 < 1; 0 < \sigma < \tau_1 < \tau_2 < 1$ and $\rho,\ \sigma$ are incomparable elements.

Let us consider: $x \sim y = (x \supset y)\&(y \supset x)$;

$\rfloor 0 = \rfloor \tau_1 = 1,\ \rfloor 1 = 0;\ f(0) = 0,\ f(\tau_1) = f(\tau_2) = \tau_2,\ f(1) = 1;\ g(0) = 0,$
$g(\tau_1) = g(1) = 1,\ g(\tau_2) = \tau_2; h(0) = 0, h(\rho) = \sigma, h(\sigma) = \rho, h(\tau_1) = h(1) = 1.$
The set of all the functions from algebra $\mathfrak{A}$, permutabled with a given function f is referred to us centralizer of the function f (denoted $\langle f \rangle$) on algebra $\mathfrak{A}$. The system Σ of formulas is called parametrically complete in logic LZ_m, if each formula is parametrically expressed in LZ_m via the system Σ.

Theorem. *A system of formulas Σ is parametrically complete in the logic LZ_5 and LZ_6, iff the system Σ is not included into the next centralizers:*

$$\langle 0 \rangle,\ \langle 1 \rangle,\ \langle x\&y \rangle,\ \langle x \vee y \rangle,\ \langle \neg x \rangle \ \textit{and}\ \langle x \sim (y \sim z) \rangle \ \textit{on}\ Z_2;$$

$$\langle \rfloor\rfloor x \rangle,\ \langle \neg\neg x \& (x \vee \neg y) \rangle,\ \langle \neg\neg x \& (x \vee \neg\neg y) \rangle \ \textit{and}\ \langle \neg\neg x \& (x \vee y \vee \neg y) \rangle \ \textit{on}\ Z_3;$$

$$\langle f \rangle \ \textit{and}\ \langle g \rangle \ \textit{on}\ Z_4;$$

$$\langle h \rangle \ \textit{on}\ Z_5.$$

The Padovan-Jacobsthal Numbers Modulo m

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The Padovan-Jacobsthal sequence and the Padovan-Jacobsthal matrix were defined by Deveci (see [1]). In this work, we consider the cyclic groups which are generated by the multiplicative orders of the the Padovan-Jacobsthal matrix when read modulo m. Also, we study the the Padovan-Jacobsthal numbers modulo m and then we obtain the relationship among the periods of the Padovan-Jacobsthal numbers modulo m and the orders of the cyclic groups obtained. In [2] and [3], the Jabosthal and Padovan numbers modulo m were studied, respectively. Now we discuss connections between the period of the Padovan-Jacobsthal numbers modulo m and the periods of the Jabosthal and Padovan numbers modulo m.

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Fixed Point Theormes in Spaces with Distance

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We establish fixed point theorems for two classes of discontinous mappings (Kannan type mappings, Bianchini type mappings) in the very general setting of a space with a distance, thus extending some results in the paper [3]. We also indicate some particular cases of our main results and present some examples to illustrate the theoretical results and show that our generalizations are effective.

Theorem 1. *Let (X, d) be a complete $H-distance$ space and let $T : X \to X$ be a mapping for wich there exists $0 < a < 1/2$ such that:*

$$d(Tx, Ty) \leq a\left(d(x, Tx) + d(y, Ty)\right), (\forall)x, y \in X.$$

Then the Picard iteration at the any point $x \in X$ is convergent. If, additionally, the limit $\overline{x}$ of the Picard sequence is a fixed point of T , then $\overline{x}$ is the unique fixed point of T.

By working in the general setting of a complete $H-distance$ space, we ob- tained significant generalizations of Kannan and Bianchini fixed point theorems in usual metric spaces.

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Finite cristallographic pseudo-minor groups of W_p-symmetry

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Ascribe to each point of the geometrical figure F with finite symmetry group G at least one "index", which means a non-geometrical feature, from the set $N = \{1, 2, ..., m\}$, and fix a certain transitive group P of the permutations of these "indexes". Let each "index" r from N have a scalar nature. We construct the direct product W of isomorphic copies of the group P which are indexed by elements of G, i.e. $W = \prod_{g_i \in G} P^{g_i}$, where $P^{g_i} \cong P$. The finite discret groups

$G^{(W_p)}$ of W_p-symmetry [1,2] are subgroups of left direct wreath product of initial group P with the generating group G, accompanied with a fixed isomorphism $\varphi : G \to AutW$ by the rule $\varphi(g) = \overleftarrow{g}$, where $\overleftarrow{g}: w \mapsto w^g$. The group $G^{(W_p)}$ with initial group P, generating group G, subset $W' = \{w|g^{(w)} \in G^{(W_p)}\}$, symmetry subgroup H and the subgroup V of W-identical transformations is called W'-pseudo-minor group, if $w_0 = V \subset W' \subset W$, but W' is not a group.
Any W'-pseudo-minor finite group of W_p-symmetry with initial group P, generating group G and symmetry subgroup H can be derived from G and P by the following steps: 1) we construct the left direct product W of isomorphic copies of the group P, indexed by elements of G; 2) we find in W so subset W' with unit (W' is not a subgroup), which verify the condition $\overleftarrow{g}\ (W')W' = W'$, for each g from group G; 3) we construct an exact natural left quasi-homomorphism μ with the kernel H [2] of the group G onto the subset W' by the rule $\mu(Hg) = w$; 4) we combine pairwise each g of class Hg with $w = \mu(Hg)$; 5) we introduce into the set of all these pairs the operation $g_i w_i \circ g_j w_j = g_k w_k$, where $g_k = g_i g_j$, $w_k = w_i^{g_j} w_j$ and $w_i^{g_j}(g_s) = w_i(g_j g_s)$.
From the non trivial crystallographic punctual groups G of order 2,3,4,6 (cyclic and non cyclicals) and group P ($P \cong C_2$), we obtained 41 pseudo-minor groups of W_p-symmetry.
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About interpretation of some classical problems of geometry in indefinite relativistic metric

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In this paper we give some geometry problems adapted to a "hyperbolic" context. For example, we will give a result similar to Pithot's theorem for hyperbola and an interpretation in the indefinite relativistic metric.

A note on a Diophantine exponential equation

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One important topic in number theory is the study of Diophantine equations, equations in which only integer solutions are permitted. The field of Diophantine equations is ancient, vast,

and no general method exist to decide whether a given Diophantine equation has any solutions, or how many solutions. The famous general equation $p^x + q^y = z^2$ has many forms. The literature contains a very large number of articles on non-linear such individual equations involving particular primes and powers of all kinds.
In this paper, we study equations: $2^x + 41^y = z^2$, $2^x + 89^y = z^2$, $2^x + 97^y = z^2$, $2^x + 83^y = z^2$, where x, y, z are non-negative integer numbers. One could cite here many articles on the equation $p^x + q^y = z^2$. We provide here only a small number of related equations which include the prime 2 in particular, such as [1,2,3,4,5,6].
Theorem 1. *The exponential diophantine equation* $2^x + 41^y = z^2$ *has exactly three integer solutions* (x, y, z)*:* $\{(3, 0, 3),\ (3, 1, 7),\ (7, 1, 13)\}$.
Theorem 2. *The exponential diophantine equation* $2^x + 89^y = z^2$ *has exactly three integer solutions* (x, y, z)*:* $\{(3, 0, 3),\ (5, 1, 11),\ (13, 1, 91)\}$.
Theorem 3. *The exponential diophantine equation* $2^x + 97^y = z^2$ *has exactly two integer solutions* (x, y, z)*:* $\{(3, 0, 3),\ (7, 1, 15)\}$.
Theorem 4. *The exponential diophantine equation* $2^x + 83^y = z^2$ *has exactly one integer solution* (x, y, z)*:* $\{(3, 0, 3)\}$.

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The Holder theorem for monoasociative right-ordered loops

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Right-ordered loop is an algebraic system $(L, \cdot, \backslash, /, \leq)$ with three binary operations $\cdot$, $\backslash$, $/$ called multiplication, left-hand division, right-hand division, and binary relationship $\leq$ so that $(L, \cdot, \backslash, /)$ is a loop again $(L, \leq)$ is a ordered set, at the same time in $(L, \cdot, \backslash, /, \leq)$ the following quasi-identities are true:

$$x \leq y \Rightarrow x \cdot z \leq y \cdot z;$$
$$x \leq y \Rightarrow x/z \leq y/z.$$

An right-ordered L bit is called archimedes if for any two positive elements $x, y \in L$ there is a natural number n, such that $x^n > y$.

Lemma 1. In an right-ordered loop L is true quasi-identity:

$$x < y \,\&\, z < t \Rightarrow x/t < y/z.$$

Lemma 2. *If in an right-ordered monoasociative loop L is arhimedian and there is the smallest positive element a, then L is the cyclic group generated by element a.*

Lemma 3. *Any right-ordered monoasociative and archimedian loop L that does not contain the smallest positive element checks the condition: for any positive element $a > e$ of L and any natural number n exists in L a positive element $c > e$ such that $c^n < a$.*

Theorem (Hölder's generalized). *The right-ordered monoasociative and archimedian loop is the Abelian group, and therefore is isomorphic ordered to a subgroup of the additive group of real numbers.*

Corollary. *If a right-ordered loop is a Moufang loop or automorphic loop, then it is an abelian group.*

7. Computer Science

Algorithms of spline-collocation and spline-quadratures methods for solving integral equations of the second kind

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In this paper, we present new steps in construction of algorithms for solving integral equations of the second kind (see Abstracts of CAIM-2017 and CAIM-2018) based on:
– approximation of the continuous function with convex and concave basic second order splines;
– algorithms of the spline-collocations and spline-quadratures methods for solving the Fredholm and Volterra IE of the second kind, which use convex and concave second order splines as basic functions;
– theoretical substantiation of the developed computing algorithms, in the space of continuous functions, proceeding from function approximation results, using convex and concave basic second order splines;
– extension of the database BKP_IE_COL (see [1]) of Intelligent Support System (ISS), destined for solving the Fredholm and Volterra integral equations (IE) of the second kind (ISS_IE), with spline-collocations method, using the convex and concave second order splines as basic functions.

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On the Annihilation Number of a Bipartite Graph

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If $\alpha(G) + \mu(G) = |V|$, then $G = (V, E)$ is a *König-Egerváry graph*, where $\alpha(G), \mu(G)$ stand for the size of a maximum independent set, and the size of a maximum matching, respectively. For instance, each bipartite graph is a König-Egerváry graph.
Let $d_1 \leq d_2 \leq \cdots \leq d_{|V|}$ be the degree sequence of the graph $G = (V, E)$. Pepper defined the *annihilation number* $h(G)$ as the largest integer k such that $\sum\limits_{i=1}^{k} d_i \leq |E|$, [4]. For $A \subseteq V$, let $\deg(A) = \sum\limits_{v \in A} \deg(v)$. Every $A \subseteq V$ satisfying $\deg(A) \leq |E|$ is called an annihilating set. An annihilating set A is *maximal* if $\deg(A \cup \{v\}) > |E|$, for every $v \in V - A$, and it is *maximum* if $|A| = h(G)$, [4]. The relation between the annihilation number and various parameters of a graph were studied, for example, in [1, 2].

Theorem 1. *[3] For a graph G with $h(G) \geq \frac{|V|}{2}$, $\alpha(G) = h(G)$ if and only if G is König-Egerváry and every maximum independent set is* ***maximum*** *annihilating.*

Conjecture 1. *[3] Let G be a graph with $h(G) \geq \frac{|V|}{2}$. Then $\alpha(G) = h(G)$ if and only if G is König-Egerváry and every maximum independent set is* ***maximal*** *annihilating.*

In this talk we disprove Conjecture 1, by proving the following.

Theorem 2. *There exist a bipartite graph G of order $2k+9, k \geq 0$ and a bipartite of order $2k+8, k \geq 0$, such that $h(G) \geq \frac{|V|}{2}$ and each maximum independent set of G is a* ***maximal non-maximum*** *annihilating set.*

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Solving of some logical mathematical problems by means of Petri Nets

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The aim of this paper is to perform theoretical research with practical applicability [1] that will contribute to solving some logical mathematical problems by applying the Petri nets [4] formalism. Their facilities offers a new approach that allows the solution to be determined by a method different from the existing ones, but which is quite suggestive and easy to understand.
Problems solving are possible by applying the Petri nets verification methods [3] like reachability graph [2]. For example the solution for well-known logical problem with wolf, goat and cabbage [5] was done.

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Index

Index

Ababii, Victor, 16, 50
Akdemir, Ahmet Ocak, 38, 41
Anghel, Cristian, 58

Baltag, Iurie, 12
Baluta, Alexandra, 30
Banaru, Galina, 58
Banaru, Mihail, 59
Barbu, Vlad Stefan, 48
Borş, Dan-Mircea, 36
Bordian, D., 50
Botnaru, Dumitru, 61
Boureanu, Maria-Magdalena, 14
Bucur, Maria Liliana, 17
Bujac, Cristina, 17
Bulat, Inga, 48

Calugari, D., 50
Carmocanu, Gheorghe, 70
Cataranciuc, Emil, 51
Celik, Baris, 36
Cernea, Aurelian, 18
Cherevko, Ihor, 18, 20
Chiriac, Liubomir, 62
Choban, Mitrofan, 8
Constantinescu, Cristian-George, 22
Constantinescu, Dana, 21
Cozma, Dumitru, 21
Croitoru, Anca, 36
Cruceanu, Stefan-Gicu, 33
Cucu, Ion, 63

Deveci, Omur, 64
Dhame, Angela, 34
Dimitriu, Gabriel, 31
Dimitrov, Julian, 30
Dmitrieva, Irina, 30
Dorosh, Andrew, 18
Dragan, Irinel, 52
Dubovoi, A., 16, 50
Dunea, Daniel, 32

Efrem, Maria Raluca, 17, 21
Erdaş, Yeter, 44

Falie, D., 30

Georgescu, Paul, 32
Gheorghiu, Călin-Ioan, 37

Gok, Omer, 37
Grünfeld, Cecil P., 22
Gurbuz, Mustafa, 38

Hancu, Boris, 52
Harvim, Prince, 32
Henderson, Johnny, 27
Hila, Ina, 53
Homorodan, Paula, 65

Ilie, Mihaela, 30
Ion, Stelian, 33
Iordache, Stefania, 32

Josu, Natalia, 62

Karagrigoriou, Alex, 48
Karaliopoulou, Margarita, 53

Limnios, N., 9
Lungu, Alexandru, 65
Lungu, Emil, 32
Lupasco, Natalia, 62
Lupu, Mircea, 22

Magdun, Oliver, 33
Makrides, Andreas, 48
Mandrescu, Eugen, 70
Marinescu, Dorin, 33
Melnic, R., 50
Minculete, Nicusor, 39
Mocanu, Marcelina, 39
Morosanu, Costica, 40
Muca, Markela, 34, 53

Neagu, Natalia, 23
Nedelcu, Otilia, 33
Novac, Ludmila, 54

Orlov, Victor, 23
Osovschi, M., 16
Ozdemir, M. Emin, 41

Pasa, Gelu, 13
Pasa, Tatiana, 34
Pavăl, Silviu, 41
Pohoata, Alin, 33
Pop, Horia, 9
Popa, Mihail, 23

Popescu, George, 42
Postolică, Vasile, 54
Predescu, Laurentiu, 32
Putuntica, Vitalie, 24

Radu, Gheorghe, 22
Rashid, Saima, 38
Repeşco, Vadim, 25
Rocsoreanu, Carmen, 26
Rotaru, Diana, 30

Salisteanu, Corneliu, 9, 33
Schlomiuk, Dana, 17
Seichiuc, Elena, 70
Seichiuc, Vladislav, 70
Set, Erhan, 36
Snizhko, N.V., 42
Stamate, Cristina, 43
Sterbeti, Catalin, 66
Sterpu, Mihaela, 26
Suba, Alexandru, 27
Sudacevschi, V., 16, 50

Taralunga, Boris, 66
Titchiev, Inga, 71
Tkacenko, Alexandra, 55
Tudorache, Rodica Luca, 27
Turcan, A., 16
Turuta, Silvia, 27
Tuzyk, Iryna, 20

Unluyol, Erdal, 44
Ursu, Vasile, 67

Vasile, Eugen, 30
Vulpe, Nicolae, 17

Zbaganu, Gheorghita, 56
Zhang, Hong, 32
Zhang, Lai, 32