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1. Real, Complex, Functional and Numerical Analysis

Some connection between Pompeiu-Hausdorff convergences of nets and other well-known hyperconvergences

Gabriela Apreutesei

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Let (X, d) be a metric space and $\mathcal{C}l(X)$ the family of closed non-void parts of X. One of the most used convergences for sequences of sets is Pompeiu-Hausdorff convergence. Sometimes is difficult to verify the properties of this convergence, so we find some conditions on sequences or nets (like monotonicity, boundedness or compactness) which assure the coincidence of Pompeiu-Hausdorff convergence with other well-known hyperconvergences: Kuratowski, Wijsman and Mosco convergences.

Properties of nonlocal reaction-diffusion equations from population dynamics

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Some models of integro-differential equations from population dynamics are presented and analyzed. The integral term describes the nonlocal consumption of resources. We are interested mainly in the bistable case. For some models, Fredholm property of the linearized operators is combined with the implicit function theorem to prove the existence of waves. This is available only for the case of small support of the integral, when the integro-differential operators are close to the differential operators. For some other models, Leray-Schauder method can be applied. This implies the construction of a topological degree for the corresponding operators and the establishment of a priori estimates of the solution.

Multilevel methods for variational and quasi-variational inequalities

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This talk is an overview of some results concerning the global convergence rate of the standard multigrid method for the variational inequalities and variational inequalities conatining a term introduced by a contraction operator. Also, we estimate the convergence rate of the one- and two-level methods for variational inequalities of the second kind and of the quasi-variational inequalities. The methods are introduced as subspace correction algorithms in a reflexive Banach space, where general convergence results are derived. These algorithms become multilevel and multigrid methods by introducing the finite element spaces. In this case, the error estimates are written in function of the number of subdomains and the overlapping parameter for the one- and two-level methods, and in function of the number of levels for the multigrid methods. For the one- and two-level methods, we comparatively illustrate the convergence rates of by numerical experiments. Also, the obtained convergence rates for the multigrid methods are compared with those existing in the literature for the complementarity problems.

About extensions with p-remainders

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We use the terminology from [5]. Any space is considered to be a non-empty completely regular T_1-space. If X is a dense subspace of a space Y, then Y is called an extension of X and the subspace $Y \setminus X$ is called remainder of X. Denote by βX the Stone-Čech compactification of the space X. Let $X^* = \beta X \setminus X$.

Fix a space X. Let Y and Z be two extensions of the space X. We consider that $Y \leq Z$ if there exists a continuous mapping $f : Z \to Y$ such that $f(x) = x$ for any $x \in X$. If f is a homeomorphism of Z onto Y, then we say that the extensions Y and Z are equivalent. We identify the equivalent extensions. Any non-empty family $\mathcal{E}$ of extensions of the space X is a partially ordered set and briefly is called a poset.

A space X is a paracompact p-space if there exist a metrizable space Y and a perfect mapping $g : X \longrightarrow Y$ of X onto Y [2].

Let X be a space and d a pseudumetric on X. For each $x \in X$ and each real number $rr > 0$ put $K(x) = \{y \in X : d(x, y) = 0\}$ and $V(x, d, r) = \{y \in X : d(x, y) < r\}$. There exist a set X/d, a metric $\bar{d}$ on X/d and a mapping $\pi_d :\longrightarrow X/d$ such that $d(x, y) = \bar{d}(\pi_d(x), \pi_d(y))$ for each $x, y \in X$. The pseudometric d is called a k-metric if the mapping π_d is perfect. If the mapping π_d is continuous, then the pseudometric d is called a continuous pseudometric on X. Any pseudometric is considered to be continuous. A pseudometric d is called a complete k-metric if d is a k-metric and$(X/d, \bar{d})$ is a complete metric space.

On a space there exists a complete k-metric if and only if it is a paracompact Čech-complete -space.

An extension Y of a space X is called a (complete) p-extension if on Y there exists a (complete) k-metric d such that $\pi_d(X) \cap \pi_d(Y \setminus X) = \emptyset$.

Proposition 1. *Let Y be a p-extension of a space X. Then X, Y and $Y \setminus X$ are paracompact p-spaces.*

Theorem 2. *For a space X the following assertions are equivalent:*

1. *X is a Lindelöf p-space.*
2. *For X there exists a p-extension Y which is a Lindelöf p-space.*
3. *Any p-extension Y of X is a Lindelöf p-space.*
4. *There exists a compactification bX of X such that $bX \setminus X$ is a paracompact p-space.*
5. *For any compactification bX of X the remainder $bX \setminus X$ is a Lindelöf p-space.*
6. *On X there exists some totally bounded k-metric.*

Theorem 3. *If d is a k-metric on a space X then there exist a complete p-extension Y of X and a complete k-metric ρ on Y such that $d(x, y) = \rho(x, y)$ for each $\rho x, y \in X$ and $\pi_\rho(X) \cap \pi_\rho(Y \setminus X) = \emptyset$. In this case we say that ρ is a completion of the k-metric d.*

We mention that a completion ρ of a k-metric d, in general, is not unique. Let $E_c(X, d)$ is the family of all complete p-extension (Y, ρ) of X, where ρ is a completion of the k-metric d. Obviously that $E_{cp}(X, d)$ is a poset.

Theorem 4. *If d is a k-metric on a space X, then $E_{cp}(X, d)$ is a complete upper-lattice with a maximal element (mX, d_m) and $X \subseteq mX \subseteq \beta X$, where βX is the Stone-Čech compactification of X.* Some similar problems of the theory of extensions of spaces were examined in [1, 3, 4, 5, 6].

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Time adaptivity for a BDF2 discretization of the wave equation

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We are interested in the space-time adaptive solution of the linear second-order wave equation when a second-order method is used in the time discretization. With this aim, we consider the full discretization of the wave equation using the variable-step second order Backward Differentiation Formula (BDF2) for the time discretization and continuous piecewise affine finite elements for the spatial discretization. As it is well-known, the BDF2 is unconditionally stable (cf. for instance [2]). We followed [1] and derived a priori error estimates for this fully discrete scheme. Then, we follow the ideas in [3] and developed an a posteriori error analysis for the time discretization.

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Product function norms and Köthe-Bochner sequence spaces

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Function norms on pozitive sequences generate (classical) Köthe spaces of sequences. The theory can be extended to Köthe-Bochner spaces (of Banach-valued sequences). Product function norms on pozitive sequences generate special cases of Köthe-Bochner spaces.

MSC 2010: 46E30, 46E50.

Keywords: sequence space, Köthe space, Köthe-Bochner space, product function norm.

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Comparison results on Gould integral

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In our talk we point out some relationships between the Gould integral and the Birkhoff integral for real bounded functions with respect to a vector measure.

Remarks on the Gould integral in Banach lattices

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We present some properties concerning continuity and total measurability for the Gould integral of real functions with respect to a non-additive set function with values in a Banach lattice.

On the numerics of Allen-Cahn type equations

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We are concerned with initial/boundary value problems of the form:

$$\begin{cases} u_t = \mathcal{L}u + \mathcal{N}(u,t),\ t>0,\ -1<x<1, \\ \mathcal{B}u(\pm 1,t) = 0,\ t>0, \\ u(x,0) = u_0(x),\ -1<x<1, \end{cases} \tag{1}$$

where $\mathcal{L}$ and $\mathcal{N}$ are linear and nonlinear operators, respectively, $\mathcal{B}$ is a trace operator which can imply Dirichlet, Neumann or mixed (Robin) boundary conditions and $u_0(x)$ stands for the initial data. Once we discretize the spatial part of the above PDE we get a system of ODEs supplied with an initial condition, i.e., the Cauchy's problem,

$$\begin{cases} U_t^M = LU^M + N(U^M,t),\ t>0, \\ U^M(0) = U, \end{cases} \tag{2}$$

where the vector $U^M(t) := (u(x_0,t),\ u(x_1,t),\ldots,u(x_M,t))^T$ contains the values of unknown solution $u(x,t)$ in the nodes of a partition

$$-1 = x_0 < x_1 < \ldots < x_M = 1,$$

applied to the interval $[-1,1]$ at various moments t.

The Allen-Cahn equation is a well-known equation from the area of reaction-diffusion (gradient) systems. The corresponding operators from (1) are respectively

$$\mathcal{L}u := \varepsilon u_{xx},\ \mathcal{N}(u,t) := u - u^3,\ 0 < \varepsilon << 1.$$

The equation exhibits difficulties due to the combination of *nonlinearity* and *stiffness*. It has three constant *steady states*, $u = 0$ and $u = \pm 1$. The middle state is unstable but the latter two states are stable (attracting).

Our main aim is to accurately detect the so called *metastability* phenomenon, i.e., the rapid transition of the initial data to a solution with just one interface between the attracting states.

We will pay a special attention to the following issues:

- we consider different spatial discretizations, i.e., finite differences, finite elements and spectral methods (see [1]), to differential operator $\mathcal{L}$ in order to get the finite dimensional operator L;
- in order to solve the Cauchy's problem (2) we will study some stable implicit schemes (fourth order time-differencing ([3], [4]));
- we will extend our analysis to the nonlinearities of the form

  $$\mathcal{N}(u,t) := u - u^{2p+1},\ p \in \mathbb{N},$$

  and observe that the metastability persists;
- we consider the case of mixed boundary conditions which is fairly different from the simple Dirichlet and Neumann boundary conditions cases. A Lyapunov functional is no longer available. In order to march in time, a fractional time step method is introduced in [2].

A lot of numerical experiments are carried out using MATLAB and their outcomes are displayed.

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On a time and space discretization for the nonlinear Boltzmann equation in unbouded domain

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Convergence properties of a time and space discretized approximation scheme for the Boltzmann equation in a bounded volume, considered in the context of the validation of the Nanbu's simulation method for the Boltzmann model, have been recently extended to the setting of the Boltzmann equation in the whole physical space. Similar results are obtained in the present work in the context of the Boltzmann equation in a general unbounded domain with smooth boundary conditions compatible with the conservation of energy.

On approximation properties of q-Baskakov-Szász-Stancu operators

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This paper provides the q-analogue of Baskakov-Szász-Stancu operators. We have estimated some approximation properties with asymptotic formula for these operators by using properties of q-calculus and also discuss the better error estimations for these operators.

Differentiability properties of Orlicz-Sobolev functions on metric measure spaces

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Given a metric measure space (X, d, μ) and a Young function $\Phi : [0, \infty) \to [0, \infty)$, we consider the Orlicz-Sobolev space $N^{1,\Phi}(X)$, which is a collection of equivalence classes of functions $u \in L^{\Phi}(X)$ having an upper gradient in $L^{\Phi}(X)$. We prove three differentiability results for functions in $N^{1,\Phi}(X)$.

If (X, d) is complete, μ is doubling and (X, d, μ) supports a $(1, \Phi)-$Poinca-ré inequality, we show that every function in $N^{1,\Phi}(X)$ is approximately differentiable almost everywhere, provided that Φ is strictly increasing.

Assume that the measure μ is $s-$Ahlfors regular, (X, d, μ) supports a $(\Phi, \Phi)-$Poincaré inequality and that Φ satisfies the $\Delta_2-$and ∇_2-conditions, as well as the Calderón-type growth conditions $\int\limits_0^1 \left(\frac{t}{\Phi(t)}\right)^{1/(s-1)} dt < \infty$ and $\int\limits_1^{\infty} \left(\frac{t}{\Phi(t)}\right)^{1/(s-1)} dt = \infty$. Under these assumptions, we

partly generalize some results on the $L^{\Phi_s}-$differentiability of functions in $N^{1,\Phi}(X)$, proved by Björn for $\Phi(t)=t^p$ and by Alberico and Cianchi for $X=\mathbb{R}^n$. Here Φ_s is the Sobolev conjugate of Φ with respect to the homogeneous dimension s of X.

Finally, suppose that the metric space (X,d) is geodesic, μ is $s-$Ahlfors regular and (X,d,μ) supports a $(1,1)-$Poincaré inequality, while the Young function Φ satisfies the growth condition $\int\limits_1^\infty \left(\frac{t}{\Phi(t)}\right)^{1/(s-1)} dt<\infty$. We prove that $Lip u(x)<\infty$ for almost every $x\in X$, in particular u is differentiable almost everywhere with respect to any strong measurable differentiable structure on X, in the sense of Cheeger.

Connection between continuous and V-continuous functions, proximative sequences and approximative sequences and flow interaction

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The required notions are given in the book of K. Borsuk and the paper of N. Shekutkovski which involves intrinsic strong shape category construction. In the paper through several definitions, properties and theorems, it is given the way how from continuous functions to associate V-continuous functions, and it is defined function from the classes of approximative sequences to the classes of proximative sequences. Natural construction of proximate sequence in a given flow will be presented.

Symbol of singular integral operators. The case of an unlimited contour

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In this work we study subalgebra Σ of algebra $L(L_p(\Gamma,\rho))$, which contains all operators of the form

$$(M\phi)(t)=\sum_{k=0}^{n-1}\left\{a_k(t)\phi(\nu_k(t))+\frac{b_k(t)}{\pi\, i}\int_\Gamma \frac{\phi(\tau)}{\tau-\nu_k(t)}d\tau\right\} \tag{1}$$

with piecewise-continuous coefficients. It is necessary to consider separately the case, when ν preserves the orientation on Γ, and the case, when ν reverses the orientation. The algebra Σ contains the set Σ_0 of all sums of compositions of operator of the form (1), and also operators, which are limits (in the sense of convergence by the norm of operator) of a sequence of operators from Σ_0. The research of the set Σ_0 is based on the suggested by I. Gohberg and N. Kripnik [1,2] method of the study of *complicated* operators, which allows to receive necessary and sufficient conditions of Noetherian property of operators from Σ.

In the present paper the existence of such an isomorphism between Σ and some algebra A of singular integral operators with a Cauchy kernel that an arbitrary operator from Σ and its image are simultaneously Noetherian or not Noetherian is proved. It allows to introduce the concept of a symbol for all operators from Σ and, using known results for algebra A, in terms of a symbol to receive conditions of Noetherian property for all operators from Σ, including for $\Sigma\backslash\Sigma_0$. Through the symbol the index of operators $A\in\Sigma$ can be also expressed. The set of values of the determinant of a symbol $A(t,\xi)$ represents a closed continuous curve, which can be oriented in a natural was. The index of this curve (i.e. the number of turns about the origin), taken with the opposite sign, is equal to the index of the operator A.

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Fractal vector measures. Support

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We extend the concept of fractal measure (Hutchinson measure - unique fixed point of the derived Hutchinson contraction). To obtain this fixed point, special metrics derived from Monge-Kantorovich-type norms are used. We work for Banach space-valued measures. Special attention is given to the structure of the support of the fractal measure.

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Some convergence estimates for abstract linear second order differential equations with two small parameters

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In a real Hilbert space H we consider the following singularly perturbed Cauchy problem

$$\begin{cases} \varepsilon\, u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases} \tag{1}$$

where $u_0, u_1 \in H$, $f : [0,T] \mapsto H$ and ε, δ are two small parameters.

We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem (1) in two different cases:

(i) when $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$, relative to solutions to the following unperturbed system:

$$\begin{cases} \delta l'_\delta(t) + Al_\delta(t) = f(t), \quad t \in (0,T), \\ l_\delta(0) = u_0, \end{cases}$$

(ii) when $\varepsilon \to 0$ and $\delta \to 0$, relative to solutions to the following unperturbed system:

$$\begin{cases} Av(t) = f(t), \quad t \in (0,T), \\ v(0) = A^{-1}f(0). \end{cases}$$

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighbourhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

Simple method for finding the numerical solution of the initial value problem of the ordinary differential equation

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Euler method is the well known numerical method for finding the numerical solution of the initial value problem of the ordinary differential equation. In this paper, the seven simple numerical formulas for finding the numerical solution of the initial value problem of the ordinary differential equation are introduced. These formulas are found in the same manner as the Euler formula. We will use these formulas to find the numerical solution of an initial value problem of the ordinary differential equation. The results will be compared with the Euler method.

A general fixed point theorem for occasionally weakly compatible hybrid mappings

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Definition 0.1. *Let X be a nonempty set. A function $m : X \times X \to \mathbb{R}_+$ is a minimal condition metric (briefly mc - metric) on X if $m(x, y) = 0$ if and only if $x = y$. The pair (X, m) is a minimal condition metric space (briefly mc - metric space).*

Remark 0.2. *1) By Definition 1 it follows that every metric, symmetric, quasi - metric, b - metric space, generalized metric space are all mc - metric spaces.*
2) The metric, symmetric, quasi - metric, b - metric spaces are all mc - metric spaces.

Definition 0.3. *Let X be a nonempty set and $f : X \to X$ and $F : X \to 2^X$. The pair (f, F) is occasionally weakly compatible if $fFu \subset Ffu$ for some coincidence point of f and F.*

We denote $M(A, B) = \inf\{m(a, b) : a \in A, b \in B\}$.

Definition 0.4. *Let ϕ_m be the set of all real functions $\phi(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying the following conditions:*
(ϕ_1) : *ϕ is nonincreasing in variables t_2, t_5, t_6,*
(ϕ_2) : *$\phi(t, t, 0, 0, t, t) > 0, \forall t > 0$.*

The main result of this paper is the following

Theorem 0.5. *Let f, h be self mappings of a mc - metric space (X, m) and F, H be maps of X into 2^X such that the pairs (f, F) and (g, G) are owc. If*

$$\begin{array}{c}\phi(m(fx, hy), M(Fx, Hy), M(fx, Fx),\\ M(hy, Hy), M(fx, Hy), M(Fx, hy)) \leq 0\end{array} \tag{1}$$

for all $x, y \in X$ for which $fx \neq hy$ and $\phi \in \phi_m$, then f, h, F and H have an unique common fixed point.

As application, two general fixed point theorems for mappings satisfying contractive conditions of integral type and for hybrid mappings in G - metric spaces are obtained.

A general fixed point theorem for mappings satisfying a cyclical contractive condition in G - metric spaces

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In 2003, Kirk et al. extended Banach's contraction principle to a case of cyclical contractive mappings. M. Păcurar, I. A. Rus, M. A. Petric and other authors extended most of the fundamental metrical fixed point theorems in literature to cyclical contractive mappings. In 2006, Mustafa and Sims introduced a generalization of metric spaces, named G - metric space.

Definition 0.6. *Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:*

(G_1) : $G(x, y, z) = 0$ *if* $x = y = z$,

(G_2) : $0 < G(x, x, y)$ *for all* $x, y, z \in X$ *with* $z \neq y$,

(G_3) : $G(x, x, y) \leq G(x, y, z)$ *for all* $x, y, z \in X$ *with* $z \neq y$,

(G_4) : $G(x, y, z) = G(y, z, x) = G(z, x, y) = ...$ *(symmetry in all three variables),*

(G_5) : $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ *for all* $x, y, z, a \in X$ *(rectangle inequality).*

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

In this paper, a general fixed point theorem for mappings satisfying a cyclical implicit contractive relation which extend some results from metric spaces to G - metric spaces is proved.

Theorem 0.7. *Let (X, G) be a complete G - metric space and let $\{A_i\}_{i=1}^p$ be a family of nonempty closed subsets of X. Let $Y = \bigcup_{i=1}^{p} A_i$ and $T : Y \to Y$ be a mapping satisfying*

$$T(A_i) \subset A_{i+1}, i = \overline{1, p}, where A_{p+1} = A_1. \tag{1}$$

If the inequality

$$\begin{array}{c} F(G(Tx, Ty, Ty), G(x, y, y), G(x, Tx, Tx), \\ G(y, Ty, Ty), G(x, Ty, Ty), G(y, Tx, Tx)) \leq 0 \end{array} \tag{2}$$

holds for all $x \in A_i$, $y \in A_{i+1}$, $i = \overline{1, p}$ and $F \in \mathfrak{F}_G$, where $\mathfrak{F}_G$ is the set of all continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ such that

(F_1) : *F is nonincreasing in variable t_5,*

(F_2) : *There exists $h_1 \in [0, 1)$ such that for all $u, v \geq 0$, $F(u, v, v, u, u + v, 0) \leq 0$ implies $u \leq h_1 v$,*

(F_3) : *There exists $h_2 \in [0, 1)$ such that for all $t, t' > 0$, $F(t, t, 0, 0, t, t') \leq 0$ implies $t \leq h_2 t'$.*

Then T has an unique fixed point in $\bigcap_{i=1}^{p} A_i$.

Finite volume method on nonuniform grids for semiconductor device simulation

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We consider the nonlinear problem for semiconductor diode based on drift-difusion equations in two dimensions. To solve the problem numerically we apply the finite volume method with Scharfetter-Gummel discretization scheme on nonuniform grids. The refinement of the grid is carried out in areas where the impurity profile function has large gradients and in the vicinity of the point where the type of boundary condition is changed. To realize such a refinement, we use so-called frontal algorithm for constructing the triangular grid. The results of numerical simulation are represented and discussed.

A coincidence between the general efficiency and the approximate dominance

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This research work represents a short synthesis on the main results obtained for the general efficiency and its immediate connections with the approximate dominance by the convex cones in ordered vector spaces. The largest class of such as these ordering cones, named by us Isac's cones in Hausdorff locally convex spaces ensures this link, together with the proved applications including the Choquet boundaries.

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Relative convexity and relative Schur-convexity

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The aim of the present paper is to discuss a rather general condition under which the inequality of Jensen works in a framework that includes a large variety of nonconvex functions. We introduce a weaker notion of convexity, namely relative convexity. This new concept allow us to generalize Jensen's inequality and Hardy-Littlewood-Polya's inequality outside the framework of convex functions. As some applications, we present many different types of inequalities which works nicely into this context. **Acknowledgement**. This work was supported by the strategic grant POSDRU/159/1.5/ S/133255, Project ID 133255 (2014), cofinanced by the European Social Fund within the Sectorial Operational Program Human Resources Development 2007- 2013.

On Intelligent Support System in solving of integral equations of second kind

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In this paper the computing algorithms for spline-collocations and spline-quadratures methods in approximate solving of integral equation Fredholm and Volterra of the second kind, are developed. Also, a theoretical substantiation in the space of continuous functions and in the Hölder spaces of the developed computing algorithms is obtained.
Next, it is developed and implemented the Intelligent Support System in approximate solving of integral equation (ISS_IE) Fredholm and Volterra of the second kind. Based on more than 3000 types offered kernels for integral equations there were created:
– the Base of Prototype of Kernels of integral equations (BPK_EI_COMP) for checking the sufficient conditions of integral equations compatibility;
– the Base of Prototype of Kernels of integral equations (BPK_EI_COL) for solving by spline-collocations method;
– the Integral Equations Solver (IES), accompanied by the convenient interface during the solving of integral equations.

Computing algorithms for approximate solving of some nonlinear singular integral equations with Carleman moved shift

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In this paper the computing algorithms for collocations and quadratures methods for approximate solving of some nonlinear singular integral equations (NSIE) with Carleman moved shift, defined on a arbitrary closed smooth contour Γ in the complex plane, are obtained.
We consider the following NSIE with Carleman moved shift $\gamma(t) : \gamma(\gamma(t)) = t$ in Hölder spaces $H_\beta(\Gamma),\ 0 < \beta < 1$:

$$\Phi\left(t, \varphi(t), \varphi(\gamma(t)), \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau, \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\tau)}{\tau - \gamma(t)}d\tau,\right.$$

$$\frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\gamma(\tau))}{\tau - t}d\tau, \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\gamma(\tau))}{\tau - \gamma(t)}d\tau,$$

$$\left.\frac{1}{2\pi i}\int\limits_\Gamma K_1(t,\tau)\varphi(\tau)d\tau, \frac{1}{2\pi i}\int\limits_\Gamma K_2(t,\tau)\varphi(\gamma(\tau))d\tau\right) = 0, \quad t \in \Gamma,$$

where $\gamma'(t) \in H_\mu(\Gamma),\ 0 < \beta < \mu \leq 1, \forall t \in \Gamma$; Φ are known function verifying certain Hölder conditions on Γ and φ is a unknown function.
Next, the substantiation theory for these computing algorithms in Hölder spaces $H_\beta(\Gamma),\ 0 < \beta < 1$ is obtained.

Intrinsic strong shape of global attractors in compact metric spaces

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The first intrinsic approach to shape is given in the papers from J.M.Sanjurjo and J. Felt. In the paper from N.Shekutkovski for the first time the construction of the strong shape category SSh for compact metric spaces was given, using the intrinsic approach. The approach combines continuity up to a covering and the corresponding homotopies of second order. We shall present the strong shape version result for global attractors in compact metric spaces using this approach.

Remarks on a Pettis-Sugeno types integral of multifunctions

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In this talk we present the principal properties, calculus rules and convergence results for the Pettis-Sugeno vector integral as well as the relationships between this integral and other known integrals for vector multifunctions by respect to vector multisubmeasures.

Local parametrization and generalized solutions for implicit functions

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We consider implicit systems and we introduce general local parametrizations under the classical assumptions. When the nondegenaracy hypothesis is removed, we introduce and study generalized solutions. Several examples clarify the notion and its properties.

PPF dependent fixed point results in A-closed Razumikhin classes

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Some PPF dependent fixed point results are given, in algebraically closed Razumikhin classes. by reducing them to standard fixed point statements over the supporting Banach space.

2. PDEs with Applications in Mechanics, Biology, etc.

On some class of nonlocal problems for parabolic equations

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In this paper we consider a class of problems arising in mathematical modeling of various phenomena of modern natural science such as heat conduction, processes in liquid plasma, dynamics of ground waters, thermo-elasticity and some technological processes with special attention to nonlocal problems with integral conditions of the first and second type for parabolic equations. We describe different approaches in the analysis of nonlocal problems and study a problem with the second type integral condition for a general parabolic equation. Equivalence of the nonlocal problem to the homogenious problem for a loaded equation is shown. Further, we introduce the notion of a generalized solution and obtain a number of apriori estimates. Finally, we show existence and uniqueness of the solution.

Numerical analysis of an iterative scheme of fractional steps type associated to the nonlinear phase-field equation in Caginalpś model endowed with non-homogeneous dynamic boundary conditions

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The paper concerns with the existence, uniqueness, regularity and the approximation of solutions to the nonlinear phase-field (Allen-Cahn) equation, endowed with non-homogeneous dynamic boundary conditions (depending both on time and space variables). It extends the already studied types of boundary conditions, which makes the problem to be more able to describe many important phenomena of two-phase systems, in particular, the interactions with the walls in confined systems. The convergence and error estimate results for an iterative scheme of fractional steps type, associated to the nonlinear parabolic equation, are also established. The advantage of such method consists in simplifying the numerical computation. On the basis of this approach, a conceptual numerical algorithm is formulated in the end.

Optimizing the position of the support of the control for some optimal harvesting problems

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We investigate the optimal position of the support of the control for some optimal harvesting problems. First we refer to a logistic model with diffusion. Our purpose is to prove the existence of an optimal control and to obtain the necessary optimality conditions for the related optimal harvesting problem. Then we obtain an iterative method to improve the position of the support of the optimal harvesting effort (for a simplified model without logistic term). In the last part numerical tests illustrating the effectiveness of the theoretical results are given.

Existence and Uniqueness of Positive Solutions for a Boundary Value Problem with a Laplacian Operator

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In this work we study the existence and uniqueness of positive and nondecreasing solutions for a fractional two-point boundary value problem with p-Laplacian operator.

3. ODEs; Dynamical Systems

A subfamily of cubic systems with invariant lines of total multiplicity 8

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The aim of this paper is to bring a classification for the family of polynomial differential cubic systems according to their geometric properties encoded in the configurations of invariant straight lines of total multiplicity eight (including the line at infinity with its own multiplicity), which these systems possess. More exactly, we consider here the subfamily of such cubic systems, which possess two distinct singularities at infinity.

According to the paper [1] we call *configuration of invariant lines* of a cubic system the set of invariant lines (which may, but not necessarily, have real coefficients) of the system, each one which is not filled up with singularities, endowed with its own multiplicity and together with all the real singular points of this system, located on these invariant lines, each isolated singularity endowed with its own multiplicity.

It is well known that the infinite singularities (real and/or complex) of cubic systems are determined by the linear factors of the polynomial $C_3 = yp_3(x, y) - xq_3(x, y)$, where p_3 and q_3 are the cubic homogeneities of these systems. So in the case of two distinct infinite singularities they are determined either by one triple and one simple real or two double factors of the polynomial $C_3(x, y)$ both real (or both complex).

We prove that a cubic system with invariant lines of total multiplicity 8 can not have infinite singularities determined by two double factors of the polynomial C_3. Moreover, if the infinite singularities are determined by one triple and one simple factors of C_3 then this system has a total of 24 such configurations which are distinguished, roughly speaking, by the multiplicity of their invariant lines and by the multiplicities of the real singularities of the systems located on these lines.

Additionally using the algebraic method of invariants of differential systems, developed by Sibirskii and his disciples, we construct necessary and sufficient affine invariant conditions for the realization of each one of these 24 configurations. We remark that cubic systems with the maximum number of invariant lines are considered in [2], whereas cubic systems with 8 invariant lines possessing either four or three infinite singularities were studied in [4] and [5].

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Some classification results for commutative algebras without derivations

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The classification up to an isomorphism of commutative algebras is mainly based on the corresponding subalgebra lattices of these algebras. The author has already used this result for getting the classification up to an isomorphism of real 3-dimensional commutative algebras having at least a derivation. In present paper are classified, up to an isomorphism, the algebras that belong to the subclass of real 3-dimensional commutative algebras having no derivation but which contain at least an one-dimensional ideal. The obtained results are used to get classification results, up to an affine equivalence, of corresponding homogeneous quadratic differential systems by Markus' correspondence.

Center conditions for a cubic system with parallel invariant straight lines and an invariant cubic

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We consider the cubic system of differential equations

$$\begin{aligned} \dot{x} &= y + ax^2 + cxy + fy^2 + kx^3 + mx^2y + pxy^2 + ry^3, \\ \dot{y} &= -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3), \end{aligned} \tag{1}$$

in which variables and coefficients are assumed to be real. The origin $O(0,0)$ is a singular point of a center or focus type for (1). Suppose system (1) has two parallel invariant straight lines $a_jx + b_jy + 1 = 0,\ j = 1, 2,\ a_1b_2 - a_2b_1 = 0$ (real or complex). Then by a rotation of axes we can make them parallel to the axis of ordinates (Oy) and the linear part of (1) preserves the form. For this to occur it is necessary and sufficient that the following coefficient conditions to be satisfied $a = f = k = p = r = 0,\ m(c^2 - 4m) \neq 0$. Then the invariant straight lines are $2 + (c \pm \sqrt{c^2 - 4m}\,)x = 0$.

The problem of the center for cubic system (1) with: at least three invariant straight lines, two of which are parallel, was solved in [1]; two parallel invariant straight lines and one invariant conic was solved in [3]. Sufficient center conditions for cubic system (1), with two distinct invariant straight lines by using the method of Darboux integrability and rational reversibility, are obtained in [2]. Sadovskii and Shcheglova [4] solved the problem of the center for a nine-parameter cubic differential system (1) that can be reduced to a Liénard type system.

In this paper we give the center conditions for cubic system (1) with two parallel invariant straight lines and one irreducible invariant cubic curve $a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{10}x + a_{01}y + 1 = 0$ and we prove that the cubic system (1) with a center having two parallel invariant straight lines and an invariant cubic curve is always Darboux integrable.

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First integrals with polynomials not higher than third order of the model of tuberculosis

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Consider three-dimensional autonomous real differential system which simulate the intrinsic transmission dynamics of tuberculosis [1], [2]

$$\frac{dS}{dt} = \tau - \mu S - \beta ST, \quad \frac{dL}{dt} = -\delta L - \mu L + (1-p)\beta ST, \quad \frac{dT}{dt} = \delta L - (\mu+\nu)T + p\beta ST. \tag{1}$$

We assume that the first integral has the form $I_q(S,L,T,t) = P_q(S,L,T)\exp(\lambda t)$, $(q \leq 3)$, where $P_q(S,L,T) = a + bS + cL + dT + eS^2 + fL^2 + gT^2 + 2hSL + 2kST + 2lLT + mS^3 + 3nS^2L + 3oS^2T + 3ySL^2 + 3rSLT + 3sST^2 + uL^3 + 3vL^2T + 3wLT^2 + zT^3$. Note that the expressions $U_1 = \beta\tau, U_2 = \mu, U_3 = \nu, U_4 = \delta, U_5 = p$, are invariants of the system (1) in relation with the continuous group admitted by the system (1) [3]. **Theorem:** *Assume that the conditions* $U_1U_2U_3U_4 \neq 0$ *and* $0 \leq U_5 \leq 1$ *holds. Then the system (1) prossing the vector* $(\tau,\beta,\mu,\delta,\nu,p)$ *has 7 first integrals (see the Table below)*

$(\tau,\beta,\mu,p\nu,\nu,p)$	$I_1^{(1)} = (L + \frac{p-1}{p}T)\exp(t(\mu+\nu))$
$(\tau,\beta,\mu,\delta,\nu,1)$	$I_1^{(2)} = L\exp(t(\delta+\mu))$
$(\tau,\beta,\mu,-\mu,\nu,1)$	$I_2^{(1)} = a + L(c+fL)$
$(\tau,\beta,\mu,-p\mu,-\mu,p)$	$I_2^{(2)} = a + (L+\frac{p-1}{p}T)(c+f(L+\frac{p-1}{p}T))$
$(\tau,\frac{\mu(\nu^2-\mu^2)}{\nu\tau},\mu,-\nu,\nu,0)$	$I_2^{(3)} = (\frac{(\nu^2-\mu^2)((L+S)^2+2T(L+S))}{2\mu\tau} + (L+S+T) + +\frac{T\nu}{\mu} - \frac{S\nu^2}{\mu^2} - \frac{\tau}{2\mu} + \frac{\nu^2\tau}{2\mu^3})\exp(2t\mu)$
$(\tau,\beta,-\nu,p\nu,\nu,p)$	$I_3^{(1)} = \frac{1}{p^3}(ap^3 + cp^2K + fK^2 + uK^3)$, where K=(Lp-T+pT)
$(\tau,\beta,\mu,-\mu,\nu,1)$	$I_3^{(2)} = a + cL + fL^2 + uL^3$,

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The absolute stabilization and the optimal control of nonlinear dynamical systems for special cases. Flight control systems in the case of rolling

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In the first part of this paper it is presented the automatic regulation methods for the absolute stability (a.r.a.s) of the nonlinear dynamical systems that have applications on the stabilization of the rolling oscillations curse for aircraft or rackets. Two methods for the absolute stability are specified: a) the A.I. Lurie method with the effective determination of the Liapunov function; b) the frequency method of the Romanian researcher V.M. Popov who has used the transfer function in the critical cases. The authors develop a new sufficient criterion for (a.r.a.s.), with efficient technique of calculus.
In the second part - there are obtained the analytic - numerical solutions and the conditions for the regulator parameters for the realization of the absolute stability for the airplane autopilot route in the case of rolling oscillations. At the end authors prove practically the theorem Kalman - Yakubovich - Popov for the equivalence of these methods. (Th. K-Y-P). In the last section of the paper it is presented the optimal control for the flight system in the case of rolling oscillations. The optimization is made using the maximum principle of Pontreaguine; the authors solve the problem of minimum time. It is determined the command function and the optimal trajectory for this system.

On the computational asynchronous systems

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The asynchronous flows are the binary models of the asynchronous circuits from digital electronics. They are generated by boolean functions that iterate their coordinates independently on each other (generalizing the usual dynamical systems, that iterate the coordinates of their generator functions at the same time). The purpose of our work is define the computational asynchronous systems, a concept that puts under the same frame several such models.

4. Probability Theory, Mathematical Statistics, Operations Research

On bayesian analysis for Birnbaum Saunders distribution. Results and applications

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Some classes of probability distributions used for lifetime modeling are extremly important in various fields, such as: industry, life insurance, medicine and many others. The Birnbaum-Saunders distribution, proposed by Birnbaum and Saunders in 1969, is used for lifetime modeling in fatigue failure caused under cyclic loading. The estimation of the parameters of this distribution using maximum likelihood method is extremly complicated due to the analytical form of the probability distribution function and solving the system obtained needs powerful numerical methods. This argument proves the importance of bayesian analysis effective method for parameter estimation. Using some apriori distributions, a bayesian approach is considered for estimating the parameters of the Birnbaum-Saunders distribution. Also, some Markov Chains Monte Carlo simulations are performed.

Entropy Optimization with Applications to Signal Processing Theory

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In this paper we consider some new entropic and optimization methods with applications to signal processing theory.

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Algorithms for solving stochastic discrete optimal control problems on networks

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In this work we consider the stochastic discrete optimal control problem on the decision network (G, X_C, X_N, c, p, x_0). This network is determined by the directed graph $G = (X, E)$ with a fixed starting state x_0, the subsets $X_C, X_N \subset X$ which satisfy $X = X_C \cup X_N$, $X_C \cap X_N = \emptyset$, the cost function $c : E \to \mathbb{R}$ and the probability distribution function $p : E_N \to [0, 1]$ on the set $E_N = \{ e = (x, y) \mid x \in X_N, y \in X\}$. The set X_C represents the set of controllable states in which the transitions of the system to the next state can be controlled by the decision maker at every discrete moment of time and X_N represents the set of states in which the decision maker is not able to control the transition because the system passes to the next state randomly.

Parallel algorithm to solve bimatrix informational extended game

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Let be given a matrices $A = ||a_{ij}||_{i\in I}^{j\in J}$, $B = ||b_{ij}||_{i\in I}^{j\in J}$ were $I = \{1, 2, .., n\}$ is the line index and $J = \{1, 2, .., m\}$ the column index and $\Gamma = \langle A, B\rangle$ be the strategic form of the bimatrix game with complete and perfect information. All players know exactly the payoff matrices and they know the sets of strategies. Players know what kind of the strategy will be chosen by each others. These condition stipulate that we can use the set of the informational extended strategies of the player 1 (respectively 2) which is the set of the functions $\Theta_1 = \{\theta_1^\alpha : J \to I\}_{\alpha=1}^{\varkappa_1}$ and respectively $\Theta_2 = \left\{\theta_2^\beta : I \to J\right\}_{\beta=1}^{\varkappa_2}$. The informational extended strategies of the player 1 are functions θ_1^α such that for all $j \in J$ there is $i_j^\alpha \in I$ such that $\theta_1^\alpha(j) = i_j^\alpha$ and it means the following: the player 1 will choose the line $i_j^\alpha \in I$ in case the player 2 will choose the column $j \in J$. Accordingly the informational extended strategies of the player 2 are functions θ_2^β such that for all $i \in I$ there is $j_i^\beta \in J$ such that $\theta_2^\beta(i) = j_i^\beta$ and it means the following: the player 2 will choose the column $j_i^\beta \in J$ in case the player 1 will choose the line $i \in I$. For all fixed $\left(\theta_1^\alpha, \theta_2^\beta\right)$ we can construct the sets of the informational non-extended strategies $I^\alpha = \left\{\left(i_1^\alpha, i_2^\alpha, ..., i_j^\alpha, ...i_m^\alpha\right) : i_j^\alpha \in I, \forall j = \overline{1,m}\right\} \subseteq I$, $J^\beta = \left\{\left(\left(j_1^\beta, j_2^\beta, ..., j_i^\beta, ...j_n^\beta\right) : j_i^\beta \in J, \forall i = \overline{1,n}\right)\right\} \subseteq J$ and bimatrix game $\Gamma\left(\theta_1^\alpha, \theta_2^\beta\right) = \left\langle I, J, A^\alpha, B^\beta\right\rangle$, $A^\alpha = ||a_{i_j^\alpha j_i^\beta}||_{i\in I}^{j\in J}$, $B^\beta = ||b_{i_j^\alpha j_i^\beta}||_{i\in I}^{j\in J}$, $i_j^\alpha \in I^\alpha$, $j_i^\beta \in J^\beta$ were be called informational non-extended game generated by the informational extended strategies $\left(\theta_1^\alpha, \theta_2^\beta\right)$.

Let be given a lot of matrices $\left\{\left(A^\alpha, B^\beta\right)\right\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$, were (α, β) denote the node of the parallel cluster. Denote by $NE\left(A^\alpha, B^\beta\right)$ the all Nash equilibrium profiles in the bimatrix game. The parallel algorithm to find the set of all equilibrium profiles $((i^*(\alpha, \beta), j^*(\alpha, \beta)) \in NE\left(A^\alpha, B^\beta\right)$ consist of the following steps.

1. Using the MPI programming model and open source library ScaLAPACK-BLACS, initialize the processes grid $\{(\alpha, \beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$,whereupon all (α, β) MPI process constract the sets of informational non-extended strategies I^α, J^β and the root prcess scatter to MPI processes grid the matrices $\left(A^\alpha, B^\beta\right)$;
2. All fixed MPI process (α, β) using the OpenMP functions and ScaLAPACK routines. eliminate from matrix A^α and B^β the lines that is strictly dominated in matrix A^α and columns that is strictly dominated in matrix B^β. Finally we obtain the matrices $\left(\widehat{A}^\alpha, \widehat{B}^\beta\right)$ where $\widehat{A}^\alpha = ||\widehat{a}_{i_j^\alpha j_i^\beta}||_{i\in\widehat{I}}^{j\in\widehat{J}}$, $\widehat{B}^\beta = ||\widehat{b}_{i_j^\alpha j_i^\beta}||_{i\in\widehat{I}}^{j\in\widehat{J}}$ and cardinals $\left|\widehat{I}\right| \le |I|$, $\left|\widehat{J}\right| \le |J|$.
3. All fixed MPI process (α, β) using the OpenMP functions and ScaLAPACK routines determine all strategy profile from $NE\left(\widehat{A}^\alpha, \widehat{B}^\beta\right)$ and construct the set $NE\left(A^\alpha, B^\beta\right)$.
4. Using ScaLAPACK-BLACS routines, the root MPI process gather from processes grid $\{(\alpha, \beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$ the set $NE\left(A^\alpha, B^\beta\right)$ of strategy profiles.

For these algorithm has been developed a C++ program using MPI functions, OpenMP directives and ScaLAPACK routines. Program has been testing on the control examples. The test results were consistent with theoretical results.

Optimization approach to Airline Disruption Management: A survey

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In this paper, we consider the aircraft recovery problem. This problem may arise due to disruption in a flight schedule. Disruption can be defined as an act of delaying or interrupting the continuity. If such disruptions are not managed properly and timely, then it will severely affect the airlines performance in terms of revenue, operational efficiency and customer satisfaction. The objective is to get feasible, cost minimizing plans that allow the airlines to recover from the disruptions and their associated delays. Aircrafts and aircrews are expensive resources that need efficient utilization. We provide a review of the problems arise in airline disruption management to solve fleet assignment problems, flight recovery, aircraft recovery, aircraft routing, passenger recovery and crew recovery problems. Finally, we discuss the future course of research for disruption management systems in the airline industry.

Estimating portfolio return using interval analysis

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In this paper we present an estimation method for the return of a financial portfolio using interval analysis. This approach uses intervals of real numbers instead of real numbers and thus the return of the portfolio can be modeled more realistically.
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Inferences on the parameters of a Burr-type distribution with conventional Type-I censoring

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We consider statistical inferences on the unknown parameters of a Burr-type distribution when the data are Type-I censored. Also some applications are presented.

Bayesian analysis of Lindley distribution with partial information

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In this paper we present some Bayesian analysis results obtained using noninformative apriori distributions.

A new family of Lindley-type distributions with applications

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We propose a family of Lindley-type distributions obtained by considering additional parameters. Some statistical properties of these distributions are investigated and some applications are presented.

Binary Options in Forex versus traditional Trading, comparison and evaluation

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Electronic trading in currency (known as online FOREX trading) nowadays has become one of the most profitable businesses thanks to computer technology and global network of information like it is Internet. In this article we will present a comparison between one of the methods which is widely used in the last decade known as Binary Options and traditional trading otherwise known as Spread Trading. The comparison will be made in statistical terms and how the different nature and different rules of trade of these methods affect the success of Traders. During the comparison analysis we will use different statistical indicators and different time frames on the same currency pair. Expected results will attempt to clarify the circumstances in which where it is better to use one method or the other one and in which conditions. What is the potential profit related with the associated risk for each of the methods.

LQG homing for jump-diffusion processes

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The problem of optimally controlling one-dimensional diffusion processes until they enter a given stopping set is extended to the case of jump-diffusion processes. We assume that the jump size is small. By making a logarithmic transformation, the optimal control problem is reduced to a first-passage problem for the corresponding uncontrolled process. A particular example is solved explicitly.

On unique perfect matchings

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Let $\alpha(G)$ denote the maximum size of an independent set of vertices and $\mu(G)$ be the cardinality of a maximum matching in a graph G. A matching saturating all the vertices is a *perfect matching*. If $\alpha(G) + \mu(G) = |V|$, then $G = (V, E)$ is called a *König-Egerváry graph* [3, 6]. To the best of our knowledge, a most efficient unique perfect matching algorithm needs $O(mlog^4(n))$ time for a graph with n vertices and m edges [4], and $O(m)$ for a bipartite graph [2]. Bartha conjectured that a unique perfect matching can be found in $O(m)$ time [1].

In this talk we show that this conjecture is true for both König-Egerváry graphs and unicylic graphs. We employ a modification of Karp-Sipser leaf-removal algorithm [5], which ends with an empty graph if and only if the original graph is a König-Egerváry graph with a unique perfect matching. It turns out that a unicyclic non-bipartite graph G may have at most one perfect matching, and this is the case where G is a König-Egerváry graph.

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Approach of the exchange rate risk for the Problem of Currency Exchange

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We consider a non-static model, and we analyse some multistage interactive situations, using the theory of extensive games. Every such a situation can be represented by a strategic game.
Thus we can construct a dynamical model for the problem of the Currency Exchange, representing a model as a multistage interactive situation. As a particular problem we can analyse the simple dynamical game of two players (the "seller" and the "buyer"). In this case the "buyer" will be considered a person who has to do some operation of the Currency Exchange and to choose some exchange point, which is the most adequate for his interests. The "seller" (in this game) will be considered the responsible person of the chosen exchange point. We can consider a more complicated situation for some exchange operations of the same participants during a period of some days or weeks, and thus we can construct a dynamical model with two or more participants. Also, we can consider a probability distribution in order to provide a description of the random movements in the game. The information partition for every player of the game will be considered the information about what has happened in the game up to that point.
For this dynamical problem we need to approach the utilities for every player. The most important in this case is to approach the exchange rate risk for every player, because every player (the "seller" and the "buyer") is interested to have a minimum loss. There are many factors that could affect the Exchange Rates. In this way, we need to approach the exchange rate risk, and thus, every participant of this game could select the optimal strategy according to his interests.
The VaR measure of exchange rate risk is used by firms to estimate the riskiness of a foreign exchange position resulting from a firm Rs activities, including the foreign exchange position of its treasury, over a certain time period under normal conditions. In order to estimate the loss of players for this game, we will apply the VaR measure of exchange rate risk for this dynamical game of the Currency Exchange problem.

New random coincidence point theorems

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We study the existence of the random coincidence points for lower semicontinuous random correspondences defined on metric spaces. The case of condensing random correspondences is treated as well. Our results extend the corresponding ones present in literature.

Exponential tilting for computing multivariate compound distributions using the Fast Fourier Transform

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The Fast Fourier Transform is an alternative approximate method used to evaluate compound distributions resulting from aggregating claims in insurance. Since this method works with truncated claims distributions, it can generate aliasing errors by wrapping around the probability mass that lies at the truncation point below this point. To eliminate this problem, a suitable change of measure called exponential tilting is usually made, forcing the tail distribution to decrease at exponential rate. This method has already been study for univariate and bivariate compound distributions. In this paper, we discuss its generalization to multivariate compound distributions and illustrate it on some numerical examples.

Iterative approach for solving fuzzy multi-criteria fractional transportation problem of "bottleneck" type

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In the paper is developed an iterative fuzzy programming approach for solving the multi-objective fractional transportation problem of "bottleneck" type [1] with some imprecise data. Minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound for each objective function iterative, we find the set of efficient solutions for all time levels [3]. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum\limits_{i=1}^{m}\sum\limits_{i=1}^{n} \tilde{c}_{ij}^k x_{ij}}{\max\limits_{ij}\{t_{ij}\,|x_{ij} > 0\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij}\{t_{ij}\,|x_{ij} > 0\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} x_{ij} = a_i, j = 1, 2, \ldots, n; \tag{3}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, r. \tag{4}$$

where: $Z^k(x) = \left\{Z^1(x), Z^2(x), \ldots, Z^k(x)\right\}$ is a vector of r objective functions; $\tilde{c}_{ij}^k$, k=1,2...r, i=1,2,...m, j=1,2,...n are unit costs or other amounts of fuzzy type, t_{ij} - necessary unit transportation time from source i to destination j, a_i - disposal at source i, b_j -requirement of destination j, x_{ij} - amount transported from source i to destination j. In order to solve the model (1)-(4) we proposed to reduce it to one of linear type, equivalent in terms of the set of solutions. Since the parameters and coefficients of transportation multi-criteria models have real practical significances

such as unit prices, unit costs and many other, all of them are interconnected with the same parameter of variation, which can be calculated by applying of various statistical methods. Thus, the model (1)-(4) can be transformed in one with deterministic type of data. It can be solved using fuzzy techniques:

$$\mu_k(Z^k)) = \begin{cases} 1, \text{if } Z^k(x) \leq L_k \\ \dfrac{U_k - Z^k(x)}{U_k - L_k}, \text{ if, } L_k < Z^k(x) < U_k; \\ 0, \text{ if} Z^k(x) \geq L_k \end{cases} \tag{5}$$

where: U_k is the worst upper bound and L_k is the best lower bound of the objective function k, respectively. Using fuzzy techniques we construct the set of optimal compromise solutions, each corresponding to the time allowed [2]. The main part of this paper develops an iterative algorithm for solving the model (1) - (4) with deterministic data, which builds its set of efficient solutions for every level of allowable time. The proposed algorithm was tested on some concrete examples and has proven to be quite effective.

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Asymptotic behavior of randomly weighted sums of dependent heavy tailed random variables

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Let $\{X_i\}$ be a sequence of Weakly Negatively dependent(WND) random variables with semiexponential varying tails. Let $\{W_i\}$ be a sequence of non-negative random variables, independent of $\{X_i\}$. We denote the weighted random sums $S_N = \sum_{i=1}^{N} W_i X_i$, and the tail probability of maximum of sums $\widetilde{S}_N = \max_{k \leq N} \sum_{i=1}^{k} W_i X_i$ where N is a non-negative integer valued random variable. Under the assumption that $\{X_i\}$, $\{W_i\}$ and N are mutually independent with some mild conditions, this paper establishes an asymptotic relationship for the tail probability $P(\widetilde{S}_N > x)$.

Study on the correlation of the remittances and the life of the Albanian families

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Our study focuses on the late trend of remittances in Albania and their correlation to the everyday life of the Albanian families. In order to come to our conclusions we have extensively made use of field studies and we have heavily relied on questionnaires and the date acquired through them. Nowadays the remittances are quite important to most of Albanian families. The fact that there are few Albanian families not benefiting in this or that way from remittances is another indication on their impact on the daily life of the Albanians. We made use of a three chapter questionnaire. They were used to bring us as much info as possible. More data to process will help us to come to sounder scientific conclusions. The data show the vast extension of remittance phenomenon and them being still important to the family budget. The data received from the questionnaire show also the remittances are feeling the pinch due to the economic crises hitting Greece and Italy, such an effect is coupled with the remittance cycle, too. In a nut shell, remittances do not consist any more the most precious "gem" in Albanians "coffers". The remittances make their presence felt on the family business, too. Yet they can't replace or mitigate the fall in FDI, (Foreign Direct Investments), as they consist of little amounts of money ramificated to a vast number of families, meaning too little to be an investment, too small to cause an impact. Only the money brought home by our migrants returning home for good from Greece and Italy due to the crises there can save as a sort of special FDI. Yet. this is an one time act, not a continuous flow as in the case of remittances.

Mathematical models of PNS control processes influenced by echoes and Ψ phenomena

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We expose the current evolution of Pareto-Nash-Stackelber (PNS) game-control theory [1, 2, 3] by referring various real dynamic processes with particular features and parameters. As a concrete example, we present results of analyses and investigation of a linear discrete-time PNS control of decision processes that evolve as Pareto-Nash-Stackeberg games with constraints (a mixture of hierarchical and simultaneous games) under the influence of echoes and Ψ phenomena. We present mathematical models, solution notions, conditions for PNS control existence and method for PNS control computing. Wolfram Mathematica applications, demonstrations and benchmarks are presented, too.

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Broken stick models in finite and infinite settings: corelation and Lorenz curves

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Let $(X_n)_n$ be a sequence of i.i.d. absolutely continuous non-negative random variables. Let $(X_{j:n})_{1\leq j\leq n}$ be the order statistics attached to it, $\Delta_{1,n} = X_{1:n}, \Delta_{2,n} = X_{2:n} - X_{1:n}, ..., \Delta_{n,n} = X_{n:n} - X_{n-1:n}$ be the n intervals generated by the first n random variables. The interval $[0, X_{n,n}]$ is the stick. It is broken in n smaller sticks We are interested in
- Computation of the covariance matrix the vector of sorted sticks $\Delta_n^{(o)} = (\Delta_{1:n} < \Delta_{2:n} < ... < \Delta_{n,n})$ estimating it for big n at least in the uniform and exponential case. In the first case all the sticks which have the length smaller than the average $\frac{1}{n}$ are positive correlated.
- Finding necessary and sufficient conditions that the Lorenz curves attached to a sequence of positive numbers $(a_n)_n$ do have a limit and when this limit is a Lorenz curve;
- Deciding if the Lorenz curves attached to the empirical distributions $F_n = \frac{1}{n+1}\sum_{k=1}^{n+1}\delta_{\Delta_{k:n}}$ do have a limit in probability and if this limit is itself a Lorenz curve of some other distribution;
- Debating the conjecture "among all the absolutely continuous distributions on [0,1], the uniform one ensures the minimum inequality among the spacings " .

5. Algebra, Logic, Geometry (with applications)

Anisotropic metric models in the Garner oncologic framework

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The present work introduces a three-fold Finsler model, which is related to the classical Garner dynamical system which describes the evolution of the cancer cell population. The obtained statistically fit metric structures are determined by the energy of the deformed field of the biological model, assuming that severe disease circumstances occur, and it is shown that the subsequently derived geometric objects are able to provide an evaluation of the overall cancer cell population growth.
The geometric background, the comparison between the constructed Randers, $m-$th root and Euclidean fit structures, and the applicative advantages of the developed geometric model are discussed.

The research of unorder tables as algebraic structures

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The purpose of the paper is research unordered tables as algebraic structures. Algebraic properties were investigated operations tables union, intersection and difference. It was proposed in C + + a generic parameterized class with abstract class records tabular representation which allows modeling work with tables arranged in terms of algebraic structures [1,2].

Let A $= \{e_1, e_2, ..., e_n\}$ the set of all tabulated records by the same key structure highlighted.

We denote by M $= \{\mathrm{T}_1, \mathrm{T}_2, ..., \mathrm{T}_m, ...\}$ – the crowd of all possible unordered tables on crowd A.

Definition 1. The operation "$\bigcap$" that are intersecting unorder tables $T_1 = T(e_{r_1}, e_{r_2}, ..., e_{r_{k_i}})$, with unorder tables $T_2 = T(e_{l_1}, e_{l_2}, ..., e_{l_{k_i}})$, is operation $(T_1(e_{r_1}, e_{r_2}, ..., e_{r_{k_i}}) \bigcap T_2(e_{l_1}, e_{l_2}, ..., e_{l_{k_i}}))$, that results in unorder tables $T(e_{j_1}, e_{j_2}, ..., e_{j_{k_i}})$ so every element belongs to the unorder tables $T_1 = T(e_{r_1}, e_{r_2}, ..., e_{r_{k_i}})$, and unorder tables $T_2 = T(e_{l_1}, e_{l_2}, ..., e_{l_{k_i}})$, for other$T_r, T_l \in$ M. Thus the set of all unordered tables the lot A with operation "$\bigcap$" is a couple denoted(M, $\bigcap$).

Theorem 1. Couple (M, $\bigcap$) has identity record.

Theorem 2. Couple (M, $\bigcap$) forms a semigroup.

Theorem 3. Couple (M, $\bigcap$) forms a monoid.

Theorem 4. Couple (M, $\bigcap$) forms a commutative monoid.

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Topological quasigroups which are direct products of Abelian topological groups

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In this paper we develop a new method of constructing medial and paramedial topological quasigroups. We study some topological properties of topological quasigroups using a special direct product of commutative groups.

A topological groupoid is a Hausdorff space equipped with a jointly continuous binary operation. A topological groupoid is said to be paramedial if $xy \cdot zt = ty \cdot zx$, for any x, y, u, v. A topological groupoid $(G, \cdot)$ is called *medial* if $xy \cdot zt = xz \cdot yt$, for any $x, y, z, t \in G$.

Let G be a set and $(\cdot)$ a binary operation on G. We say that $(G, \cdot)$ is a quasigroup if $(\cdot)$ is left and right cancellable, i.e., for every $a, b, c \in G$,

$$a \cdot b = a \cdot c \text{ or } b \cdot a = c \cdot a \Longleftrightarrow b = c.$$

If, in addition, G is a topological space and $(\cdot)$ is continuous with respect to the topology of G, we say that $(G, \cdot)$ is a topological quasigroup.

The results we establish here are related to [1], [2], [3].

Theorem 1. *Let $(G, +)$ be a commutative topological group. The topological space $G \times G$ equipped with the continuous operation*

$$(x_1, y_1) \circ (x_2, y_2) = (-x_2 - y_2 + x_1, -x_1 - y_1 + y_2)$$

is a paramedial, non-medial and non-associative topological quasigroup. Moreover, the following statements are true:
1. If $(G, +)$ is compact, then $(G \times G, \circ)$ is also compact.
2. If $(G, +)$ is connected, then $(G \times G, \circ)$ is also connected.
3. If $(G, +)$ is Hausdorff, then $(G \times G, \circ)$ is also Hausdorff.
4. If $(G, +)$ is completely regular, then $(G \times G, \circ)$ is also completely regular.

Theorem 2. *Let $(G, +)$ be a commutative topological group. The topological space $G \times G$ equipped with the continuous operation*

$$(x_1, y_1) \circ (x_2, y_2) = (-x_1 - y_1 + y_2, -x_2 - y_2 + x_1)$$

is a medial, non-paramedial and non-associative topological quasigroup. Moreover, the following statements are true:
1. If $(G, +)$ is compact, then $(G \times G, \circ)$ is also compact.
2. If $(G, +)$ is connected, then $(G \times G, \circ)$ is also connected.
3. If $(G, +)$ is Hausdorff, then $(G \times G, \circ)$ is also Hausdorff.
4. If $(G, +)$ is completely regular, then $(G \times G, \circ)$ is also completely regular.

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The semireflexive product

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In the category $\mathcal{C}_2\mathcal{V}$ of the locally convex topological vector Hausdorff spaces we examine some properties of the semireflexive product of two subcategories [2]. Let $\mathcal{L}$ be a relative subcategory of the category $\mathcal{C}_2\mathcal{V}$ with the reflector functor $l\colon \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{L}$. We denote

$$v(\mathcal{L}) = \{v^X \mid X \in| \mathcal{C}_2\mathcal{V} |\}, \quad \varepsilon\mathcal{L} = \{e \in \mathcal{E}pi\,\mathcal{C}_2\mathcal{V} \mid l(e) \in \mathcal{I}so\}.$$

For $X \in| \mathcal{C}_2\mathcal{V} |$ let be $l\colon X \longrightarrow lX$ $\mathcal{L}$-replique, and $v^X\colon lX \longrightarrow \pi\mathcal{X}$ Π-replique for the respective objects, where Π is the subcategory of complete spaces with weak topology. We denote $v(\mathcal{L}) = \{v^X \mid X \in| \mathcal{C}_2\mathcal{V} |\}$, $(\mathcal{P}'(\mathcal{L}), \mathcal{I}'(\mathcal{L})) = ((v(\mathcal{L}))^{\urcorner}, (v(\mathcal{L}))^{\urcorner\llcorner})$. $(\mathcal{P}'(\mathcal{L}), \mathcal{I}'(\mathcal{L}))$ is a factorisation structure in the category $\mathcal{C}_2\mathcal{V}$ (see [6]).

Definition [2]. *Let $\mathcal{A}$ be a subcategory, and $\mathcal{L}$ a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$. The full subcategory of all objects for which $\mathcal{L}$-replique belongs to the subcategory $\mathcal{A}$ was called the semireflexive product of the subcategory $\mathcal{L}$ and $\mathcal{A}$ is denoted $\mathcal{L} \times_{sr} \mathcal{A}$.*

Theorem 1. *Let $\mathcal{A}$ be a subcategory which is closed under the products, and $\mathcal{L}$ - a reflective subcategory. Then the subcategory $\mathcal{L} \times_{sr} \mathcal{A}$ is closed under the products, $(\varepsilon\mathcal{L})$ - subobjects and $(\varepsilon\mathcal{L})$ - factorobjects.*

Theorem 2. *Let $\mathcal{L}$ and $\mathcal{A}$ be two reflective subcategories of the category $\mathcal{C}_2\mathcal{V}$ and let that the reflector functor $l\colon \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{L}$ posesses the properties*

$$l(\mathcal{M}_f) \subset \mathcal{I}'(\mathcal{A}).$$

Then $\mathcal{L} \times_{sr} \mathcal{A}$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$. In particularly, if the functor l is left exact (see [6]), then $\mathcal{L} \times_{sr} \mathcal{A}$ is a reflective subcategory.

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About of the right product of two subcategories

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In the category $\mathcal{C}_2\mathcal{V}$ of the vectorial topological locally convex Hausdorff spaces we examined some proprieties of the right product. Let be $\mathcal{K}$ a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory with $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ the coreflector and respectively reflector functor, and $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi \mid r(e) \in \mathcal{I}so\}$, $\mu\mathcal{K} = \{m \in \mathcal{M}ono \mid k(m) \in \mathcal{I}so\}$.

Always there is a functor $v : \mathcal{C}_2\mathcal{V} \to \mathcal{V}$, where $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$ is the right product of subcategory $\mathcal{K}$ and $\mathcal{R}$. It is not always a reflector functor [1]. If $\mathcal{A} \subset \mathcal{C}_2\mathcal{V}$, and $\mathcal{B}$ is a class of bimorphisms, then $\mathcal{S}_{\mathcal{B}}(\mathcal{A})$ (respectively $\mathcal{Q}_{\mathcal{B}}(\mathcal{A})$) is the full subcategory of all $\mathcal{B}$-subobjects (respectively $\mathcal{B}$-factorobjects) of objects of the subcategory $\mathcal{A}$.

Theorem 1. *Let Σ be the coreflective subcategory of the spaces with the strongest locally convex topology, and Π - the reflective subcategory of complete spaces with a weak topology. Then the left product $\Sigma *_s \Pi$ is not a the coreflective subcategory and the right product $\Sigma *_d \Pi$ is not a the reflective subcategory in the category $\mathcal{C}_2\mathcal{V}$.*

Theorem 2. *Let $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$, and $\mathcal{M} = \mu\mathcal{K} \cap \varepsilon\mathcal{R}$ be. Then, the following affirnations are equivalent*:

1. *The functor $v : \mathcal{C}_2\mathcal{V} \to \mathcal{V}$ is a reflector functor.*
2. $\mathcal{V} = \mathcal{S}_{\mathcal{M}}(\mathcal{R})$.
3. $|\mathcal{V}| = \{A \in |\mathcal{C}_2\mathcal{V}| : kA = krA\}$.
4. $|\mathcal{V}| = \{A \in |\mathcal{C}_2\mathcal{V}| : r^A \in \mu\mathcal{K}\}$.

Theorem 3. *Let be $\mathcal{V} = \mathcal{K} *_d \mathcal{R}$, and $v : \mathcal{C}_2\mathcal{V} \to \mathcal{V}$ is a reflector functor. Then*:

1. *The subcategory $\mathcal{V}$ is closed with respect to $\varepsilon\mathcal{R}$-subobjects. In particulary $k(\mathcal{V}) \subset \mathcal{V}$.*
2. *The subcategory $\mathcal{V}$ is closed with respect to $\varepsilon\mathcal{R}$-factorobjects.*
3. *Let be $(E,t) \in |\mathcal{V}|$, $(E,k(t))$ - $\mathcal{K}$-coreplique, $(E,r(t))$ - $\mathcal{R}$-replique by (E,t), and u - locally convex topologies on the space E, with the property $r(t) \leq u \leq k(t)$. Then $(E,u) \in |\mathcal{V}|$.*
4. $\mathcal{V} \subset \mathcal{Q}_{\mu\mathcal{K}}(\mathcal{K}(\mathcal{R}))$.
5. *The subcategory $\mathcal{V}$ is closed with respect to $\varepsilon\mathcal{R}$-subobjects iff $\mathcal{V} = \mathcal{C}_2\mathcal{V}$.*

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Convex covers of undirected graphs

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Graph covering and partitioning are mostly NP-hard problems, but with numerous significant applications. My study is focused on some aspects of undirected graphs covering, in particular convex sets problem (CCS) and partitioning undirected graph into convex sets problem (PCS). Are proved several theorems regarding existence of graphs with fixed number of convex sets which serve as solutions to CCS and PCS problems and some properties of graph covers and graph

partitions. Also, is studying the complexity of these problems and is proved the NP-completeness of some of them.

Selections and fixed points theorems for mapping defined on convex spaces

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Mathematical structures are successfully used during modeling of different social-economic processes. The more general mathematical structures have more possibilities, because every condition from the characterization of the structures puts concrete restrictions on the object of the modeling.

From the formal point of view, in the basis of the conception of the optimization there is a supposition that for every pair of alternatives it is possible to elucidate which of them is better, or, at least, that there is not a better alternative for one of them. In other words, on a set of alternatives it is given a binary relation of preference and the optimal choice is the maximal or the greatest element of the relation.

Let $\mathbb{I} = [0, 1]$ is the closed unit interval. A space X is called a convex space if the set X is non-empty and is given a continuous mapping $c_X : \mathbb{I} \times X \times X \longrightarrow X$ such that $c_X(1, x, y) = x$, $c_X(0, x, y) = y$ and $c_X(t, x, x) = x$ for all $x, y \in X$ and $t \in \mathbb{I}$. The mapping c_X is called a convexor on a the space X. If $Y \subseteq X$ and $c_X(Y) \subseteq Y$, then Y is called a convex subset of the convex space X. The notion of convex structure is due to I.N. Herstain and S. Milnor (see [1]). More general structures were studied in V. P. Soltan. For convex spaces is true the Kuratowski - Knaster - Mazurkievicz Abstract Principle.

The following assertion improved the Yannelis-Prabhakar's Theorem from ([3], p. 47).

Theorem 1. *Let X be a paracompact space, E be a convex space and $\theta : X \longrightarrow exp(E)$ be a set-valued mapping for which $X = \cup\{Int_X\theta^{-1}(y) : y \in E\}$. Then:*

1. There exists a upper semicontinuous mapping $\psi : X \longrightarrow exp(E)$ and a lower semicontinuous mapping $\varphi : X \longrightarrow exp(E)$such that $\varphi(x) \subseteq \psi(x) \subseteq \theta(x)$ and $\psi(x)$ is a finite non-empty set for each $x \in X$.

2. If E is a convex space, then there exists a continuous function $f : X \longrightarrow E$ such that $f(x) \in co\theta(x)$ for each point $x \in X$.

Corollary 2. *Let E be a convex space and $\theta : E \longrightarrow exp(E)$ be a set-valued mapping for which $E = \cup\{Int_E\theta^{-1}(y) : y \in E\}$ and $\theta(E) \subseteq \Phi$ for some compact convex subset of E. Then $x \in co\theta(x)$ for some $x \in X$.*

A set-valued mapping $\theta : X \longrightarrow exp(E)$ is called sub-lower semicontinuous if for each point $x \in X$ and each neighborhood V of 0 in E there exist $z \in \theta(x)$ and a neighborhood Ux of x in X such that $z \in \theta(y) + V$ for each $y \in Ux$.

The following theorem is a specification of the W. K Kim theorem ([3], Theorem 2.7.7.3).

Theorem 3. *Let X be a non-empty compact convex subspace of a topological vector space E and $\theta : X \longrightarrow exp(X)$ be a sub-lower semicontinuous mappings with convex values. Assume that if $x \notin \theta(x) + U$ for some neighbouhood U of 0 in E, then there exists an open convex subset V of E such that $0 \in V$ and $x \notin cl_E\{y \in X : y \in \theta(y) + V\}$. Then the set $\{x \in X : x \in cl_X\theta(x)\}$ is closed and non-empty.*

For convex spaces are true many theorems of the preference theory, obtined by K. Fan, P.C.Fishburn, H.Sonnenenschein, T. C. Bergstrom, A. Borglin and H. Keiding, D. Gale and A. Mas-Colell, W. Shafer, A. Ia. Kiruta, A. M. Rubinov and E. B. Yanovskaia, etc (see [2, 3, 4, 5]).

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On almost periodic functions on dynamical systems

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A topological monoid is a topological space A with a continuous mapping $p : A \times A \to A$ for which there exists a point $1 \in A$ such that $p(1, x) = p(x, 1) = x$ for each $x \in X$. The element 1 is the unity of monoid A and we say that $xy = p(x, y)$ is the product of x, y.

A dynamical system is a triple (S, X, m), where S is a topological monoid, X is a Tychonoff space and $m : S \times X \to X$ is a continuous action on X, i. e. $m(s.m(t, x)) = m(st, x)$ and $m(1, x) = x$ for all $s, t \in S$ and $x \in X$. In theory of finite state machines and in automata theory the dynamical system (S, X, m) is called a semiautomaton, where S is called the input alphabet, X is called the set of states and m is the transition function.

Fix a discrete monoid S and a dynamical system (S, X, m).

for any real valued function $f : X \to \mathbb{R}$ and any $s \in S$ we consider the function $sf : X \to \mathbb{R}$, where $sf(x) = f(m(s, x))$ for each $x \in X$, and put $S(f) = \{sf : s \in S\}$.

Let $F(X)$ be the Banach algebra of all bounded function $f : X \to \mathbb{R}$ with the sup-norm $\| f \| = sup\{|f(x)| : x \in X\}$.

a function $f : X \to \mathbb{R}$ is called almost periodic if $f \in F(X)$ and $cl_{F(X)}S(f)$ is a compact subset of $F(X)$. Denote by $F_{ap}(X)$ the class of all almost periodic functions on X. Let $C(X)$ be the BAnach algebra of all bounded continuous functions on X and $C_{ap}(X) = C(X) \cap F_{ap}(X)$.

A pseudometric $d : X \times X \to \mathbb{R}$ is stable if $d(m(s, x), m(s, y)) \leq d(x, y)$ for all $x, y \in X$ and $s \in S$.

If $f : X \to \mathbb{R}$ is a function, then $d_f(x, y) = sup\{|sf(x) - sf(y)| : s \in S\}$.

Theorem 1. *If $f \in C_{ap}(X)$, then $S(f) \subseteq C_{ap}(X)$. Moreover, $F_{ap}(X)$ and $C_{ap}(X)$ are closed subalgebras of the algebra $F(X)$.*

Theorem 2. *If $f \in F_{ap}(X)$, then d_f is a totally bounded stable pseudometric. Moreover, if $f \in C_{ap}(X)$, then d_f is a continuous pseudometric on X.*

Theorem 3. *If d is a totally bounded stable pseudometric and $f_{(d,a)}(x) = d(a, x)$ for all $a, x \in X$, then:*

1. $f_{(d,a)} \in F_{ap}(X)$ for each $a \in X$.

2. If the pseudometric d is a continuous, then $f_{(d,a)} \in C_{ap}(X)$ for each $a \in X$.

Theorem 4. *The set $\{f_{(d,a)} : a \in X, d$ is a totally bounded stable pseudometric$\}$ is dense in $F_{ap}(X)$.*

Distance function of n-dimensional abstract cubes complex

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The notion of n-dimensional abstract cubes complex, introduced for the first time in [1], is a particular case of the G-complex of multi-ary relations [2], and appears in many applicative problems [3, 4].

Let $C_1, C_2, \ldots, C_m$ be the classes of parallel edges [3] of the cubical complex $T^n = \{I^m, 0 \leq m \leq n\}$ and p_i is the edges length of the class C_i, $1 \leq i \leq m$. We denote by Q^r the set of all r-dimensional cubes, by $L-$ the set of all linear chains of T^n, $0 \leq r \leq n$ and define the function $d : L \to R^+$ as follows

$$d(l) = \sum_{k=1}^{m} \varepsilon_k p_k, \tag{1}$$

for every chain $l \in L$. Here $\varepsilon_k = 0$, if l intersects C_i an even number of times and $\varepsilon_k = 1$, if l intersects C_i an odd number of times. If $I_p^0, I_q^0 \in Q^0$ are extremities of l, then we use the notation: $d(l) = d_l(I_p^0, I_q^0) = \sum_{k=1}^{m} \varepsilon_k p_k$. This number represents the distance between I_p^0 and I_q^0.

The following statements are true:

a) if l_1 and l_2 are two distinct linear chains that connect the vertices $I_p^0, I_q^0 \in Q^0$, then

$$d_{l_1}(I_p^0, I_q^0) = d_{l_2}(I_p^0, I_q^0);$$

b) for the set Q^0 of all 0-dimensional cubes of the abstract cubic complex T^n, the function defined by (1) represents the univocal metrics.

Function (1) is useful in the development of special methods for solving applied problems, particularly, for location problems on discrete structures.

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Some generalization of completeness in 5-valued pseudo-Boolean algebra with two incomparable elements

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The function f of algebra is called parametrically expressed via a system of functions of , if there exists the function which are expressed explicitly via using superpropositons such the predicate is equivalent to on Let us consider the pseudo-boolean algebra where is pseudocompliment and is compliment. We examine the 5 valued pseudo-Boolean algebra definied Evidently the algebra a subalgebra of Z5. The function and are called permitable if the identily. The set of all the functions from algebra permutasbled with a given function is reffered to us centalizator of the function f (dented by) on algebra. Let us define the new function on z5 iff is parametrically complete in subalgebra z3 and the system is not included into the centralizator f on algbra z5. examine chain logics $C_2, C_3, \ldots$, which are intermediary between classical and intuitionistic logics. The formula F is called to be implicitly expressible in logic L by the system Σ of formulas if the relation $L \vdash (F \sim q) \sim ((G_1 \sim H_1) \,\&\, \ldots \,\&\, (G_k \sim H_k))$ is true, where q do not appear in F, and formulas G_i and H_i, for $i = 1, \ldots, k$, are explicitly expressible in L via Σ. The formula F is said to be implicitly reducible in logic L to formulas of Σ if there exists a finite sequence of formulas $G_1, G_2, \ldots, G_l$ where G_l coincides with F and for $j = 1, \ldots, l$ the formula G_j is implicitly expressible in L by $\Sigma \cup \{G_1, \ldots, G_{j-1}\}$. The paper contains the researches of completeness with respect to implicit reducibility in the logic C_m, for any $m = 2, 3, \ldots$.

Some classic and special Lie group structures on some algebraic plane cubic curves with singularities

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It is well known that an elliptic curve (i.e. an algebraic projective *non-singular* plane cubic curve) admits a group composition law such that the curve becomes an algebraic (Lie) group (abelian variety). There are numerous applications of this fact in Geometry, Number Theory, as well in Applied Mathematics (such as Code Theory)

In this talk we deal with an algebraic plane cubic curve (i.e. plane algebraic subset defined by a polynomial equation of degree 3) having *a singularity,* called *Descartes Folium.*

Let K be a field with $char.K \neq 3$ and the affine *Descartes Folium*

$$DF : x^3 + y^3 - 3axy = 0 \subset A^2_K, \text{ with } a \in K - \{0\},$$

having the singular point $O = (0, 0)$. Its projective closure $\overline{DF} \subset P^2_K$ is the projective *Descartes Folium.*

The aim of this talk is to present some natural group composition laws on $\overline{DF}$ or on "good" subsets of $\overline{DF}$.

Suppose that K is algebraically closed (e.g. $K = C$, the complex number field)

Starting with the fact that the normalization of $\overline{DF}$ is the projective line P^1_K and on some very simple Zariski-open subsets $U \subset P^1_K$ appear natural group composition laws, we can transport some of them on some "good" subsets of $\overline{DF}$ via the normalization morphism.

This idea can be extended to an arbitrary base field K with $char.K \neq 3$.

We obtain in this way natural group composition laws on

1) $\overline{DF} - \{0\}$,
2) $\overline{DF}$,
3) DF, if $K = R$ (the real number field).

In this talk we deal mainly with case 1. We can describe all K-algebraic group structures appearing on $\overline{DF} - \{0\}$ if K is algebraically closed (in particular for $K = C$), as well as all C-Lie group structures if $K = C$. If $K = C$ each C-Lie group structure is canonically associated to a unique C-algebraic group structure (An algebraization fact). For all these structures we have concrete algebraic formulae of the group composition laws and a pure geometric description of them which astonishes by a complete analogy with the case of the famous elliptic curves.

For case 2 if $K = R$ or C, as well as for case 3, the found group composition law does not produce a K-Lie, resp. a R-Lie group structure if we consider $\overline{DF} \subset P^2_K$, resp.$DF \subset A^2_R$ endowed with the induced real or complex topology of P^2_K, resp. A^2_R . In this situation we suggest to modify this topology around the singular point O, by defining two finer topologies , corresponding to the branches of the singularity, such that $\overline{DF}$ and DF to become a K-Lie, resp. a R-Lie group ("exotic" Lie structures).

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Location of Zeros of Polynomials

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In this talk, we show classical and new results for the location of zeros of polynomials. In particular, we present upper bounds to estimate the moduli of the complex and real zeros of polynomials. Also, we discuss results and examples when studying graph polynomials.

The Pell-Padovan p-Lengths of Some Special Metacyclic Groups

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In [1], Deveci et.al defined the Pell-Padovan p-orbit $PeP_p(G; x_1, x_2, \cdots, x_p)$ of a group G as following:
Let $(x_1, x_2, \cdots, x_p)$ be a generating p-tuple of the group G. Then the Pell-Padovan p-orbit $PeP_p(G; x_1, x_2, \cdots, x_p)$ with respect to the generating p-tuple $(x_1, x_2, \cdots, x_p)$ to be the sequence $\{b_i\}$ such that

$$b_0 = e,\ b_1 = x_1,\ b_2 = x_2 \cdots,\ b_p = x_p,\ b_{p+1} = e,\ b_{n+p+1} = b_{n-1} \cdot (b_{n+p-1})^2,\ \ n \geq 1.$$

In this work, we obtain the periods of the Pell-Padovan p-orbits of the Fox groups $G_{n,l}$ for $l > n > 1$, $(l, n) = 1$ and the Fibonacci groups $F(r, 2)$, (r odd).

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Topology of spongy nanostructures

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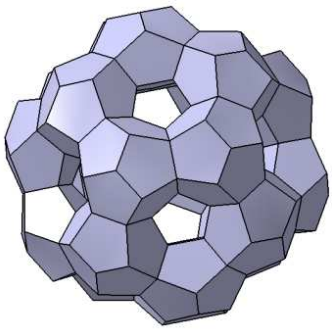
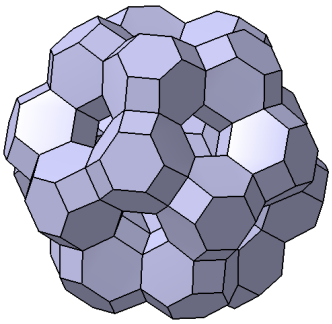
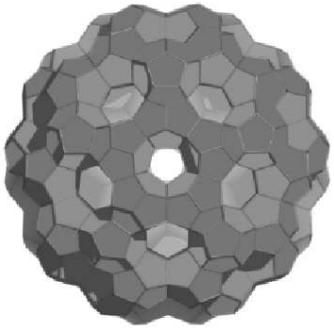
Spongy materials are encountered in nature in zeolites, used as molecular sieves. There are also synthetic compounds like spongy carbon, metal-organic frameworks MOFs, etc, with hollow structure [1], [2]. The design and topological study of some hypothetical spongy nanostructures is presented in terms of map operations and genus calculation [3] of their associated graphs, respectively. Several novel spongy polyhedra (see Figure), that can evolve with 1-periodicity or radially, to provide multi-shell cages, are presented. Filling the space inside these nanostructures can be realised by small fullerenes that self-arranje in aggregates with a well-defined geometry. The way of space filling differ function of the dimensions and shape of composing small fullerenes [4], [5]. The design of nanostructures was performed by original software packages.



C_{60}



$C_{60}(Do_{60})\_750$



$TR(Du(C_{60}(Do_{60})750)662)\_5040$

Spongy polyhedral nanostructures

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Finslerian homotheties and isometries

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We consider a Finsler manifold (M, F) and define a isometry of it as a diffeomorphism f of M whose differential f_* keeps the Finsler function. We say that the diffeomorphism f is an affine transformation if f leaves invariant the (horizontal) distribution defining the standard nonlinear connection of (M, F). The diffeomorphism f is a proper homothety if f multiplies F by a constant $\neq 1$. We give several properties of the affine transformations, we show that the homotheties and the isometries are affine transformations and prove that if the Finsler manifold is forward complete, then any proper homothety has an unique fixed point.

On the properties of structures induced on a product manifold of two metallic Riemannian manifolds

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The main aim of our paper is to give some properties of structures induced on a product manifold of two metallic Riemannian manifolds. A metallic structure on a Riemannian manifold (M, g) is a structure defined as a polynomial structure determined by an $(1, 1)$ tensor field J which satisfies $J^2 = p \cdot J + q \cdot I$, where I is the identity operator on the Lie algebra $\chi(M)$ of the vector fields on M, p, q are fixed positive integer numbers and the metric g is *J-compatible* (i.e. $g(JX, Y) = g(X, JY)$ for every $X, Y \in \chi(M)$).

The semi-minor and pseudo-minor groups of $\bar{6}$-symmetry, generated by tablet groups

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The groups $G^{(P)}$ of $\overline{P}$-symmetry are subgroups, which verify certain conditions, of the right semi-direct products of the defining group P with the generating group G, accompanied by a homomorphism $\varphi : G \to AutP$, where $Ker\varphi = H$ and $G/H \cong \Phi \leq AutP$. The set $P' = \{p | g^{(p)} \in G^{(P)}\}$ is a subset of the group P. Moreover, $e \subseteq P' \subseteq P$.

Note that for P'-semi-minor (respectively, for P'-pseudo-minor) groups P' is a nontrivial subgroup of P (respectively, a subset containing the group unit, which is not a subgroup).

Any P'-pseudo-minor (respectively, any P'-semi-minor) group $G^{(P)}$ of $\overline{P}$-symmetry can be derived from its generating group G and the defining group P, when the kernel H of accompanying homomorphism φ is known, by the following steps: 1) we find in G all proper subgroups H' with the index equal to the power of subset P' (respectively, the nontrivial subgroups P') and for which there is an isomorphism μ from the quotient group H/H'' to P'' ($\mu : H/H'' \to P''$ defined by the rule $\mu(gH'') = p$), where $e \leq P'' \subset P'$ and $H'' = H' \cap H$; 2) we construct a right quasi-homomorphism ψ of the group G onto the P' defined by the rule $\psi(gH') = p$ and which preserves the correspondence between H and P'', using the isomorphism μ; 3) we combine pairwise each g' of gH' with $p = \psi(g')$; 4) we introduce on the set of all these pairs the operation $p_i g_i * p_j g_j = p_k g_k$, where $g_k = g_i g_j$, $p_k = p_i \overrightarrow{\varphi_{g_i}}(p_j)$, $\overrightarrow{\varphi_{g_i}} = \varphi(g_i)$ and $\overrightarrow{\varphi_{g_i}}(p_j) = g_i p_j g_i^{-1}$.

From the 31 generating crystallographic tablet groups G are obtained 59 2-semi-minor, 7 3-semi-minor and 352 P'-pseudo-minor nonequivalent groups of $\bar{6}$-symmetry ($P = \{(123456)\} \cong C_6$). For P'-semi-minor and P'-pseudo-minor inferred groups the polynomial symbols are elaborated. These symbols have the following form: $G|H'[(P, P_i)|P'|P''; H/H'''/H'']$, where G is the generating group, H' is the subgroup of symmetry, (P, P_i) is the symbol of defining group P with the stationary subgroup P_i, $H/H'''/H''$ is the symbol of the subgroup of P-symmetry.

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Quasi-homogeneous time–dependent geometry

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The so-called time-dependent geometry was intensively studied by M. Anastasiei and H. Kawaguchi (see [1, 2, 3, 4]). A survey on this topic can be found in [8] and the case of multi–parametrized Lagrangians was also studied in [5]. In particular, the geometry of time-dependent Lagrange spaces gives few models for both the gravitational and electromagnetic fields in a very natural blending of the geometrical structure of the space with the characteristic properties of the physical fields, cf. [8].

The homogeneous lift of a Riemannian metric was introduced and studied by R. Miron in [7] and the corresponding geometrical model was used in [9] in order to develop a homogeneous gauge theory on the total space of the tangent bundle of a Riemannian manifold. Furthermore, in [6] some notions from [2, 3, 4] were extended to the case of a generalized reonomic metric. Namely, the geometry of generalized reonomic Lagrange spaces were taken into consideration instead of the geometry of reonomic Lagrange spaces.

This paper is a continuation of our previous works [6, 9]. Let M be a differentiable manifold endowed with a Riemannian metric γ, and let TM be its tangent bundle. Here, the construction of the time-dependent geometrical model on $TM \times \mathrm{R}$ is based on the homogeneous lift of the Riemannian metric γ which determines a generalized reonomic metric on $TM \times \mathrm{R}$. This one is not homogeneous but has the "essential" part homogeneous, so that it is natural to call it "quasi-homogeneous". With this new metric, the geometry of the space $TM \times \mathrm{R}$ is further developed.

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Topological symmetry of spongy nanostructures

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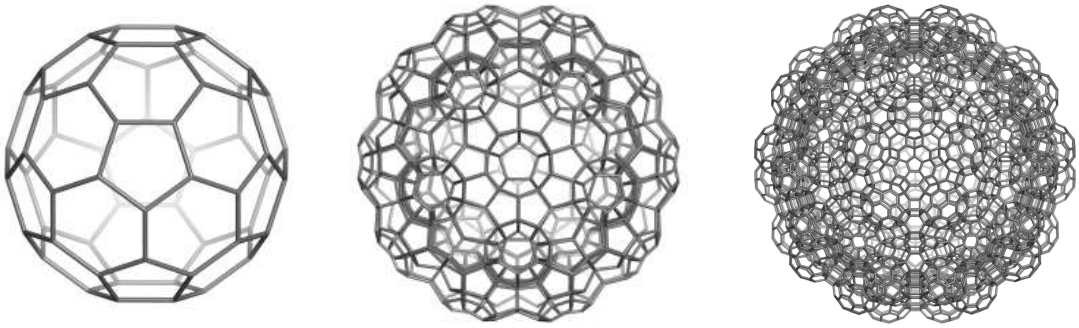
Spongy structures are hollow-containing materials, encountered in natural or synthetic zeolites, spongy carbon, etc. [1], [2]. The design of some hypothetical spongy nanostructures is performed by using some topological transformations, called map operations [3], [4]. Topological symmetry is referred to the maximum possible symmetry achievable by a given molecular structure; it can be performed either by permutations on the adjacency matrix or by calculating the value of some topological indices [5]. The equivalence classes of vertices/atoms, edges/bonds and faces of the nanostructures in figure are solved by using an index of centrality, computed on the layer matrix of distances. The design of nanostructures was performed by original software packages.



C_{60} $C_{60}(Do_{60})\_750$ $TR(Du(C_{60}(Do_{60})750)662)\_5040$

Spongy polyhedral nanostructures

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Combinatorial Optimization Algorithms for P_4-sparse graphs

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The class of P_4-sparse graphs was introduced by Hong as the class of graphs for which every set of five vertices induces at most one P_4 and Hong also gave a number of characterizations for these graphs. The class of P_4-sparse graphs generalizes both the P_4-free and the P_4-reducible graphs. Class of cographs (P_4-free) was introduced by Lerchs and P_4-reducible graphs were introduced by Jamison and Olariu as those in which no vertex belongs to more than one induced P_4. Both cographs and P_4-reducible graphs can be recognized in linear time. Jamison and Olariu gave a constructive characterization asserting that P_4-sparse graphs are exactly the graphs constructible from single-vertex graphs by three graph operations. This result leads to a linear time recognition algorithm for this class. The classes of P_4-sparse graphs, cographs and P_4-reducible graphs have applications in many areas of applied mathematics, computer science and engineering, mainly because of their good algorithmic and structural properties. We give a characterization of P_4-sparse graphs using weak decomposition. We also give recognition algorithms for P_4-sparse graphs and we determine the combinatorial optimization numbers in efficient time.

Mathematical Aspects of Molecular Structure Descriptors

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Vlado Prelog just before winning the Nobel Prize in Chemistry (1975) made a simple but profound comment about Graph Theory in a foreword for the book *Chemical Applications of Graph Theory* (A. T. Balaban Ed.). He said: "Pictorial representations of graphs are so easily intelligible that chemists are often satisfied with inspecting and discussing them without paying to much attention to their algebraic aspects, but it is evident that some familiarity with the theory of graphs is necessary for deeper understanding of their properties." Graph invariants that are useful for chemical purposes were named *topological indices* or, less confusing, *molecular structure-descriptors*. Their main use is for designing so-called quantitative structure-property relations, *QSPR* and quantitative structure-activity relations, *QSAR*. Here structure means molecular structure, property some physical or chemical property, and activity some pharmacologic, biologic, toxicologic, or similar property, both the properties and the activities are understood to be measurable numerical quantities. A large number of various topological indices was proposed and considered in the chemical literature. This talk presents some new extremal results and open problems concerning degree distance, general sum-connectivity index and general Randić index, which are defined using the distances between vertices and the degrees of the vertices of a graph.

About Some Properties of the Group $G^{\square}$

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A topological group G is said to be ω - narrow if for every open neighborhood U of 1_G there exists a countable subset S such $SU = G$ [1]. The symbol $G^{\square}$ denotes an Abelian group G equipped with the maximal ω - narrow group topology. Some properties of the group $G^{\square}$ were investigated in [2].

A space X is called to be realcompact (hereditarily realcompact) if X can be embedded as closed subspace of a power of the real line (if every subspace of X is a realcompact space). Below $d(X)$, $\psi(X)$ denotes the density and pseudocharacter of a space X.

Theorem 1. *For a discret Abelian group, the following conditions are equivalent*:
1) $|G| \leq 2^{\aleph_0}$;
2) $\psi(G^{\square}) \leq \aleph_0$;
3) $G^{\square}$ *is hereditarily realcompact*;
4) $G^{\square}$ *admits a continuous isomorphism onto a second-countable topological groups.*

Theorem 2. *Let G be an infinite discrete Abelian group. Then*:
1) $\psi(G^{\square}) = Ln|G|$;
2) $d(G^{\square}) = |G|$.

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On Quasi-Equational Teory of Finite Algebraic System

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One of the important problem in the quasivarieties theory area is the problem of existing the finit base of the real quasiidentities in a given algebra, actual in the present. It is well known that have no finit base the quasiidentities of noncommutative nilpotent finit group [1], of a nonassociative or associative ring finit nilpotent with the no null produce ([2], [3]), of noncommutative or nonassociative finit nilpotent Moufang loop ([4],[5]). At this moment it is necessary to investigate the finit base existance of the quasiidentities of a finit number fixed of variables of a finit algebra. In the present paper this problem is solved for the finit algebric system of finit signature.

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A Method for Constructing DNA Codes from Additive Self-Dual Codes over $GF(4)$

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Coding theory has several applications in Genetics and Bioengineering. Every DNA molecule consists of two complementary strands which are sequences of four different nucleotide bases. These are called adenine (A), cytosine (C), guanine (G) and thymine (T). The problem of designing DNA codes (sets of words of fixed length n over the alphabet {A, C, G, T} that satisfy certain combinatorial constraints) has applications for reliably storing and retrieving information in synthetic DNA strands. There are four certain constraints for DNA codes [1]: the Hamming distance constraint, the reverse constraint, the reverse-complement (RC) constraint and the fixed GC-content constraint.

In this work we translate in terms of coding theory constraints that are used in designing DNA codes for use in DNA computing. We focus in particular on additive codes over $GF(4)$, and we propose a new method for constructing DNA codes satisfying the Hamming distance constraint, the RC-constraint, and the fixed GC-content constraint. Our method is based on additive self-dual codes with circulant generator matrix in graph form (that is a generator matrix $G = \Gamma + \omega I$ where I is the identity matrix and Γ is the adjacency matrix of a simple undirected graph, which must be symmetric with 0's along the diagonal). An additive circulant graph (ACG) code is a code corresponding to graph with circulant adjacency matrix. It is easy to see that such matrix has the following property: $b_i = b_{n-i}$, for all $i = 1, \ldots, n-1$, and $b_0 = \omega$. We can restrict our search space to the codes over $GF(4)$ of length n corresponding to graphs with circulant adjacency matrices.

The graph codes are proper to construct DNA codes with fixed GC-content u that satisfy the Hamming distance constraint for given d. If we know already that the minimum distance of the code is at least d, the Hamming distance constraint is satisfied for this value of d. Any codeword that is a sum of u rows of the generator matrix G has GC-weight u. Also, about RC-constraint we can see that if R is a reverse permutation then any codeword that is a sum of even number of rows of G with '*symmetric*' indexes (i.e. i^{th} row and $(n-i+1)^{th}$ row, for any $1 \leq i \leq n/2$) is fixed by R. In this way it is easy to determine the set C_0 (the set of codewords fixed by R). This set is very important in order to find the codewords that satisfy the RC-constraint. By $A_4^{GC,RC}(n,d,u)$ we denote the maximum size of a DNA code of length n with constant GC-content u that satisfies the Hamming distance constraint for a given d and also satisfy the RC-constraint for these parameters. In [2] lower bounds for $A_4^{GC,RC}(n,d,u)$ are given. In this work we improve some of these bounds. For example, in [2] $A_4^{GC,RC}(30,12,u) = 281928$. By our construction method we obtain $A_4^{GC,RC}(30,12,15) = 77558760$. Also, by this method we obtain some results for $n > 30$ (all of them are new bests).

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6. Mathematical Modeling

Three-dimensional hydromagnetic flow arising in a porous elliptic slider

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The problem considered here is the injection of a viscous fluid through a moving elliptic plate in the presence of a transverse uniform magnetic field. The solution of such a flow model has applications in fluid-cushioned porous sliders, which are useful in reducing the frictional resistance of moving objects. The governing equations are reduced to a system of nonlinear ordinary differential equations by means of appropriate transformations for the velocity components. The resulting boundary value problem is solved numerically using the Matlab routine bvp4c. The solutions presented in this research include the special cases corresponding to a porous circular plate and a porous flat plate. The influence of the magnetic field on the velocity components, load-carrying capacity and friction force are discussed in detail with the aid of graphs and tables.

Non-inertial flow of a third order fluid in a corner region with a moving wall

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The problem dealing with the two dimensional steady and slow flow of a third grade fluid between intersecting planes, one of which is fixed and the other moving, has been analysed. Such flows are of considerable practical interest as they locally appear at the edge of a blade used to scrape up liquid from a surface, in a cylinder with a moving piston, or in a screw extruder. Using a truncated series expansion for the stream function, the governing equations of the problem are reduced to linear ordinary differential equations. These equations have been solved analytically subject to the relevant boundary conditions. Our current results reduce the special cases corresponding to a Newtonian fluid, a Maxwell fluid, an Oldroyd-B fluid, and an Oldroyd 6-constant fluid for different values of involving parameters. The effects of the non-Newtonian parameters on the flow pattern are carefully delineated. There is, unlike the case of Newtonian fluid, a secondary flow near the corner.

Adaptive-robust control of spatial robot arm

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Development of technology in recent years, the use of robotic manipulators have expanded their fields. Nowadays, robots operate in many fields such as industrial, medical, automotive and military industry. With the expansion of the use of robots in the area, the robots are required to be worked with minimum error. Hence the aim of this study is to increase the yield of the robot

and minimize the trajectory tracking error. The robots parameters can @ Yt be calculated exactly because of changing the weight and the shape of the object being handled. And these systems are under the influence of external disturbance like wind, force and torque. Because of these external disturbances, robot parameters can not be calculated exactly. In this study, parameters of robot are assumed to be unknown. In such systems, adaptive and robust controllers give better results than current controllers such as PI and PID. In this study, adaptive and robust control laws were examined and a new control system was designed. First forward and inverse kinematic analyses of the three-axis robot model are made and motion equations are obtained. Then a new Lyapunov function is defined and this adaptive-robust controller was derived from new Lyapunov function. And this new control system applied to a three-axis robot model. Finally it is understood that, the results of the new developed controller is closed to expected results and they are satisfactory.

Mathematical theory of M.H.D. power generators

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We use an asymptotic expansion with respect to the small number $N = Rm \cdot Rh$ (Rm stands for the magnetic Reynolds ' number and Rh stands for the magnetic pressure) and solve the first approximation of the M.H.D generators problem. To this aim we use L. Dragoş 's analytic solution for the electric field [1] and then we employ numerical calculations to obtain the generator power and to represent the electric current density.

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Qualitative analysis for delayed recurrent neural networks

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Many scientific studies have proven that an animal continuously senses its environ- ment via different perceptual means and integrates the sensory information to adapt its behavior. The temporal aspect of this integration is fundamental for the sensory perception.

A population of neurons makes a success of this dynamic integration by an intricate combination of synchronization of potential of action and recurring connections. Inspired by this biological mechanism, recurrent neural networks (RNNs) are believed to be a powerful sequence processing method. Recurrent interactions among large populations of neurons are expected to yield collective phenomena adapted for dealing with temporal behavior.

Besides, the stability of dynamical systems in presence of time-delay is a problem of big interest since the presence of a time-delay may induce instabilities, and complex behaviors for the corresponding schemes. In particular, the problem becomes even more di cult in the case when the delays are distributed.

As we all know, many phenomena in nature have oscillatory character and their mathematical models have led to the introduction of certain classes of functions to describe them. Such a class

form pseudo almost periodic functions which a natural generalization of the concept of almost periodicity. This paper is concerned with the existence and uniqueness of pseudo almost-periodic solutions and/or pseudo almost-automorphic solutions to some delayed neural networks. Several conditions guaran- teeing the existence and uniqueness of such solutions are obtained in a suitable convex domain. Furthermore, several methods are applied to establish sufficient criteria for the globally exponential stability of the considered models.

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Investigation of electromagnetic field problems by systems of partial differential equations

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Symmetrical differential Maxwell system as mathematical simulation of homogeneous isotropic expofunctionally excited media [1] is given

$$\begin{cases} \mathbf{rot}\vec{H} = (\sigma \pm \lambda\varepsilon_a)\vec{E} + \varepsilon_a\partial_0\vec{E} + \vec{j}^{OS} \\ \mathbf{\text{-}rot}\vec{E} = (r \pm \lambda\mu_a)\vec{H} + \mu_a\partial_0\vec{H} + \vec{e}^{OS}, \end{cases} \tag{1}$$

where: $\partial_0 = \frac{\partial}{\partial t}$; $\vec{E}, \vec{H} = \vec{E}, \vec{H}(x, y, z, t)$ are the unknown vector field electromagnetic intensities with scalar components $E_k, H_k = E_k, H_k(x, y, z, t)$ $(k = \overline{1,3})$; $\sigma, \mu_a, \varepsilon_a = const > 0$ represent the specific conductivity, absolute magnetic and dielectric permeability of the medium;

$\lambda = const > 0$ is the parameter of the signal excitative the medium, and "$\pm$" in front of λ mean the absorption of signal and activity of a medium respectively. Theoretical constant $r > 0$ is added for the system symmetry, and can be deleted at the end of computing not infringing the original statement. The known functions $\vec{j}^{OS}, \vec{e}^{OS} = \vec{j}^{OS}, \vec{e}^{OS}(x, y, z, t)$ with scalar components $j_k^{OS}, e_k^{OS} = j_k^{OS}, e_k^{OS}(x, y, z, t)$ $(k = \overline{1,3})$ are responsible for the outside current sources and intensities.

System (1) describes quite a lot of phenomena in technical electrodynamics, such as wave or signal propagation in various media, multidimensional circuits and filters with the distributed parameters as well.

More interesting case of (1) for heterogeneous media when $\sigma, \mu_a, \varepsilon_a$ are not constants but depend on the spatial coordinates (x, y, z), is considered completely in [2]. Jointly with [3], solvability conditions in the class of non generalized functions are found, and respective criterion is proved in the meaning of heterogeneity of (1) equivalence to the general wave equation. The latter depends on all unknown electromagnetic field intensities simultaneously. Explicit solution of this PDE (partial differential equation) is got irrespectively of specific boundary conditions [2].

In spite of the heterogeneous general statement for (1), its specific case of mathematical simulation of electromagnetic wave propagation in heterogeneous lines is rather interesting too. Corresponding system of PDEs is responsible for the mentioned process and looks like

$$\begin{cases} \partial_1 H = (\sigma \pm \lambda\varepsilon_a)E + \varepsilon_a \partial_0 E + j^{OS} \\ -\partial_1 E = (r \pm \lambda\mu_a)H + \mu_a \partial_0 H + e^{OS}. \end{cases} \tag{2}$$

In (2): $\partial_1 = \frac{\partial}{\partial x}$; $H, E = H, E(x, t)$ and $j^{OS}, e^{OS} = j^{OS}, e^{OS}(x, t)$ are unknown and given scalar analogies with respect to (1); λ and r remain the same as in (1); $\sigma, \varepsilon_a, \mu_a = \sigma, \varepsilon_a, \mu_a(x, t)$ are of the same physical meaning as in (1), but here they determine heterogeneity of lines.

Using the same technique as for (1) in the presence of heterogeneity [2, 3], system (2) is reduced to the general wave PDE, and respective boundary problems are formed. They represent mathematical models of electromagnetic wave propagation in various heterogeneous expofunctionally excited lines during arbitrary time intervals. For finite ones, complete analytic solutions are got in [4], though more peculiar and complicated case of any heterogeneous line and arbitrary large time gaps was not considered till now. So, explicit solution of the last formulated problem and comparative analysis between finite or infinite temporal values are done in the given article in detail.

In conclusion, it should be noted crucial importance of arbitrary numerical (especially non positive) values of $\sigma, \varepsilon_a, \mu_a = \sigma, \varepsilon_a, \mu_a(x, t)$ in the heterogeneous statement for application to metamaterials [5], [6].

Moreover, suggested results are specific parts of two more or less new general operator methods dealing with finite dimensional systems related to vector field functions and systems of PDEs, in particular.

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The heat diffusion in a thin alveolar structure

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The paper presents the propagation of heat in a thin, tall, alveolar structure, in which the material of the alveoli and the material of the rest of the structure's surface have different conductibility coefficients. The problem contains a parabolic equation with nonzero conditions on the alveoli border. For solving the problem we use homogenization methods following two small parameters and, finally, we obtain a mixed problem for one-dimensional heat equation.

Fuzzy linear programming approach for determining the manufacturing amount of different parts in automotive supply industry

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In recent years, rapid and correct decision making is crucial for both people and enterprises. However, uncertainty makes decision-making difficult. Fuzzy logic is used for coping with this situation. Thus, fuzzy linear programming models are developed in order to handle uncertainty in objective function and the constraints. In this study, a problem of a factory in automotive supply industry is investigated, required data is obtained and the problem is figured out as a fuzzy linear programming model. The model is solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of manufacturing for each part type in order to gain maximum profit.

Fuzzy linear programming approach and an application in a food company

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In recent years, rapid and correct decision making is crucial for both people and enterprises. However, uncertainty makes decision-making difficult. Fuzzy logic is used for coping with this situation. Thus, fuzzy linear programming models are developed in order to handle uncertainty in objective function and the constraints. In this study, a problem in a food company is investigated, required data is obtained and the problem is figured out as a fuzzy linear programming model. The model is solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each product type in order to gain maximum profit.

Estimating oil reserves in Kucova oilfield by using stochastic methods and Monte Carlo simulation

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Reserves estimation is always affected by technical uncertainty and errors. The first level of ancertainty is associafed with one-dimensional data (well logs, cores, well tests, analyses etc). These data provide reservoir properties such as porisity, oil and gas saturation, oil viscosity etc. The second devel of uncertainty arises when the first data one- dimensional reservoir properties are extrapolatel to two and three dimensisns with the help of geologis, seismic and production tests data. Reserves-estimation methods are usually classified as unalogy, volumetric and performance types. Volumetric and performance methods are the more often used technicues and the main difference between the two is the type of data we use. The two established volumetric approved are deterministic and stochastic. In case of deterministic method, mathematicas vormulas are used to estimate volumes or reserves. In different, the stochastic approach considers the fact that all the parameters are not presented with a single value, but are included in an interval of values. Each parameter follows a fit statistical distribution which is to be found out and used properly. The results from stochastic calculations are summarized generally by a descending (reverse) cumulative probability function commonly known as expectation curve. We use both methods, deterministic and stochastic in estimating reserves in the case of Kucova oilfield, Driza sector.
Keywords: Reserve, prebibility, oilfield, stochastic, analogy, volumetric, simulation.

Multiresolution analysis of spatial enviromental data

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In this talk we provide a tool to analyse discrete spatial data. The method is based on an interpolation method; the distribution of the data is irregular but with some restrictions. The interpolant function is a bicubic spline which can exactly reconstruct polynomials up to degree three in each variable. We validate the method by analyzing errors in some theoretical case studies followed by other studies with real terrain elevation data.

Soil erosion on a vegetated surface

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In this talk we present a general partial differential equations based model of water flow and soil erosion on a hill covered by plants. The model is of hyperbolic type with source terms. We analyze different scenarios concerning mechanical interactions among the three different media implied in the process: plants, water and soil. Some numerical results are also presented.

Low radiation pulse formation in a semiconductor photo-detector

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In semiconductors photo-detectors the collection of mobile electrical charges in the region with fixed spatial electrical charge of a p-n junction, revere biased, it depends on the initial distribution of electrical charges generated either by thermal (the only one in darkness) or by optical radiation absorption (usually exponentially) with its energy quantified at minimal photon level. The produced electrical signal is affected in the first by the own photo-detector electronic noise. In very low irradiance conditions (specific to cosmic radiation or long-distance optical communications) presents interest the detection in photon counting mode. This paper examines, by the particularization to the one-dimensional case of generalized Ramo?s theorem, the generation of pulses into "single photon detector" mode also taking into account the avalanche multiplication phenomenon. Numerical simulations are performed in the MathCAD software.

Modelling of the gradient-force in the optical trap

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The optical trap uses the electromagnetic field of a laser focused beam in order to immobilize and manipulate with high accuracy a nonconductive microparticle. For particles with dimensions smaller than the laser wavelength the force responsible for the particle movement is that one due to the nonuniformity of the beam intensity in its section (the gradient force). Its study allows the monitoring of the particle migration with respect to the laser beam.
In the paper, starting from existing evaluations for a focused single beam optical trap, the gradient force acting on a water immersed polysaccharide microparticle is studied. In the physical modelling the relationships between the optical parameters of the studied microparticle (atomic or molecular polarizability, refractive index, etc.) and the physical characteristics of the laser beam are systematized.
In the mathematical modelling and the numerical simulation the atraction of the microparticle towards the beam axis under the gradient force was revealed. The software environment MathCAD with standardized units and relationships (dedicated to the engineering design) as well as normalized representations are used. The modelling allows the laboratory study of the influence of both model parameters and experimental conditions upon the microparticle positioning monitoring in a single beam optical trap.

Surface modeling of the left ventricle of the heart

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The surgical treatment of mitral valve disease has dramatically improved with the recent development of repair techniques that avoid valve replacement with artificial valve. A better understanding of the dynamics of the normal and diseased mitral valve is necessary in order to design and test new repair maneuvers that would increase the scope of this surgery.
In this talk I will explain how to produce wireframe images (model) of the left ventricle and mitral valve using Sonomicrometry distance data. The same data is used to construct the pressure-volume loops as the most reliable load independent index of left ventricular contractility. I will also discuss a proposed three-dimensional surface model that is based on three-dimensional (3D) Sonomicrometry data.

An approach about robustness using non-linear scalarizing functionals

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In paper we will try to solve an economic problem using mathematical tools. We will show that many concepts of robustness can be described as being special cases of a general method for nonlinear scalarization by choosing properly the parameters involved and the corresponding sets which appear in the problem as well. Solving the robust counterpart of the problem we will obtain the solution of the economic problem approached using non-linear scalarizing functionals.
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Research of dynamics and stability of elastic elements of system of wing profiles

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At design of constructions and the devices which are in interaction with a flow of gas, it is necessary to solve the problems connected with research of stability, demanded for their functioning and reliability of operation. Determination of the stability of an elastic body corresponds to the concept of stability of dynamical systems by Lyapunov. The dynamics and dynamic stability of the elastic ailerons of a wings taking into account a flow by a subsonic stream of gas or fluid (in model of the ideal incompressible environment) is investigated. The model are described by the related system of the partial differential equations for unknown functions - the potential of velocity of gas and deformations of an elastic ailerons.

Let on xOy plane in which there are joint fluctuations of elastic ailerons and a subsonic flow of ideal gas (liquid), to wings there correspond on Ox axis pieces $[a_1, b_1]$ $[a_2, b_2]$, and to ailerons - pieces $[b_1, c_1]$ $[b_2, c_2]$, $a_2 > c_1$. In an infinitely far point the velosity of gas is equal V and has the direction coinciding with the direction of an axis of Ox. We will enter designations $w_1(x,t)$, $w_2(x,t)$ $\varphi(x,y,t)$ – respectively deflections of ailerons and potential of velosity of a gas flow, where x, y – the Cartesian coordinates, t – time. The mathematical definition of the problem has an appearance:

$$\varphi_{xx} + \varphi_{yy} = 0, \quad (x,y) \in G = R^2 \setminus ([a_1, c_1] \cup [a_2, c_2]), \tag{1}$$

$$\left(\varphi_x^2 + \varphi_y^2 + \varphi_t^2\right)_\infty = 0, \tag{2}$$

$$\varphi_y^{\pm} = V f_k^{\pm\prime}(x),\ x \in (a_k, b_k),\ \varphi_y^{\pm} = v_k,\ x \in (b_k, c_k),\ k = 1, 2, \tag{3}$$

$$M_k \ddot{w}_k + D_k w_k'''' + N_k w_k'' + \delta_k \dot{w}_k'''' + \beta_k \dot{w}_k + \gamma_k w_k = \rho\left(\varphi_t^+ - \varphi_t^-\right) + \tag{4}$$

$$+\rho V\left(\varphi_x^+ - \varphi_x^-\right),\ x \in (b_k, c_k),\ y = 0,\ k = 1, 2,$$

where $\varphi_y^{\pm} = \lim\limits_{y \to 0 \pm 0} \varphi_y(x, y, t)$; $v_k(x,t) = \dot{w}_k(x,t) + V w_k'(x,t)$, $k = 1, 2$; functions $f_k^{\pm}(x)$ defines a form of profiles; the point designates a derivative on t, and a stroke – a derivative on x; ρ, M_k, D_k, N_k, δ_k, β_k, γ_k – some constant parameters of mechanical system.

The solution of aerohydrodynamic part of a problem is constructed by methods of the theory of functions complex variable, thus aerohydrodynamic loading (pressure of liquid or gas) is defined by the functions describing unknown deflections of ailerons. Then the solution of an initial and boundary problem for the related system of the integro-differential equations with private derivatives for unknown functions of deformations of elastic ailerons and potential of speed of liquid (or gas) is consolidated to research of the related systems only for functions of deformations of elements. The research of dynamics and stability is conducted on the basis of Bubnov-Galerkin's method for last system.

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Adsorption of fluorine on siliceous adsorbent: kinetics modelings

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For the design and effective operation of adsorption processes necessary to the equilibrium data were related to the kinetic model and mass transfer. Adsorption equilibrium models linked to the solid phase with a volume concentration as a function of the properties of the solution and the sorbent, such as fluorine concentration, pH, and temperature of the solution, the presence of foreign ions and heterogeneity centers. In preliminary experiments on the adsorption of fluorine on modified trepel the optimal conditions for its removal from the solution, depending on the solution pH, mixing rate of the trepel with a solution, processing time, the ratio of solid:liquid were determined. Under these optimal parameters adsorption isotherms and kinetics curves were obtained. The kinetics of adsorption of fluorine from aqueous solution on trepel surface modified have been studied at room temperature. For evaluation of adsorption properties and mechanism the adsorption isotherms of fluorine have been constructed Adsorption data have been fitted by the models Langmuir and Freundlich. Adsorption capacity of trepel for fluorine in the studied range of concentrations (0-10mmol/L) is found to reach 25mg/g. The Langmuir model is found to be the best for experimental data description with coefficient correlation R2= 0.9973. In order to examine the controlling mechanism of the sorption process of F^- ions on modified trepel at 20°C, three kinetic models were used:
intraparticle diffusion model of Weber-Morris

$$a_t = k_d t^{0.5} + C$$

pseudo-first order model Lagergren

$$\ln(a_e - a_t) = \ln a_e - K_1 t$$

and pseudo-second order model Mackay and Ho

$$t/a_t = (1/K_2)(1/a_e^2) + t/a_e,$$

where:
a_m and a_t are the amounts of F^- ions adsorbed at equilibrium and at time t, respectively (mg/g),

k_1 [min-1], is the rate constant of the pseudo-first order kinetics
k_2 [g/(mg min)] is the rate constant of the pseudo-second order kinetics
k_d [g/mg min0.5] is the intraparticle diffusion rate constant
C is a constant that gives an idea about the effect of boundary layer thickness.
The equations parameters have been determined and it has been shown that pseudo-second order model is the best for fitting to experimental kinetic curves, indicating the chemical character of interaction of fluorine with active sites of the trepel surface. It is also noted that the mechanism of fluorine removing is controlled by intraparticle diffusion and superficial adsorption.

7. Computer Science

Automated system of digital processing of interferograms for high accuracy characterization of thin films

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There are a lot of human activity domains that use information technologies. Area of research is not an exception. There are various integration possibilities of information technologies in the research domain. One of them is creating applications used in processing of the experimental data coming from different optical instruments. Of particular interest in this regard is the digital processing of data containing in the object Rs holograms and interferograms obtained by the interferometric methods. Processing of interferogram Rs images can give us high precision data regarding the thickness or the refraction of the thin films used in semiconductor technology. In order to increase the accuracy of some parameter measurements, the calculation can be performed many times in different areas of the image. The result of the measurement session can be saved in a database for statistical processing and future utilization. In article described is the interferograms processing software for high precision measurements of the submicron films thickness and their refractive index. The measurements are conducting by the conventional microinterferometer MII-4 with attached web camera. The processing algorithm is based on the method of list-squares fitting of interference fringes in image. In contrast to the methods of profilometry or scanning force microscopy, this method is much cheaper, noncontact, and does not require complicated specimen preparation.

Knowledge-based recognition of objects in aerial images

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The purpose of the project is to build a fast and reliable software for recognition of objects in aerial images. The preliminary stages consist of an edge detection by the analysis of a color gradient vector. After edge detection and feature extraction of the three-dimensional color histogram a set of parameters is extracted. The parameters taken into account to recognize regions are the histograms of the R, G, and B color components of the pixels and characteristics on the homogeneity and the shape. Principal Component Analysis was performed for examining relationships among several quantitative variables. The programmation language used operationally to translate is Java, Software Development Environment was Eclipse platform with Swing components for the Human Machine Interface.

The software was applied to 400*400 pixels images. Real images have been tested to evaluate the performance concerning translation, rotation, zoom, and random noise addition. For reason of speed, the analysis can be made on only a part of the image, until 1/8 of the image.

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Computational study on solving a dynamic problem with uncertain data using an Ant-based system

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The current real-life problems are difficult in several ways: the solution space is extremely large, many restrictions are hard to meet, and some uncertainty, inconsistency and/or dynamicity are manifested. The results provided by algorithms designed for classic, academic problems are biased when approaching these rich problems and to a-priori find or to estimate their quality are not easy tasks. This paper studies the resilience of an Ant Colony Optimization implementation when approaching a rich problem, with several complexity features. The goal of this work is to assess the outcome of a biologically-inspired solving model when the complexity of the problem approached is enriched. The results show that, at a reasonable level, the new challenges are over-came by the application, as it provides results of similar quality.

Parallel algorithms for solving informational extended games

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We propose a parallel backward induction algorithm that solves informational extended games. As base to this algorithm was taken the Zermelo's well known algorithm [1].

Algorithm:

1. Use the MPI functions $MPI\_comm\_group, MPI\_Group\_incl$ and MPI_Comm_create to create the new communicator and new group of processes;

2. Use the function $MPI\_Scatter$ to send the data of the sequence games to different processes in the communicator, that processes should determine the set of penultimate nodes $P_t = \{m_i \in M : s(m_i) \in T\}$ and for every penultimate node processes should construct the set of successor nodes $S(m_i) = \{m \in M : s(m) : M \to \{m \cup \emptyset\}\} \subset T$. All this will be done using a set of verifications and comparisons;

3. For every set of successor nodes ,determined earlier, use the reduction function MPI_Reduce with reduce operation MPI_MAXLOC to determine the maximum payoff of the player who is to choose and the action that leads to it. In this way for every penultimate node determine a node and the action that leads to it which maximizes the payoff of player who is to choose in that penultimate node;

4. The root node will perform the replacements: $T \leftarrow (T \setminus S(m)) \cup \{m\}$, $X \leftarrow X \setminus \{m\}$, $u(m) \leftarrow u(\nu(a|m))$;

5. If the initial nodes of every sequential game were reached the root process will calculate the payoff of every player as an average value of payoffs obtained in all subgames $u_i = \sum p \cdot u_i(m)$ else will go to *step 2*.

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Soft Computing vs. Hard Computing

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Hard computing is conventional computing and requires a precisely stated analytical model. But many analytical models are valid for ideal cases only. The real world problems exist in a non-ideal environment. The term Soft Computing was coined by L.A. Zadeh in the early 90?s. In according with Zadeh?s definition, Soft Computing is based on Fuzzy Logic, Neural Networks, Support Vector Machines, Evolutionary Computation, Machine Learning and Probabilistic Reasoning. Soft computing can deal with ambiguous and noisy data. Soft computing is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for Soft Computing is the human mind. The paradigm shift ?from Kelvin to Zadeh? becomes urgent to keep pace with a rapidly changing e-world. The goal of this paper is to present, analyze, compare and discuss a few of the benefits and limits of these two computational paradigms.
Keywords and phrases:well-defined problem, fuzzy ?defined situation, deterministic, nondeterministic, precision, imprecision. %

Principal Component Analysis applied to Real Images

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The project consists in determining regions in aerial images with use of statistical methods. The statistical methods are the Principal Component Analysis and the HAC (Hierarchical Ascending Classification). For this purpose, the preliminary stages consist of a fast and reliable edge detection, and a construction of regions with analysis of the three-dimensional color histogram. Principal

Component Analysis was performed for examining relationships among several quantitative variables. The result of the edge pixel detection procedure is used for clustering-based segmentation in order to extract meaningful regions representing the objects in the scene. SAS (Statistical Analysis System) was used extensively and adaptations and specializations of some procedures were made with SAS/IML package. The software was applied to 400*400 pixels images .The performance has been evaluated with real images concerning translation, rotation, zoom, and random noise addition. A practical operationnal system has been implemented, with Java language.

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[8] ***, Matlab (interactive Matrix Laboratory).

Modern technology for the efficiency data processing

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This research aimed at the possibility of using modern technology for the efficiency data processing.The efficiency time and space processing of parallel algorithms can provide significant advantages in various fields such as medicine, biology, astronomy, forensics, which are based on the use and interpretation of specific images, also they can be used in the training of specialists (education) from various fields. In this paper we describe the opportunities for object detection and recognition of images. Recognition is performed in following way: In the detection phase of the Viola @ SJones[1] object detection framework, a window of the target size is moved over the input image, and for each subsection of the image the Haar-like feature[2] is calculated.

Used formula for image search is

$\sum_{i=0,j=0}^{W,H} \mid I(x+i, y+j) - J(i,j) \mid$

Where W and H - dimensions of image, I - original image , J - Haar Cascade x,y - coordonates of object. The sum should be minimal. Reducing time of image processing, would entail reducing costs by using these processing techniques of information.

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Modelling business processes by Petri nets

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Petri nets are a mathematical model used for the specification and the analysis of complex systems, like parallel or distributed systems ([1]).

They were introduced by C. A. Petri in 1964 as a graphical and mathematical tool for modelling and validation of such systems, and they are now in widespread use for a very wide variety of applications because of their generality and adaptability ([2]). As a graphical tool, Petri nets can be used as a visual-communication aid similar to flow charts, block diagrams, and networks. As a mathematical tool, it is possible to set up state equations, algebraic equations, and other mathematical models governing the behaviour of systems.

Because of their numerous applications in areas like engineering, economics, medicine, education and science, Petri nets became a very prolific research field soon after their introduction, such that today the Petri nets research is materialized in a large number of publications in prestigious journals and conference proceedings.

The concept of a business process was developed in 1990s in order to better organize and optimize the activity in large organizations. A business process is a conceptual way of organizing work and resources, defining "a specific ordering of work activities across time and place, with a beginning, an end, and clearly defined inputs and outputs" ([3]).

A workflow is an abstraction of a business process, which supports automated manipulation, such as modelling or enactment by a software system for workflow management. Such software systems work with models of the workflows, expressed in a modelling language.

There exists several workflow modelling languages, like event-driven process chains, UML activity diagrams, flowcharts, BPEL, or BPMN, but these don't have formal semantics. In order to reduce ambiguity and developing techniques for analysis, formal methods have been proposed for workflow specification: process algebra (π-calculus, CSP, CSS), temporal logic (LTL, CTL, TLA), transactional logic, or Petri nets ([4]).

Today there exists several extensions of Petri nets used to model workflows, and many analysis techniques have been developed to analyze them, e.g. for proving certain desirable properties of workflows ([5]).

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8. Education

Elaborarea automată a itemilor individuali

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Cercetarea se referă la elaborarea unei noi metodologii şi tehnologii de asistare a profesorilor în elaborarea oricărui număr necesar, arbitrar de mare, de itemi individuali, destinaţi activităţilor de (auto) trening şi/sau de evaluare, cu importarea lor pe o platformă de e-Learning/e-testing pentru desfăşurarea multiplu repetată a sesiunilor de e-training/e-testing.

Asupra formării intuiţiei probabiliste

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Care-i legătura dintre realitate şi modelele realităţii? Cât de exact corespund concepţiile noastre teoretice despre lumea care ne înconjoară cu ceea ce se întâmplă în această lume? La aceste întrebări, dar şi la altele, răspunde Teoria Probabilităţilor. Unul din scopurile studierii teoriei probabilităţilor este formarea şi dezvoltarea intuiţiei şi gândirii probabiliste la studenţi (elevi etc.). Există numeroase probleme care, în cadrul lecţiilor practice (seminare), pot contribui la atingerea acestui scop. Se cere doar să selectăm cu atenţie aceste probleme. Rezolvând o astfel de problemă şi stabilind răspunsul printr-o metodă analitică, le putem propune studenţilor să-l argumenteze doar cu ajutorul unor raţionamente intuitive. De exemplu, fie că într-o urnă se conţin n bile şi toate ipotezele privind numărul de bile albe din urnă se consideră echiprobabile. Din urnă se extrage la întâmplare o bilă. Care este probabilitatea ca bila să fie albă? Aplicând formula probabilităţii totale găsim că probabilitatea este egală cu 0,5. Având acest răspuns le vom cere studenţilor să-l deducă doar prin raţionamente de ordin intuitiv. De exemplu, se poate raţiona astfel: dacă toate ipotezele despre numărul bilelor albe din urnă sunt echiprobabile, atunci ipotezele despre numărul bilelor de altă culoare din urnă, de asemenea sunt echiprobabile. Prin urmare, bila extrasă poate fi albă sau de altă culoare cu aceeaşi probabilitate, care nu poate fi alta decât 0,5. Fireşte, nu pentru orice problemă răspunsul, obţinut pe căi analitice, poate fi stabilit prin raţionamente intuitive. Totuşi, în legătură cu orice problemă de probabilitate este loc de discuţii la nivel intuitiv despre experimentul aleator corespunzător, evenimentele legate de acesta şi relaţiile posibile dintre evenimente.

Analysis of the academic performance in the basic matters of the first year of two Degrees in Engineering in relation with the profile of entry of students

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In order to improve the academic performance of first year students of the two degrees offered at the Technical School of Mining and Civil Engineering (University of Basque Country), a detailed study of the results obtained in each of the subjects based on the academic requirements for the new degrees has been conducted. In addition, the weak areas that the students have in each subject have been analyzed. To accomplish this, some tests at the beginning and at the end of the courses have been done. In this way, valuable information about the difficulties that students face has been compiled. From this study weak points in the students? curriculum have been identified which will be used to design academic activities to strengthen areas where weaknesses has been detected.
Keywords: academic performance, entrance profile, core subjects.

Design of a zero virtual course to improve the academic performance in Engineering Degrees

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A study about the academic yield of the first course students of two Engineering Degrees (Civil Engineering Degree and Mines and Energy Technology Engineering Degree) has been developed, which has allowed the detection of diverse formative lacks in the students that start these degrees. As a consequence, a zero virtual course has been designed as the support for the acquisition of the necessary contents and skills for obtaining the specific competences of the mentioned subjects. A working group formed by the educational teams of the eight subjects of the first academic year has been created, and they have designed theoretical and practical activities for the different areas of knowledge. These activities have been incorporated into an agent of contents web that the students will use in their learning process before the beginning of each subject. This project will help to correct their deficiencies as well as reinforce the acquired knowledge.
Keywords: self-learning, basic subjects, zero course, virtual course

Sisteme complexe din perspectiva funcţiilor complexe

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Lucrarea noastră îşi propune să realizeze o abordare interdisciplinară a două domenii aparent bine circumscrise, unul matematic, altul fizic, dar care de fapt sunt implicate în a descrie ceea ce astăzi se numeşte ştiinţa complexităţiii sau teoria sistemelor complexe. Această teorie cuprinde o serie de fenomene descrise în a doua jumătate a secolului XX de teoria haosului, geometrie fractală, dinamică neliniară şi topologie, care încearcă să modeleze realitatea fizică cu o mai mare acurateţe generată de evoluţia cunoaşterii în mecanica cuantică şi motivată de nediferenţiabilitatea şi neliniaritatea fenomenelor fizice. Descrierea matematică a acestor fenomene impune folosirea aparatului specific analizei complexe. Fizicienii consideră funcţiile complexe ca un instrument indispensabil pentru a descrie procesele din cadrul sistemelor complexe, în timp ce matematicienii consideră spaţiul complex ca fiind un spaţiu abstract, convenabil pentru eleganţa şi frumuseţea construcţiilor matematice. Aplicarea teoriei sistemelor complexe dincolo de domeniul strict al fizicii, în domeniul biologiei, psihologiei sau sociologiei impune o altă perspectivă asupra modului în care cele două domenii sunt implicate. Terminologia comună nu mai este doar o coincidenţă, întrucat teoria sistemelor complexe impune considerarea spaţiului complex ca un spaţiu fizic, care integrează astfel teoriile legate de substanţă, energie şi informaţie.

Challenges and innovations in math student research motivating

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Student research experience at all levels of higher education enriches university potential and provide a profitable base for creating research environment which is the essential question for newly established universities. The research activity influences on valuable educational goals having long-term effect as students study for their further professional career. Increasing student interest in science entails motivation for doing research. One of the major tools in research stimulation process implies involving undergraduate students in different research actions including math competitions, projects and math students organizations. In the present work we focus on different aspects of forming a potential researcher in mathematics, arising challenges and possible adaptations of innovative techniques in certain conditions. Special attention is given to the various features of math student research motivating in East African countries. The analysis is illustrated with some findings from the practical applications.

Folosirea modelului figurativ în predarea-învăţarea matematicii în învăţământul preuniversitar

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Articolul prezintă una dintre cele mai eficiente modalităţi de schematizarea a strategiei rezolutive în predarea-învăţarea matematicii, folosirea modelului figurativ. În prima parte sunt prezentate o serie de aspecte teoretice privind conceptul de model figurativ, tipurile de modele figurative folosite în matematică şi modalităţile de învăţare abordate. Partea a doua este consacrată exemplificării practice de folosire a modelului figurativ în rezolvarea problemelor şi în prezentarea unor concepte matematice. Astfel prin folosirea modelului figurativ este stimulată valoarea formativă a învăţării şi este schematizat actul rezolutiv, ca act de construcţie mintală. Articolul se încheie cu câteva concluzii privind importanţa folosirii modelului figurativ în asigurarea transferului specific de capacităţi intelectuale şi de scheme operatorii nou formate. Cuvinte cheie:strategie rezolutivă, model figurativ şi învăţământ preuniversitar.

The straight line in the three-dimensional Euclidean space studied with Maple

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An approach using Maple environment of teaching and learning aspects of analytic geometry aspects concerning the straight line in the three-dimensional Euclidean space is presented. First, the problems are resolved in a classical mode, following the application of the formulas presented in the course. Then, the same problems are resolved with help of Maple environment. Finally, the graphics representation of the solution is shown.
Keywords: straight-line, distance

Transformation of the object-oriented approach to the subject-oriented approach in the process of the information systems development

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Socialization of the Internet made actual recognition of the interests and features of a Individual, while rapid increase of the information volume is the matter of its importance for a Individual. Apparently, the frames of the object-oriented approach which was popular in the recent years are insufficient to solve the problems faced by the applied software developers mainly because of its natural constraints. We suggest to focus at the fundamentals of subject-event-oriented approach

with a usage of decision table methods and mechanisms of R-technology to transform the object-oriented approach in the direction of subject orientation, visibility, and event-oriented information systems. Earlier we delivered a report about an experience of subject-oriented approach based on an example of developing a wine internet portal [1]. Now it's a time to present subject-event-oriented approach. It is the further development of object oriented approach and subject-oriented approach with an emphasis on event-oriented component activity of the Individual. Applying the method of decision table we achieve the possibility for administration automation of the individual event fields of the subject, and hereafter solution for the problems of correlation and optimization of individual plans with external event-oriented field of the Individual. The tangible embodiment of this approach is our Higher Education Support System for the Institution activity. Hence we can see the extension of the scope of its event-oriented part as well as of the object/objective oriented part. We observe the simplification of the search process and the increase of the educational process efficiency with a prominent economy of the span time of the students and teachers needed to reach the necessary level of the education quality through the taking into consideration the subjective particularities of the students and adaptation to these particularities education processes.

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Discipline didactics in the context of the educational systems

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First of all, we have identified the position of discipline didactics within the sciences of education, undergoing constant expansion, which led to discovering a stable and flexible position within the frame of the taxonomy of the sciences of education.
The stable position may be identified, according to the first epistemological criterion operating in the process of classifying the sciences of education, by relation to the research object characteristic of pedagogy, namely education and training. In the terms used by Garrido [9], it is a differential pedagogy resulted from relating certain educational disciplines (the Didactics of Mathematics, Physics, Chemistry) to the issues of general didactics, which remains a priority. Inside this reference system, the didactics of the discipline should also be associated with another category of pedagogic sciences related to the psychological age of the trainees (the psychology of the preschool students, the psychology of the lower-elementary student, the psychology of the preadolescent, the psychology of the adolescent, the psychology of the university student, the psychology of the adult etc.).
The flexible position of discipline didactics can be identified according to the criterion of the research methodology, involved with two types of significant intradisciplinary and interdisciplinary analyses:

1. the intradisciplinary origin of discipline didactics shows the fact that it evolves inside and with a view to completing, deepening, widening the issues of general didactics, understood as a general theory of training, as a fundamental pedagogic science.

2. the interdisciplinary origin of discipline didactics enables its development at multiple moments, identified in relation to school stages (the Didactics of Mathematics in primary, secondary, high-school, pre-university, university education).

We have identified the continuity and integration lines for two models of discipline didactics:
a) the model of applied didactics/the methodology of teaching scientific contents;
b) the model of science didactics/didactics for elaborating cognitive structures.
We have analyzed the issue of general pedagogy, especially the training theory, searching for all the possibilities and implications relevant to the teaching-learning-evaluation of the discipline.

The Formation of Spatial Representations in the Process of Constructing Sections in Polyhedra

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The formation of spatial representations is a major problem in the mathematical education, frequently tackled by specialists in the area of the theory and methodology of instruction and by the didactic staff. Experience proves that high-school graduates show anxiety towards the compartments of geometry related to the relative position of lines, of lines and planes, of planes in the three-dimensional space. Problems regarding the construction of sections in polyhedra allow the consolidation of students? theoretical knowledge in these areas and the formation of abilities of solving some real-life problems. The emphasis of the module "Sections in polyhedra" in the evaluations at different stages would lead to the systemic approach of this valuable material by the didactic staff.
The examination of the methodical aspects regarding the formation of abilities of constructing sections in polyhedra in the high-school mathematical course allowed us to formulate a number of conclusions and recommendations related to the facilitation of the educational process by using modern didactic technologies, of software and digital instruments. In this context, the module "Theory and methodology of the formation of spatial representations in the process of constructing sections in polyhedra" was designed, implemented within a workshop with the mathematics teachers involved in the activities of the Municipal Center of Excellency "ICAR" from the Chisinau municipality.

Metode de rezolvare a unor probleme de concurs din teoria numerelor bazate pe relaţia $ab = cd$

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În articol vom prezenta o metodă de rezolvare a unor probleme de tipul: Fie $a, b, c, \cdots$ numere naturale, astfel încât $P(a, b, c, \cdots) = 0$. Să se arate că numărul $Q(a, b, c, \cdots)$. este compus. Analizând subiectele propuse la diverse concursuri, constatăm prezenţa problemelor de acest tip cu diferite grade de dificultate. Metoda poate fi aplicată mai ales atunci, când $P(a, b, c, \cdots)$ şi $Q(a, b, c, \cdots)$ sunt expresii pătratice în $a, b, c, \cdots$. Idea de bază provine din următoarea:
Lemă. *Dacă a, b, c, d sunt numere naturale cu proprietatea $ab = cd$, atunci există numerele naturale m, n, p, q astfel, încât $(n : p) = 1$ şi $a = mn$, $b = pq$, $c = mp$, $d = nq$.* O consecinţă imediată

a lemei este următoarea problemă clasică:
Problema 1. *(Olimpiada Naţională, Republica Moldova) Fie a, b, c, d numere naturale, astfel încât $ab = cd$. Să se arate că $a + b + c + d$ este număr compus.*
În esenţă, lema precedentă oferă o parametrizare generală a soluţiilor ecuaţiei $ab = cd$. Prin urmare, este oarecum firesc ca $a + b + c + d$, fiind oricând compus, să aibă o descompunere generală dată de această parametrizare. Aceste idei pot servi şi la rezolvarea ecuaţiilor diofantice.
Problema 2. *(Baraj Moldova, 2004) Fie $\{a, b\}$şi $\{c, d\}$ două mulţimi distincte de numere naturale astfel, încât $a^2 + b^2 = c^2 + d^2 := x$. Arătaţi că x este compus.*
Problema 3. *Rezolvaţi în $\mathbb{N}$ ecuaţia $x^2 - xy + y^2 = z^2$.*
Problema 4. *Rezolvaţi în $\mathbb{N}$ ecuaţia $x^2 + y^2 = 2z^2$.*
Problema 5. *(OIM 2001) Fie $a > b > c > d$ numere naturale, astfel încât $a^2+c^2-ac = b^2+d^2+bd$. Să se demonstreze că $ab + cd$ este număr compus.*

Folosirea ordinului unui număr în probleme de olimpiadaă

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În acest articol vom prezenta cum pot fi folosite proprietăţi mai avansate din teoria numerelor şi anume, ordinul unui număr, la rezolvarea problemelor de concurs.
Definiţie. *Fie a, n numere prime între ele. Definim prin $\widetilde{d}_n(a)$ cel mai mic număr d astfel încît $a^d \equiv 1 \bmod n$.*
Teorema lui Euler ne asigură că aşa numere există. Cea mai importantă proprietate a ordinului este următoarea:
Dacă $(a, n) = 1$, atunci $n \vee a^k - 1$ dacă şi numai dacă $\widetilde{d}_n(a) V k$. Alt corolar important este:
Dacă $a^k \equiv 1 \bmod n$ şi $a^l \equiv 1 \bmod n$, atunci $\widetilde{d}_n(a) V\ (k, l)$, deci $a^{(k,l)} = \equiv 1 \bmod n$. Aceste proprietăţi sunt utile la rezolvarea problemelor de tipul:
Problema 1. Găsiţi toate numerele naturale nenule n pentru care $n \vee 2^n - 1$.
Problema 2. Găsiţi toate numerele naturale nenule n pentru care $n^2 \vee 2^n + 1$.
Problema 3. Fie p un număr prim. Demonstraţi că există un număr prim q care nu divide $n^p - p$ pentu orice n natural.
Problema 4. (concurs Mongolia) Fie m şi n numere naturale astfel încît $(2m + 1, 2n + 1) = 1$. Determinaţi $\left(2^{2m+1} + 2^{m+1} + 1, 2^{2n+1} + 2^{n+1} + 1\right)$.
Prin (a, b) s-a notat cel mai mare divizor comun al numerelor a şi b.
Problema 5. (concurs Rusia) Să se determine toate perechile de numere naturale $\{a, b\}$ pentru care $2^a - 1 \vee 2^b + 1$.

Conexiunile în aspect istoric ale Matematicii şi Informaticii şi rolul lor în predare învăţare

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Conexiunile matematicii cu alte ştiinţe sunt prezente tuturor perioadelor, de dezvoltare ale ei din punct de vedere practic şi metodologic.
Toate activităţile intelectuale ale omenirii exercită influenţe asupra matematicii, de altfel reciproce,

cu anumite intensităţi în diferite perioade. Tradiţional, în decurs de secole, în acest sens se menţionează ştiinţele naturale, fizica, tehnica. Însă, situaţia se schimbă pe la mijlocul secolului XX când se produc două evenimente decisive: primul - elaborarea Teoriei Matematice a Informaţiei de C. Shannon, cel de al doilea - apariţia şi dezvoltarea tehnicii electronice care a finisat procesul de creare a maşinilor de calcul, ce a parcurs lent, parcă fără deosebite pretenţii, de la anticul *abac* cu şănţuleţe şi pietricele (de la latinele *calcarius, calculi* - calcar, pietre provine termenul calculator), proces în care au fost antrenaţi aşa savanţi ca Gerbert (930-1003), R. Lull (1235-1315), W. Schickard (1592-1635), B. Pascal (1623-1662), G. Leibniz (1646-1716), C. Babbage (1792-1871), A. Turing (1912-1954), J. von Neimann (1903-1957), D. Knuth (1938-) ş.a. Calculatoarele electronice cu mult au întrecut ceia ce au visat predecesorii. Aceste maşini de calcul împreună cu noua Teorie a Informaţiei, schimbă fizionomia ştiinţelor şi în mare măsură a matematicii. Cunoaşterea constă în reflectarea, reprezentarea, memorizarea celor din jur prin diferite modalităţi. În Teoria informaţiei, ca o teorie a cunoaşterii, care cuprinde şi stabilirea *relaţiilor* între obiectele lumii reale, un rol deosebit îl joacă *codarea* informaţiei care se realizează prin intermediul unor *simboluri* şi combinări de simboluri care pot fi de orice natură. Formarea limbilor naturale reprezintă rezultatul codificării informaţiei prin simboluri *sonore* (cuvinte). Au urmat simbolizări grafice realizate prin diferite mijloace. Astfel, printre cele mai primare noţiuni sunt cele de relaţii, codări, simboluri, combinări, compoziţii de diferite tipuri, operaţii, calcule. În secolul XXI - secolul prelucrării informaţiei cu maşinile electronice de calcul, se cer unele accente noi în fundamentarea celor ce urmează să se studieze.

9. Supplement

Probability Theory, Mathematical Statistics, Operations Research

Algorithms for solving stochastic discrete optimal control problems on networks

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In this work we consider the stochastic discrete optimal control problem on the decision network (G, X_C, X_N, c, p, x_0). This network is determined by the directed graph $G = (X, E)$ with a fixed starting state x_0, the subsets $X_C, X_N \subset X$ which satisfy $X = X_C \cup X_N$, $X_C \cap X_N = \emptyset$, the cost function $c : E \to \mathbb{R}$ and the probability distribution function $p : E_N \to [0, 1]$ on the set $E_N = \{ e = (x, y) \mid x \in X_N, y \in X \}$. The set X_C represents the set of controllable states in which the transitions of the system to the next state can be controlled by the decision maker at every discrete moment of time and X_N represents the set of states in which the decision maker is not able to control the transition because the system passes to the next state randomly.
A linear programming approach for finding the optimal stationary strategy for the stochastic control problem on network with average cost criterion and infinite time horizon is proposed. We formulate an algorithm for determining the optimal solution for the control problem on perfect network, in which an arbitrary strategy s generates a strongly connected graph $G_s = (X, E_s \cup E_N)$, where $E_s = \{ e = (x, y) \in E \mid x \in X_C, y = s(x) \}$. Then we show that the proposed algorithm can be extended to the unichain problem in which the strategy s generates a graph G_s, which may not be strongly connected, but it contains a unique isolated strongly connected component $G'_s = (X'_s, E'_s)$ (such strongly connected component which do not contain a leaving directed edge $e = (x, y)$, where $x \in X'_s$ and $y \in X \backslash X'_s$) that can be reached from any vertex of $X \backslash X'_s$. Also, we consider the case when an arbitrary strategy s generates a Markov multichain, in which the graph G_s consists of several strongly connected components $G^1 = \left(X^1, E^1\right), G^2 = \left(X^2, E^2\right), ..., G^k = \left(X^k, E^k\right)$, where $\bigcup_{i=1}^{k} X^i = X$. Among these components, there are $k' < k$ isolated strongly connected components $G^{i_r} = \left(X^{i_r}, E^{i_r}\right)$, $r = 1, 2, ..., k'$. We describe an approach for determining the optimal solution which is based on a reduction procedure of the multichain problem to the unichain case.

Education

Experimental research on the effects on learning geometry

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The aim of this study is to evaluate the efficiency of using computers with GeoGebra software in the teaching-learning of Geometry in middle-school, by the students from the specialization of Mathematics, ?Vasile Alecsandri? University of Bacu. During the pedagogical practice stage, the students in the 3rd year, although better trained in Mathematics than the students from other departments, face various problems related to their practical skills in using the computer in teaching, as well as to their lack of teaching experience. The research was conducted at the National

Pedagogical College ?tefan cel Mare? from Bacu and consisted in assisting and observing 24 lessons of Mathematics and 24 lessons of Information and Communication Technologies, involving a group of 180 students from grades 1-4, 160 middle-school students and 30 teachers of various specializations. The applied tests and questionnaires have shown the efficacy of using the computer in building active thought and competences in the graphical representation of geometrical figures and shapes, as well as in solving problems of concurrence in plan. In relation to these problems, we are looking for a solution to comprise the best teaching-learning strategies using the calculus technique. With nearly 5000 functions, the GeoGebra menu provides techniques for acquiring knowledge in an electronic format, calculus techniques, explanatory mathematical texts, graphs, sounds and diagrams, e-learning solutions, including online testing and evaluation, as well as web-based learning tools designed for Mathematics (www.maplesoft.com).

Folosirea mediului de programare LAZARUS ca o alternativă a lui DELPHI în studierea programării

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Lucrarea conţine un scurt istoric ce ţine de apariţia unor limbaje de programare precum şi asemănările şi deosebirile dintre mediile de programare LAZARUS şi DELPHI. Folosirea mediului de programare LAZARUS ca o alternativă a mediului DELPHI, va duce la economisirea unor surse financiare a instituţiilor în care va fi folosit. LAZARUS, spre deosebire de DELPHI, este gratuit, şi poate fi folosit liber şi legal în orice instituţie de învăţământ, publică sau de producţie, ori la domiciliu.

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