CAIM 2015

17th–20st SEPTEMBER, 2015
SUCEAVA, ROMANIA

Book of Abstracts

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Contents

1. Real, Complex, Functional and Numerical Analysis

Internal stabilization for a class of spatial structured epidemic model

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We investigate the stabilizability of some reaction-diffusion systems with state constraints, when the control acts on a small subdomain or on a curve. We show the relationship between the stabilizability (with state constraints) and the magnitude of the principal eigenvalue of a certain related elliptic operator (corresponding to the complement of the support of the control and to appropriate boundary conditions). In case of stabilizability we provide a simple feedback stabilizing control.

Semi-linearity of some spaces of multifunctions

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We study the semi-linearity for some spaces of multifunctions. We establish sufficient and characteristic conditions for a translated topology (on the space of sn-bounded multifunctions) to be semi-linear. We also introduce two metrics on a space of multifunctions and compare the induced topologies.

Invariance results for a graph with respect to differential inclusions

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Let X be a real Banach space and $I \subset \mathbb{R}$ a nonempty and bounded interval. Let $K : I \rightsquigarrow X$ be a multi-function with the graph $\mathcal{K}$. In this talk we provide some invariance properties for $\mathcal{K}$ with respect to differential inclusions. As applications, we study a Lipshitz regularity of the solution set and we give a relaxation result. Further, we deduce a sufficient conditions for null conrollability for evolution equations.

On different types of near linear spaces

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Different types of near linear spaces have been defined and studied in the literature related to certain problems in optimization, functional analysis, computer science, set-valued analysis. We present a survey on different types of near linear spaces and some considerations on weak linear spaces.

Stable Spectral Collocation Algorithms for Nonlinear Wave Equations with Linear Dispersion

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We are concerned with non-periodic *spectral collocation solutions* for *initial-boundary value problems* attached to *nonlinear wave equations* of the form

$$u_t = N(u) + L(u) + g(x,t), \tag{1}$$

and restrict the description to one space dimension for notational simplicity. The term $N(u)$ is a genuinely nonlinear one and may also depend on u_x, u_{xx}, etc. and the linear part is of the form $L(u) := c(t)\, i^{m+1} (\partial^m u/\partial x^m)$ but more general *dispersive terms* are also treatable. The real function $c(t)$ is often a constant and the *forcing term* $g(x,t)$ can be embedded into $N(u)$.
The Korteweg-de Vries (KdV), Benjamin Bona Mahony (BBM), modified BBM and Benjamin Bona Mahony-Burgers (BBM-B) and nonlinear Schrödinger (NLS) initial value problems on the real axis are important examples of such problems. The spectral collocation is based alternatively on the *scaled Hermite* and *sinc functions*. This spatial discretization approach avoids the empirical domain truncation and open up the possibility to visualize the evolution of the waves on the largest intervals.

In order to march in time we use several one step and linear multistep finite difference schemes such that the method of lines (MoL) involved is *stable* in sense of Lax. The method based on Hermite functions ensures the correct behavior of the solutions at large spatial distances and in long time run. In order to prove the *numerical stability* we use the *pseudospectra* of the linearized spatial discretization operators.
The extent at which the *energy (first) integral* of BBM model is conserved over time is analyzed for Hermite collocation along with various finite difference schemes. This analysis has been fairly useful in optimizing the scaling parameter.
The effectiveness of our approach has been confirmed by some numerical experiments. Among them *two soliton* type solutions for KdV and NLS are displayed as MATLAB short movies.

Considerations on the problem of atomicity in non-additive set-valued integrability

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In this talk, different problems concerning atoms/non-atomicity related to Gould and Birkhoff integrals are treated in the non-additive set-valued case.

On the center of Banach C(K)-modules

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In this talk, we deal with the centers of Banach C(K)-modules and the Banach f-modules. We characterize the dual center of Banach f -modules and Banach C(K)-modules.

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Approximation by Szász operators involving Brenke type polynomials

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The aim of the present paper is to give a Chlodowsky type generalization of Szász operators defined by means of the Brenke type polynomials. We obtain convergence properties of our operators with the help of universal Korovkin-type property and also establish the order of convergence by using a classical approach, the second order modulus of continuity and Peetre's K-functional. We also give a Voronoskaja type theorem. Furthermore, we study the convergence of these operators in a weighted space of functions on a positive semi-axis. With the help of the matlab some graphical examples for the convergence of our operators and to compute error estimation are also given.

Applications of summability methods in approximation

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Korovkin type approximation theorems are useful tools to check whether a given sequence $(L_n)_{n\geq 1}$ of positive linear operators on $C[0,1]$ of all continuous functions on the real interval $[0,1]$ is an approximation process. That is, these theorems exhibit a variety of test functions which assure that the approximation property holds on the whole space if it holds for them. Such a

property was discovered by Korovkin in 1953 for the functions 1, x and x^2 in the space $C[0,1]$ as well as for the functions 1, *cos* and *sin* in the space of all continuous 2π-periodic functions on the real line. In this talk, we use the methods of almost convergence and statistical convergence to prove the Korovkin type approximation theorems for the test functions $1, e^{-x}, e^{-2x}$.

Some convergence estimates for abstract linear second order differential equations with two small parameters and depending on time operators

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In a real Hilbert space H consider the following singularly perturbed Cauchy problem

$$\begin{cases} \varepsilon\, u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + A(t)u_{\varepsilon\delta}(t) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases} \tag{1}$$

where $A(t) : V \subset H \to H$, $t \in [0,\infty)$, is a family of linear self-adjoint operators, $u_0, u_1 \in H$, $f : [0,T] \mapsto H$ and ε, δ are two small parameters.
We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem (1) in two different cases:

(i) when $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$, relative to solutions to the following unperturbed system:

$$\begin{cases} \delta l'_\delta(t) + A(t)l_\delta(t) = f(t), \quad t \in (0,T), \\ l_\delta(0) = u_0, \end{cases}$$

(ii) when $\varepsilon \to 0$ and $\delta \to 0$, relative to solutions to the following unperturbed system:

$$\begin{cases} A(t)v(t) = f(t), \quad t \in (0,T), \\ v(0) = A^{-1}(0)f(0). \end{cases}$$

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighbourhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

A general fixed point theorem for a sequence of mappings in G_p - complete metric spaces

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In this paper, a general fixed point theorem for a sequence of mappings in G_p - complete metric spaces is proved.

Efficiency in the Infinite Dimensional Ordered Vector Spaces

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This research paper is focused on the common concepts of the efficiency and set-valued map. After a short introduction, we propose some questions regarding the notion of efficiency and we emphasize the Pareto type optimality as one of the first finite dimensional illustrative examples. We present the efficiency and the multifunctions in the infinite dimensional ordered vector spaces following also our recent results concerning the most general concept of approximate efficiency, as a natural generalization of the efficiency, with implications and applications in vector optimization and the new links between the approximate efficiency, the strong optimization - by the full nuclear cones - and Choquet's boundaries by an important coincidence result. In this way, the efficiency is strong related to the multifunctions and Potential theory through the agency of optimization and conversely. Significant examples of Isac's cones, pertinent considerations on spline functions in H-locally convex spaces and several important references conclude this study.

Methods for Studying Hybrid Systems

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In this talk, we present several methods used in literature in the study of hybrid systems, namely of systems whose continuous behavior is perturbed at discrete moments. Unlike the classical theory of impulsive differential equations, these tools allow one to cover the case where the perturbation moments accumulate in a finite amount of time.
One of these methods is the theory of measure differential equations, where the main tool is the measure theory. Another extensively used method is the time scale analysis, introduced by S. Hilger in his PhD thesis in 1988. This category of dynamical systems can also be studied using the theory of generalized differential equations, initiated by J. Kurzweil in the 50's.
We finally present some situations where these theories can be shown to be equivalent.

On convergence in Lebesgue and Hölder spaces of direct-approximate methods in solving some nonlinear singular integral equations

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We consider the following nonlinear singular integral equation (NSIE) with Carleman moved shift $\gamma(t), \gamma(\gamma(t)) = t$, defined on an arbitrary closed smooth contour Γ in Hölder spaces $H_\beta(\Gamma),\ 0 < \beta < 1$:

$$\Phi\left(t, \varphi(t), \varphi(\gamma(t)), \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau, \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\tau)}{\tau - \gamma(t)}d\tau,\right.$$
$$\frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\gamma(\tau))}{\tau - t}d\tau, \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\gamma(\tau))}{\tau - \gamma(t)}d\tau,$$
$$\left.\frac{1}{2\pi i}\int\limits_\Gamma K_1(t,\tau)\varphi(\tau)d\tau, \frac{1}{2\pi i}\int\limits_\Gamma K_2(t,\tau)\varphi(\gamma(\tau))d\tau\right) = 0, \quad t \in \Gamma,$$

where $\gamma'(t) \in H_\mu(\Gamma),\ 0 < \beta < \mu \leq 1, \forall t \in \Gamma$; Φ is a known function verifying certain Hölder conditions on Γ and φ is a unknown function.
In this paper the computing algorithms of collocations, quadratures and reduction methods for approximate solving of NSIE with Carleman moved shift in the complex plane, are obtained. Next, the sufficient conditions for compatibility and convergence of these computing algorithms in Hölder and Lebesgue spaces were determined

G-integrals for vector multifunctions

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In this paper we present a generalization for the Pettis integral for vector multifunctions relative to a multisubmeasure. Between the integrals presented here we recall the G-Choquet, the G-Aumann and the G-Sugeno integral. Some properties and some comparative results on this integrals are established.

2. PDEs with Applications in Mechanics, Biology, etc.

Existence and uniqueness of generalized solutions to nonlinear nonlocal problems for hyperbolic equations

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In 1963, Cannon [1] considered a problem with integral condition for a parabolic equation which led to the foundation of a large class of nonlocal problems with integral conditions. In [2], J. L. Lions studies problems for nonlinear hyperbolic equations arising in relativistic quantum mechanics with nonlinear terms $|u|^\rho u$ and $|u_t|^\rho u_t$ on manifolds. In [3], [4], initial boundary-value problems for hyperbolic equations with nonlinear components of the same type in boundary conditions are investigated. Motivated by these works, we study problems for evolution equations which contain both nonlinear and nonlocal integral terms in boundary conditions.

In this paper, we start with an introduction to nonlocal problems and discuss their physical applications. Based on this, we then extend the area of our concentration to the class of nonlinear problems with nonlocal integral conditions for hyperbolic equations associated with vibrating string modeling and simulation of gas flows in channels. Further, we demonstrate results on existence and uniqueness of generalized solutions to the above problems and describe the ideas of proofs.

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Computational issues for a regularized extension of the Patlak-Keller-Segel model

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This talk presents the results of the numerical simulations of a two-species chemotaxis model. This model defines a regularized extension of the Patlak-Keller-Segel (PKS) system to the case of the chemotaxis motion of two noncompeting species that produce the same chemoattractant. Several numerical experiments are performed by applying a second-order positivity preserving central-upwind scheme to study the behaviour of the spiky solutions.

Landau-Lifshitz equation whose spectral parameter varies on a hyperelliptic surface

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Improved rigorous method of homogeneous vector boundary Riemann problem [1 - 3] with permutation coefficient on an algebraic surface and the finite set of the open contours is applied to solution of the Landau-Lifshitz equation (L-Le). The latter is mostly important in ferromagnetic dielectrics and magnetic resonance theory.
In [4], this equation was solved from the geometrical and physical viewpoints by the pure selective process. The statement concerned complete anisotropy and variation of the spectral parameter on a torus. As it was fairly noted in [4], only the number of contours less than 3 could be considered. All other, more complicated cases, including hyperelliptic surface, remained unattainable.
In [5], investigation of the generalized version of the L-Le was done only approximately, and it dealt with so interesting new phenomenon as the spin transfer in magnetic nanostructures.
In [6], the aforesaid exact algorithm [3] has solved explicitly L-Le [4] in the case of arbitrary finite set of contours greater than 2 and has found unexpected corollaries for the complex plane.
In the present paper, the same approach [3] is used for the L-Le analytic solution whose spectral parameter varies on a hyperelliptic surface. Some delicate facts related algebraic functions theory are proved here basing on the non commutative compound results obtained for the complex plane.

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Double scale convergence and exact internal controllability with Fourier conditions

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The paper presents a wave problem exact internal controlled on an alveolar structure which is periodic along the axis ox_2 and it is embedded at both ends. Two problems emerge: the problem on structure and the problem of the exact intern control. For homogenization we apply the double scale convergence. The result is formed from two limit problems: one of them is for control with homogeneous wave equation, and the other problem is controlled by a new control which contains the limit of the initial control. These two limit equations are waves equations with one spatial variable - ox_2. Finally, these two interconnected problems are solved with classical methods.

Analysis of electromagnetic propagation on stratified lossy inhomogeneous medium using the SPPS method

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In this paper we study the case of oblique incidence of a plane wave polarized perpendicularly in an inhomogeneous medium. We use the SPPS method for calculating the index of reflection and transmission in lossy media. In the inhomogeneous media, we use the lineal and exponencial perfils for refraccion index
We will use the SPPS (Spectral Parameter Series) to solve the problem. In the next theorem we obtain a general solution of a Sturm-Liouville equation in the form a spectral parameter series.

Theorem 1. *Assume that on a finite interval* $[a, b]$*, the equation [2],[3]*

$$(pu_0')' + qu_0 = 0$$

possesses a particular solution u_0 *such that the functions* $u_0^2 r$ *and* $\frac{1}{u_0^2 p}$ *are continuous on* $[a, b]$*. Then the general solution of the equation*

$$(pu')' + qu = \lambda ru$$

on (a, b) *has the form*

$$u = c_1 u_1 + c_2 u_2$$

where c_1 *and* c_2 *are arbitrary complex constants,*

$$u_1 = u_0 \sum_{k=0}^{\infty} \lambda^k \widetilde{X}^{(2k)} \quad \text{and} \quad u_2 = u_0 \sum_{k=0}^{\infty} \lambda^k X^{(2k+1)} \tag{1}$$

with $\widetilde{X}^{(n)}$ *and* $X^{(n)}$ *being defined by the recursive relations* $\widetilde{X}^{(0)} \equiv 1$, $X^{(0)} \equiv 1$

$$\widetilde{X}^{(n)}(z) = \begin{cases} \int\limits_{z_0}^{z} \widetilde{X}^{(n-1)}(s) u_0^2(s) r(s) ds & n \text{ odd} \\ \int\limits_{z_0}^{z} \widetilde{X}^{(n-1)}(s) \frac{1}{u_0^2(s)p(s)} ds & n \text{ even} \end{cases}$$

$$X^{(n)}(z) = \begin{cases} \int\limits_{z_0}^{z} X^{(n-1)}(s) \frac{1}{u_0^2(s)p(s)} ds & n \text{ odd} \\ \int\limits_{z_0}^{z} X^{(n-1)}(s) u_0^2(s) r(s) ds & n \text{ even} \end{cases}$$

The main results was to find R and T

$$R = \frac{-ju_1(d)k_2 - k_1k_2u_2(d) - k_2u_2(d) - u_1^{'}(d) + jk_1u_2^{'}(d) + ju_2^{'}(d)}{ju_1(d)k_2 - k_2k_1u_2(d) + k_2u_2(d) + u_1^{'}(d) + jk_1u_2^{'}(d) - ju_2^{'}(d)}, \tag{2}$$

and

$$T = \frac{2jk_1\left(u_1(d)u_2^{'}(d) - u_2(d)u_1^{'}(d)\right)}{\left[ju_1(d)k_2 - k_2k_1u_2(d) + k_2u_2(d) + u_1^{'}(d) + jk_1u_2^{'}(d) - ju_2^{'}(d)\right]e^{-ik_2d}}. \tag{3}$$

where $u_1(d)$ and $u_2(d)$ have been calculated from (1).

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Mathematical Modeling of Subsurface and Surface Water Flow

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The water flow in a hydrographic basin is a very complex phenomenon that is influenced by many physical-chemical-biological factors. To set up a solvable mathematical model, one needs to identify the determinant factors affecting this phenomenon and ignores the rest of them. In this talk we examine some mathematical models that take into account variation in soil porosity, the cover plants and variation in the soil surface gradient.

Airplanes or Ballistic Rockets Optimal Control and Flight Absolute Stabilization in Vertical Plane by Using the Criterion of Minimal Time or Fuel Consumption

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In this paper the study of airplanes or ballistic rockets vertical flight stabilization, directing and controlling in case of pitch perturbations will be approached. In the first part the automate control (pilot) is studied, obtaining the Liapunov function and absolute stability parameters. The second part of this paper deals with getting a relay-type regulator which performs an optimal control by using the "minimum time" or "minimum fuel consumption" criteria. This optimal control method is based on the extreme principle of Pontreaghin, finding theoptimal trajectories by precising the control functions and parameters. The results are accompanied numerical and graphic examples in phase-space. The pitch angle variation is described by a third order equation, so the study is performed in a phase-space of the same order.
MSC2010: 34H05, 49K35, 93C15, 93C73, 93D10.

Well-posedness for a phase-field transition system endowed with a classical regular potential and a general class of nonlinear and non-homogeneous dynamic boundary conditions

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The work is concerned with the study of a phase-field transition systems of Caginalp type, endowed with a classical regular potential and a general class of nonlinear and non-homogeneous dynamic boundary conditions (in both unknown functions). The existence, uniqueness and regularity of solutions is established. Here we extend several results which have been proven by some of the authors, including the already studied boundary conditions, which makes the present mathematical model capable to reveal the complexity of a wide class of physical phenomena (phase change in Ω and at the boundary of Ω, for instance).
MSC: 35Bxx; 35K45; 35K55; 35K57; 35K60; 35Qxx; 47H30; 74A15; 80Axx

About global solutions of partial differential equation with deviating argument in the time variable

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We studied the method of constructing global solutions and their structure for nonhomogeneous partial differential equation with deviating argument in the time variable in a form

$$u_t(x,t) = p(t)u_{xx}(x,t+\mu) + q(x,t),\ (x,t) \in Q, \tag{1}$$

with zero boundary conditions

$$u(0,t) = u(l,t) = 0,\ t \in R, \tag{2}$$

where $Q = \{(x,t) : 0 < x < l, t \in R\}, \mu \in R \setminus \{0\}$.
It is assumed that the function $q(x,t)$ can be represented as a sum of the first n terms of the Fourier series:

$$q(x,t) = \sum_{k=1}^{n} q_k(t) sin\frac{k\pi x}{l}, \quad q_k(t) = \frac{2}{l}\int_0^l q(\xi,t) sin\frac{k\pi\xi}{l} d\xi.$$

We found a global solution $u(x,t)$ of the (1), (2) as a sum

$$u(x,t) = \sum_{k=1}^{n} T_k(t) sin\frac{k\pi x}{l},$$

where $T_k(t)$ is a solution of the linear equation

$$T_k'(t) + \bar{p}_k(t)T_k(t) - \bar{q}_k(t) = 0, \tag{3}$$

with coefficients

$$\bar{p}_k(t) = \lambda_k p(t) e^{\int_{t+\mu}^{t} \bar{p}_k(s)ds},$$

$$\bar{q}_k(t) = q_k(t) + \lambda_k p(t) \int_{t+\mu}^{t} \bar{q}_k(\tau) e^{\int_{t+\mu}^{\tau} \bar{p}_k(s)ds} d\tau, \quad k \le n, \quad t \in R.$$

It was used a method of successive approximations to find coefficients $\bar{p}_k$ and $\bar{q}_k$ of the equation (3). We discovered the conditions which allow application of this method. And the offered method was considered for finding and researching the global solutions of some other differential equation types [1, 2, 3].

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Exact study in electromagnetic field theory using systems of partial differential equations

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Two methods of the explicit solution in electrodynamics are proposed here basing on the mathematical models in terms of the finite dimensional systems of PDEs (partial differential equations). Both algorithms reduce the original matrix problem to the general wave equation regarding all unknown scalar components of electromagnetic field vector intensities. Detailed analysis of restrictions in the framework of classical Maxwell theory is suggested as well using the aforesaid analytic technique.
Actually, the given approach can be applied to any type of the finite dimensional square system of operator equations with piecewise constant coefficients and invertible terms commutative in pairs. Efficiency of those research trends is shown in the case of the slow-guided structures dealing with electromagnetic wave propagation in various coordinate systems.

3. ODEs; Dynamical Systems

The role of the informational energy in wave-particle duality

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The classical experiment from Young's classical mechanics concerning the deviation of electrons through two slits is reconsidered, highlighting the role of the informational energy in the domination of the corpuscular character or the spectral one. Practically speaking, the Young type slits act as an external constraint on the informational stock the electrons possess. These electrons are considered to be fundamental elements and thus matter's pseudo-inteligence is highlighted.

Nonlocal Boundary Value Problem for Multifrequency System with Delay

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Systems of coupled oscillators in the form

$$\frac{d^2u_i}{d\tau^2}+\omega_i^2(\tau)u_i=\varepsilon f_i(\tau,u,\frac{du}{d\tau}),\qquad i=\overline{1,m},$$

where $\varepsilon\in(0,\varepsilon_0)$ — small parameter, $\tau=\varepsilon t\in[0,L]$, $\omega_i(\tau)$ — natural frequencies, $u=(u_1,\dots,u_n)$, are mathematicals models of many oscillating processes.
By change of variables

$$u_i=a_i\cos\varphi_i,\qquad \frac{du_i}{dt}=-a_i\omega_i(\tau)\sin\varphi_i$$

the system comes to the form

$$\frac{da}{d\tau}=A(\tau,a,\varphi),\qquad \frac{d\varphi}{d\tau}=\frac{\omega(\tau)}{\varepsilon}+B(\tau,a,\varphi),\tag{1}$$

where A and B — 2π-periodic functions with variables φ_i.
Systems of equations in the form (1), that called multifrequtncy, with initial and boundary conditions were investigated in work [1] and other.
In this paper multifrequency system of equations with delay is considered

$$\frac{da}{d\tau}=A(\tau,a_\Lambda,\varphi_\Theta),\qquad \frac{d\varphi}{d\tau}=\frac{\omega(\tau)}{d\tau}+B(\tau,a_\Lambda,\varphi_\Theta),\tag{2}$$

where $a_\Lambda=(a_{\lambda_1},\dots,a_{\lambda_r})$, $\varphi_\Theta=(\varphi_{\theta_1},\dots,\varphi_{\theta_s})$, $a_{\lambda_i}(\tau)=a(\lambda_i\tau)$, $i=\overline{1,n}$, $\varphi_{\theta_j}(\tau)=\varphi(\theta_j\tau)$, $j=\overline{1,m}$, $0<\lambda_1<\dots<\lambda_r\le 1$, $0<\theta_1<\dots<\theta_s\le 1$.
For system (2) nonlocal boundary value conditions are given

$$a(\tau_1)=y_1,\qquad \int\limits_{\tau_1}^{\tau_2}g(\tau)\varphi(\tau)d\tau=y_2,\tag{3}$$

where $0 \le \tau_1 < \tau_2 \le L$, $y_1 \in D$, $y_2 \in \mathbf{R}^m$, $g : [\tau_1, \tau_2] \to \mathbf{R}^m$.

The boundary problem of this type for second order ordinary differential equation was learned in [2]. Multifrequency systems with delay were researched in [3] and other.
The averaged on fast variables φ system with boundary condition like (3) was built

$$\frac{d\overline{a}}{d\tau} = A_0(\tau, \overline{a}_\Lambda), \qquad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + B_0(\tau, \overline{a}_\Lambda).$$

The existance of solution of boundary problem (2), (3) was proved and conditions under wich

$$\|a(\tau, \varepsilon) - \overline{a}(\tau)\| + \|\varphi(\tau, \varepsilon) - \overline{\varphi}(\tau, \varepsilon)\| \le c\varepsilon^\alpha,$$

were found. Here $c = const > 0$, $0 < \alpha \le (ms)^{-1}$.

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Cubic systems with invariant lines of total multiplicity eight possessing exactly one infinite singularity

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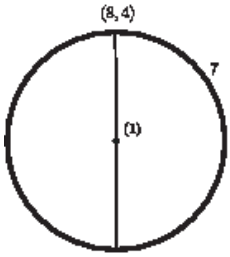
Here we consider the problem of classifying all configurations of invariant lines of total multiplicity eight (including the line at infinity) of real planar cubic differential systems. The classification was initiated in [3] where the cubic systems which possess 4 distinct infinite singularities are considered and 17 such distinct configurations were detected. That work was continued in [4] where the classification was done for the family of cubic systems with three distinct infinite singularities and where the existence of 5 distinct configurations was proved. The cubic systems with 2 infinite singularities are considered in [2] and [5] where 25 configurations were constructed.
This paper deals with the last remaining subfamily of cubic systems. We prove here a classification theorem of real planar cubic vector fields which possess exactly one infinite singular point and eight invariant straight lines, including the line at infinity and including their multiplicities. We show that these systems could possess the following 4 configurations:
We underline that the classifications of all the above mentioned families of cubic systems are taken modulo the action of the group of real affine transformations and time rescaling and are given in terms of invariant polynomials. These invariant polynomials are constructed following the theory of algebraic invariants of polynomial differential systems, developed by Sibirsky and his disciples. The algebraic invariants and comitants allow one to verify for any given real cubic system whether



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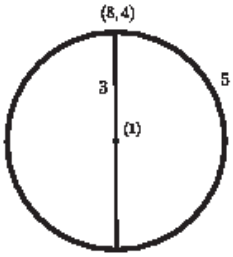

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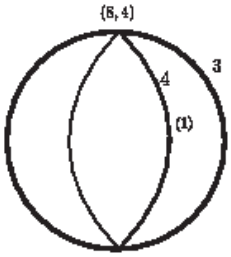

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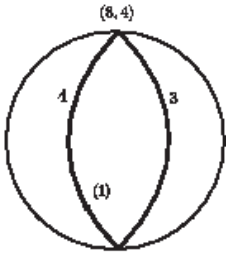

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or not it has invariant lines of total multiplicity eight, and to specify its configuration of lines endowed with their corresponding real singularities of this system. The calculations can be implemented on computer and the results can therefore be applied for any family of cubic systems, given in any normal form.

We remark that this paper is a continuation of the work [6] where the classification of cubic systems possessing invariant lines of total multiplicity 9 (including the line at infinity) have been done. We observe that in [6] there is omitted one configuration of invariant lines as it is shown in [1].

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Existence and approximation of a solution of the boundary value problems for delay integro-differential equations

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We consider the following boundary value problem

$$y''(x) = f(x, y(x), y(x-\tau_0(x)), y'(x), y'(x-\tau_1(x))) + \\ + \int_a^b g(x, s, y(s), y(s-\tau_0(s)), y'(s), y'(s-\tau_1(s)))\, ds, \tag{1}$$

$$y^{(i)}(x) = \varphi^{(i)}(x), \quad i = 0, 1, \quad x \in [a^*; a], \quad y(b) = \gamma, \tag{2}$$

where $a^* = \min\left\{\inf\limits_{x\in[a;b]}(x-\tau_0(x)), \inf\limits_{x\in[a;b]}(x-\tau_1(x))\right\}$, $\gamma \in R$, $\tau_0(x) \geq 0$, $\tau_1(x) \geq 0$.
A delay $\tau_1(x)$ is a continuous on $[a;b]$ function such that the set $E = \{x_i \in [a,b] : x_i - \tau_1(x_i) = a,\ i = \overline{1,k}\}$ is finite.
A function $y = y(x)$ is called a solution of the problem (1)-(2) if it satisfies the equation (1) on $[a;b]$ (with the possible exception of a set E) and boundary conditions (2).
In this work we study a method of solving boundary value problems for delay integro-differential equations based on approximating the solution by cubic splines with defect two. A delay $\tau_1(x)$ is a continuous on $[a;b]$ function such that the set $E = \{x_i \in [a,b] : x_i - \tau_1(x_i) = a,\ i = \overline{1,k}\}$ is finite.
A function $y = y(x)$ is called a solution of the problem (1)-(2) if it satisfies the equation (1) on $[a;b]$ (with the possible exception of a set E) and boundary conditions (2).
In this work we study a method of solving boundary value problems for delay integro-differential equations based on approximating the solution by cubic splines with defect two.
Existence and uniqueness of a solution of delay boundary value problems in various function spaces were considered in [1-2]. Applying spline functions for solving differential-difference equations was investigated in [3-4].

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Integrability conditions for a cubic system with a bundle of two invariant straight lines and one invariant cubic

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We consider the real cubic system of differential equations

$$\dot{x} = y + P_2(x,y) + P_3(x,y),\ \dot{y} = -x + Q_2(x,y) + Q_3(x,y), \tag{1}$$

where $P_j(x,y)$, $Q_j(x,y)$ are homogeneous polynomials of degree j. The origin $O(0,0)$ is a singular point of a center or focus type for (1). The problem of determining when a cubic differential system has a center at a singular point $O(0,0)$ is not completely solved. Necessary and sufficient conditions for the origin to be a center were obtained in some particular cases for such systems. In [1] it was solved the problem of the center for cubic system (1) with: four invariant straight lines; three invariant straight lines; two invariant straight lines and one invariant conic.
In this talk we present the solution of the problem of the center for cubic system (1) with two invariant straight lines $l_j \equiv 1 + a_j x - y = 0$, $a_j \in C$, $j = 1,2$, $a_2 - a_1 \neq 0$ and one real irreducible invariant cubic curve $\Phi(x,y) \equiv x^2 + y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 - y^3 = 0$ passing through the same singular point $(0,1)$, i.e. forming a bundle. Our main results are the following two theorems:

Theorem 1. *Let the cubic system (1) have a bundle of two invariant straight lines $l_{1,2} = 0$ and one irreducible invariant cubic curve $\Phi = 0$. Then the origin is a center if and only if the first three Lyapunov quantities vanish.*

Theorem 2. *The cubic system (1) with a center having a bundle of two invariant straight lines $l_{1,2} = 0$ and one invariant cubic curve $\Phi = 0$ is always Darboux integrable.*

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On the computation of an approximation of the center manifold for delay differential equations, with a single constant delay

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We present a review of some of our previous results concerning the computation of an approximation of a local center manifold for delay differential equations or systems of such equations, with a single constant delay. Usually the function whose graph is the local center manifold is written as a series of powers. Some of the first coefficients of this series should be found in order to have a certain approximation of the manifold. Usually, a local center manifold is considered when a

Hopf bifurcation is studied - and to approximate the center manifold in order to deduce the type of the Hopf bifurcation is a simple matter. If the problem presents a degenerate Hopf bifurcation, or a Bautin type bifurcation, some higher order approximation of the center manifold is required. When computing the coefficients of the series for this better approximation, we see that one of the coefficients cannot be uniquely determined. Our study gives a method to chose in a proper manner this coefficient.

On the Feedback Linearization for the Prey-Predator Dynamical Systems

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The qualitative analysis of the mathematical models associated to prey-predator interactions is very important in determining long-time dynamics, together with the study of the main sizes of these models. They are based on the application of various notions from the theory of dynamical systems to the numerical approximation of initial value problems over long-time intervals.
Between dynamical systems theory and computation analysis of dynamical systems there is a strong interplay. In this paper we take into account the problem of finding a state feedback control law for the 2D Lotka-Volterra system, in order to obtain a linear time invariant form of this system. The results will be used both for 3D Lotka-Volterra models, and for further generalizations.

Ternary generalized Lyapunov-Darboux system and some polynomial-exponential first integrals

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We will examine the ternary differential system with quadratic nonlinearities

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta} x^\alpha x^\beta \ (j,\, \alpha,\, \beta = 1,2,3), \tag{1}$$

where a tensor $a^j_{\alpha\beta}$ is symmetric in lower indexes in which the total convolution is done. The coefficients and variables in (1) take values from the field or real numbers $\mathbb{R}$.
In [1] there were determined the necessary and sufficient conditions with respect to $GL(2,\mathbb{R})$ group, when the system (1) can be brought to the Darboux generalized form

$$\dot{x}^j = a^j_\alpha x^\alpha + x^1 p_\alpha x^\alpha \ (j,\, \alpha, = 1,2,3), \tag{2}$$

where p^j_α are real numbers and in α the convolution is done.
From (2) we distinguish the following generalized Lyapunov-Darboux system

$$\begin{gathered} \dot{x}^1 = -\lambda x^2 + x^1 p^1_\alpha x^\alpha,\ \dot{x}^2 = \lambda x^1 + x^1 p^2_\alpha x^\alpha, \\ \dot{x}^3 = x^2 + n x^3 + x^1 p^3_\alpha x^\alpha \ (\alpha = 1,2,3,\ \lambda^2 = -m,\ m < 0,\ n \neq 0), \end{gathered} \tag{3}$$

which play an important role in the problem of the center [2] for system (2).
For system (3) there were examined the conditions of the existence and built the polynomial-exponential first integrals $F = F_2(x^1, x^2, x^3)e^{\mu t}$, where F_2 is a polynomial of degree at most two in the phase variables x^j $(j = 1,2,3)$.

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Applications of Lotka-Volterra equations in ecology

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In this paper we present some applications of Lotka-Volterra equation (including systems of equations) in ecology. We use the mathematical way in treating them, classifying each one in a type of equation or system of equations.

The common Hilbert series of Sibirsky algebras for differential system $s(1,7)$

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We examine the two-dimensional polynomial autonomous differential system of normal form with nonlinearities of seventh degree $s(1,7)$.
From [1] it is known a method of computing common Hilbert series of invariants ring using the residues. We adapted this method for the common Hilbert series for Sibirsky algebras of comitants and invariants of differential systems. In contrast to the construction methods of this series, exposed in [2], with the help of residues [1], the primary generating functions [2], we obtained a common Hilbert series for Sibirsky graded algebras of examined differential system.

Theorem 1. *For the differential system $s(1,7)$ the following common Hilbert series for Sibirsky*

algebras of comitants $S_{1,7}$ and invariants $SI_{1,7}$ was obtained

$$H_{S_{1,7}}(t) = \frac{1}{(1+t)^3(1-t^2)^3(1-t^3)^5(1-t^4)^3(1-t^5)^4(1-t^7)^3(1-t^9)}$$
$$(1+4t+7t^2+7t^3+17t^4+85t^5+331t^6+1009t^7+2657t^8+$$
$$+6368t^9+14278t^{10}+30208t^{11}+60574t^{12}+115441t^{13}+$$
$$+209688t^{14}+363888t^{15}+604838t^{16}+965096t^{17}+1481667t^{18}+$$
$$+2193216t^{19}+3135942t^{20}+4337738t^{21}+5811835t^{22}+7550176t^{23}+$$
$$+9518852t^{24}+11655892t^{25}+13872730t^{26}+16058633t^{27}+$$
$$+18089130t^{28}+19836497t^{29}+21182751t^{30}+22032184t^{31}+$$
$$+22322579t^{32}+22032184t^{33}+21182751t^{34}+19836497t^{35}+$$
$$+18089130t^{36}+16058633t^{37}+13872730t^{38}+11655892t^{39}+$$
$$+9518852t^{40}+7550176t^{41}+5811835t^{42}+4337738t^{43}+3135942t^{44}+$$
$$+2193216t^{45}+1481667t^{46}+965096t^{47}+604838t^{48}+363888t^{49}+$$
$$+209688t^{50}+115441t^{51}+60574t^{52}+30208t^{53}+14278t^{54}+6368t^{55}+$$
$$+2657t^{56}+1009t^{57}+331t^{58}+85t^{59}+17t^{60}+7t^{61}+7t^{62}+$$
$$+4t^{63}+t^{64}),$$

$$H_{SI_{1,7}}(t) = \frac{1}{(1+t)^5(1-t)^3(1-t^3)^5(1-t^4)^4(1-t^5)^3(1-t^7)^2}(1+3t+$$
$$+4t^2+2t^3+9t^4+53t^5+196t^6+525t^7+1214t^8+2558t^9+$$
$$+5097t^{10}+9569t^{11}+16975t^{12}+28396t^{13}+44981t^{14}+67577t^{15}+$$
$$+96665t^{16}+131839t^{17}+171920t^{18}+214631t^{19}+257063t^{20}+$$
$$+295599t^{21}+326684t^{22}+346880t^{23}+353937t^{24}+346880t^{25}+$$
$$+326684t^{26}+295599t^{27}+257063t^{28}+214631t^{29}+171920t^{30}+$$
$$+131839t^{31}+96665t^{32}+67577t^{33}+44981t^{34}+28396t^{35}+16975t^{36}+$$
$$+9569t^{37}+5097t^{38}+2558t^{39}+1214t^{40}+525t^{41}+196t^{42}+53t^{43}+$$
$$+9t^{44}+2t^{45}+4t^{46}+3t^{47}+t^{48}).$$

From this theorem result that the Krull dimension [2] of Sibirsky graded algebra $S_{1,7}$ (respectively $SI_{1,7}$) is equal to 19 (respectively 17). We note that the Krull dimension plays an important role in solving the center-focus problem for the differential system $s(1,7)$ [3].
We mention that this method of computing the common Hilbert series for the algebras of comitants and invariants for the differential systems was verified on the example of already known Hilbert series from [2].

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Krull dimension and common Hilbert series of Sibirsky algebras for differential system $s(7)$

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In contrast to the construction methods of Hilbert series, exposed in [3], using the residues, we obtained a new method for computing a common Hilbert series for Sibirsky graded algebras of differential systems.
The Theorem 4.5 [1] about Hilbert series of invariants ring, was adapted for Sibirsky graded algebras of comitants and invariants of differential systems. Here were used the primary generating functions [3] and the residue theorem [Th. 4.6.13, 2]. By this method were obtained the common Hilbert series of Sibirsky graded algebras for the system of differential equations $s(7)$,

Theorem 1. *For the differential system $s(7)$ the following common Hilbert series for Sibirsky graded algebras of comitants S_7 and invariants SI_7 was obtained*

$$H_{S_7}(t)=\frac{1}{(1+t)^5(1-t)(1-t^3)^4(1-t^4)^2(1-t^5)^4(1-t^7)^3(1-t^9)}(1+4t+$$
$$+7t^2+6t^3+6t^4+28t^5+112t^6+325t^7+788t^8+1719t^9+3499t^{10}+$$
$$+6716t^{11}+12225t^{12}+21205t^{13}+35194t^{14}+56030t^{15}+85698t^{16}+$$
$$+126023t^{17}+178425t^{18}+243697t^{19}+321789t^{20}+411501t^{21}+510260t^{22}+$$
$$+613944t^{23}+717118t^{24}+813553t^{25}+896906t^{26}+961309t^{27}+1002042t^{28}+$$
$$+1015982t^{29}+1002042t^{30}+961309t^{31}+896906t^{32}+813553t^{33}+$$
$$+717118t^{34}+613944t^{35}+510260t^{36}+411501t^{37}+321789t^{38}+243697t^{39}+$$
$$+178425t^{40}+126023t^{41}+85698t^{42}+56030t^{43}+35194t^{44}+21205t^{45}+$$
$$+12225t^{46}+6716t^{47}+3499t^{48}+1719t^{49}+788t^{50}+325t^{51}+112t^{52}+$$
$$+28t^{53}+6t^{54}+6t^{55}+7t^{56}+4t^{57}+t^{58}),$$

$$H_{SI_7}(t)=\frac{1}{(1+t)^4(1-t)(1-t^3)^4(1-t^4)^3(1-t^5)^3(1-t^7)^2}(1+3t+4t^2+$$
$$+2t^3+2t^4+15t^5+59t^6+150t^7+312t^8+578t^9+1011t^{10}+1673t^{11}+$$
$$+2631t^{12}+3917t^{13}+5541t^{14}+7450t^{15}+9551t^{16}+11651t^{17}+13543t^{18}+$$
$$+15011t^{19}+15933t^{20}+16238t^{21}+15933t^{22}+15011t^{23}+13543t^{24}+$$
$$+11651t^{25}+9551t^{26}+7450t^{27}+5541t^{28}+3917t^{29}+2631t^{30}+1673t^{31}+$$
$$+1011t^{32}+578t^{33}+312t^{34}+150t^{35}+59t^{36}+15t^{37}+2t^{38}+2t^{39}+$$
$$+4t^{40}+3t^{41}+t^{42}).$$

From [3] was abtained

Theorem 2. *The Krull dimension of Sibirsky algebra* S_7 (*respectively* SI_7) *is equal to* 15 (*respectively* 13).

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Approximations of the homoclinic solutions for the FitzHugh-Nagumo system

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The FitzHugh-Nagumo system modeling the electrical potential of the heart is considered. The Bogdanov-Takens bifurcation is detected using the normal form method. Then the local representation of the curve containing homoclinic bifurcation values is obtained. For parameters situated on this curve, we obtain second-order approximations for the homoclinic orbits. The solutions obtained theoretically are compared with those obtained numerically for several values of the parameters.

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On a Class of Eigenvalue Problems for Ordinary Differential Equations

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Eigenvalue problems for which some of the eigenvalues render the assumed solution invalid are considered. Homogeneous linear ordinary linear differential equations involving a parameter with homogeneous boundary conditions normally give an eigenvalue equation for the parameter. The solution of the ordinary differential equation is assumed to be of exponential type and the characteristic equation involves the eigenvalue. In some cases, the eigenvalue equation obtained after applying the boundary conditions may involve multiple roots for a certain eigenvalue. In this case the assumption of an exponential solution breaks down. This type of problems occur in studies of mechanical or electrical vibrations.

Cubic Differential Systems With Two Real Invariant Straight Lines of Maximal Multiplicity

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We consider the real polynomial system of differential equations

$$\dot{x} = P(x,y), \qquad \dot{x} = Q(x,y), \quad GCD(P,Q) = 1. \tag{1}$$

Denote $n = \max\{\deg(P), \deg(Q)\}$. If $n = 3$, then (1) is called the cubic system. A real straight line $\alpha x + \beta y + \gamma = 0,\ \alpha, \beta, \gamma \in \mathbb{R},\ (\alpha, \beta) \neq 0$ is called invariant for (1) if there exists a polynomial $K(x,y)$ such that the following identity $\alpha P(x,y) + \beta Q(x,y) \equiv (\alpha x + \beta y + \gamma)K(x,y)$ holds.
In this paper we show that in the class of cubic systems with two parallel real invariant straight lines the maximal multiplicity of one of these lines is equal to four and of other is equal to three.

Theorem 1. *Any cubic system having two parallel real invariant straight lines with multiplicities $m_1 = 4$, $m_2 = 3$, via an affine transformation and time rescaling can be brought to the form:*

$$\dot{x} = x(x-1)^2, \quad \dot{y} = x^3 + y(x-1)^2. \tag{2}$$

The system (2) admits the invariant straight lines $x = 0$ of multiplicity $m_1 = 4$ and $x = 1$ of multiplicity $m_2 = 3$.
For system (2) the line at infinity cannot have multiplicity greater than one.
The perturbed system of (2)

$$\begin{aligned} &\dot{x} = x(x-1)(x+2x\epsilon-1), \quad \dot{y} = x^3 + y(x-1)^2 - 2xy\epsilon + 2x^2y\epsilon + \\ &+x^2y\epsilon^2 - xy^2\epsilon^2 - 2xy^2\epsilon^3 - xy^2\epsilon^4 - y^3\epsilon^4 - 2y^3\epsilon^5 - y^3\epsilon^6 \end{aligned} \tag{3}$$

has seven affine distinct invariant straight lines: $l_1 = x$, $l_2 = x + y\epsilon^2$, $l_3 = x - y\epsilon - y\epsilon^2$, $l_4 = x + y\epsilon + y\epsilon^2$, $l_5 = x - 1$, $l_6 = x + 2x\epsilon - 1$, $l_7 = x + x\epsilon + y\epsilon^2 + y\epsilon^3 - 1$.
If $\epsilon \to 0$, then (3) tends to (2) and $l_1, l_2, l_3, l_4 \to l_1$, $l_5, l_6, l_7 \to l_5$.
Acknowledgement. *The authors acknowledges the support of FP7-PEOPLE-2012-IRSES-316338.*

The Time-Reversal Symmetry and the Technical Condition of Proper Operation of the Asynchronous Flows

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The asynchronous flows are generated by Boolean functions $\Phi : \{0,1\}^n \to \{0,1\}^n$ whose coordinates $\Phi_i, i = \overline{1,n}$ are iterated independently on each other. The order and the time instants of these iterations are not known. The flows are the models of the asynchronous circuits from the digital electronics. Time-reversal symmetry is one of the fundamental symmetries discussed in natural science and our main purpose is to adapt this concept, by analogy with mechanics, to the asynchronous flows. The technical condition of proper operation, also known as race-freedom, is a special case of work in asynchronicity which 'softens' the non-determinism of the models. We prove that when it is fulfilled, the flow behaves like a dynamical system. Finally, we relate the time-reversal symmetry and the fulfillement of the technical condition of proper operation.

4. Probability Theory, Mathematical Statistics, Operations Research

Evaluation and Assessment of Multiple Partitions for Determining the Adequated Load Profiles in a Power Distribution Station

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The paper addresses the problem of evaluating the multiple partitions issued from various clustering algorithms or from the various running of the same algorithm with different parameters. The aim of the evaluation and assesment is to find an optimal and consistent partition of the data set. The problem of finding the optimal partition is NP complete, and various heuristic algorithms have been published. The paper is focused on the concept of central partition and suitable similarity metrics that may be used for finding it.
The experiments used a data set containing time series of power consumptions measured hourly in a distribution station. The clusters taken into consideration were load profiles that are very important to be known in the electricity market. These load profiles may vary from the day of the week, the season, or under the influence of economic or demographic factors. The optimal partition is useful to discover the adequate models of load profiles.

Nontrivial convex 2-covers of simple undirected graphs

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The concept of *convex p-cover* of a graph, for $p \geq 2$, is defined in [1], as a cover of graph by p convex sets. Some basic ideas about the Euclidean concepts of convex sets to graph theory can be found in [4]. In particular, a family of sets $\mathcal{P}_2(G) = \{X_1, X_2\}$ is called *convex 2-cover* of a graph $G = (X; U)$ if $X_1 \not\subseteq X_2$, $X_2 \not\subseteq X_1$ and $X_1 \cup X_2 = X$, where X_1, X_2 are convex sets in G.
A *convex* $(2,t)$*-cover* of G is a convex 2-cover of G such that at least one convex set S is trivial, i.e, S consist of one or two vertices. Similarly, a *convex* $(2,nt)$*-cover* of G is a convex 2-cover of G such that both sets are nontrivial.
Deciding if a graph has a convex 2-cover was declared an open problem in [1]. After, we proved its NP-completeness [3]. We know that verifying if a set is convex can be done in polynomial-time [2]. Consequently determining if there exist a convex $(2,t)$-cover also can be done in polynomial-time. Thus it is NP-complete to decide whether a graph G has a convex $(2,nt)$-cover.
We establish existence of a convex $(2,nt)$-cover in dependency of existing convex $(2,t)$-covers. Also, we distinguished cases where existence of convex $(2,t)$-covers implies existence of a convex $(2,nt)$-cover. Generally, we obtained the following result.

Theorem 1. *It is NP-complete do decide whether a graph that has convex* $(2,t)$*-covers also has a convex* $(2,nt)$*-cover.*

In addition, we identified some classes of graphs for which there exist a convex $(2,nt)$-cover.

Theorem 2. *A chordal graph* G *on* $n \geq 4$ *vertices has a convex* $(2,nt)$*-cover.*

Theorem 3. *A power of cycle* C_n^k *has a convex* $(2,nt)$*-cover if and only if* $n \geq 4$, $C_n^k \neq C_4$, *and* $n \leq 2k+2$ *or* $n \equiv 0,1,2(mod\ 2k)$.

Theorem 4. *A cactus graph* G *on* n *vertices has a convex* $(2,nt)$*-cover if and only if* $n \geq 4$ *and* $G \neq C_4$.

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Modelling of Birth-Death Process

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The main aim of this work is focused analyzation of the Keller-Segel and Murray models. Subsequently the relative stability of stationary states are analysed using the Keller-Segel system for the chemotaxis with linear diffusion and the Murray model for birth-death process. When we analysed two models, we used some techniques to find the solution and then we showed some graphics.

Optimization tools for selecting wine routes

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The information resource TourGourMania (http://tourgourmania.com/) focuses on people who want to travel slowly, with pleasure. One section of this resource is devoted to wine topics and wine routes. It's often difficult to use routes offered by the guides due to lack of time, money or other reasons. In this case, the user is forced to plan out an individual route and solve the problem of optimal selection of wine routes. At the present time within the framework of TourGourMania resource the service with optimization tools for selecting and building wine routes is being developed to facilitate the selection of an individual wine route and to reduce the time to prepare for the journey.
The basis of these optimization tools is the problem of finding the shortest path, passing through a given number of nodes of a directed graph. The nodes of the graph are the wineries with their characteristics (time for tasting, time for possible sightseeing tours and accommodation, as well as a description of the wine produced). The arcs, which connect the nodes, are associated with such characteristics as distance, travel time and priority. The priority takes into account the sightseeing attraction and the possibility of selection of wines in accordance with user preferences.

The problem is formulated in the form of Boolean Linear Programming problem and is implemented as AMPL modeling language code. AMPL code was tested for data of Malopolska Wine Road. We considered the 20 most visited wineries along direction of Lviv-Wroclaw-Lviv. To solve the Boolean Linear Programming problem Gurobi program from NEOS server (http://www.neos-server.org/neos/solvers/) was used. It takes a few seconds for Gurobi program to find optimal route, passing through all 20 wineries.

Bayes solutions of the bimatrix informational extended games

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Consider the bimatrix game in the informational extended strategies

$$I^{\alpha}=\left\{\left(i_1^{\alpha},i_2^{\alpha},...,i_j^{\alpha},...i_m^{\alpha}\right):i_j^{\alpha}\in I,\forall j=\overline{1,m}\right\}\subseteq I,$$
$$J^{\beta}=\left\{\left(\left(j_1^{\beta},j_2^{\beta},...,j_i^{\beta},...j_n^{\beta}\right):j_i^{\beta}\in J,\forall i=\overline{1,n}\right)\right\}\subseteq J$$

that generate the bimatrix incomplete information game $\widetilde{\mathbf{\Gamma}}$ on the set of strategies I,J and payoffs are one of the matrix $\left\{AB(\alpha,\beta)=\left\|\left(a_{ij}^{\alpha\beta},b_{ij}^{\alpha\beta}\right)\right\|_{i\in I}^{j\in J}\right\}_{\alpha=\overline{1,\kappa_1}}^{\beta=\overline{1,\kappa_2}}$, $\left\|a_{ij}^{\alpha\beta}\right\|_{i\in I}^{j\in J}=\left\|a_{i_j^{\alpha}j_i^{\beta}}\right\|_{i\in I}^{j\in J}$, $\left\|b_{ij}^{\alpha\beta}\right\|_{i\in I}^{j\in J}=\left\|b_{i_j^{\alpha}j_i^{\beta}}\right\|_{i\in I}^{j\in J}$ for all $\alpha=\overline{1,\kappa_1}$, $\beta=\overline{1,\kappa_2}$ [1]. For game $\widetilde{\mathbf{\Gamma}}$ we construct the normal form of the bimatrix Bayesian game $\mathbf{\Gamma}_{Bayes}$ where $\mathbf{A}(\alpha)=\left\|\sum\limits_{\beta=1}^{\kappa_2}p(\beta/\alpha)a_{i_\beta j_\alpha}^{\alpha\beta}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$, $\mathbf{B}(\beta)=\left\|\sum\limits_{\alpha=1}^{\kappa_1}q(\alpha|\beta)b_{i_\beta j_\alpha}^{\alpha\beta}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$ are player's expected payoffs,

$$\widetilde{\mathbf{I}}(\alpha)=\left\{\widetilde{\mathbf{i}}=i_1i_2...i_\beta...i_{\kappa_2}:i_\beta\in I,\forall\beta=\overline{1,\kappa_2}\right\}$$

$$\widetilde{\mathbf{J}}(\beta)=\left\{\widetilde{\mathbf{j}}=\ j_1j_2...j_\alpha...j_{\kappa_2}:j_\alpha\in J,\forall\alpha=\overline{1,\kappa_1}\right\}$$

are the sets of pure strategies of the players. We can demonstrate the following theorem.

Theorem. *If the player* 1 *chooses the information extended strategy* θ_1^{α} *(respectively the player* 2 *choose the information extended strategy* θ_2^{β}*) and assumes that the player* 2 *for all* β *will choose the information extended strategies* θ_2^{β} *with probability* $p\left(\theta_2^{\beta}|\theta_1^{\alpha}\right)$ *(respectively the player* 2 *assumes that for all* α *the player* 1 *will choose the information extended strategies* θ_1^{α} *with probability* $q\left(\theta_1^{\alpha}|\theta_2^{\beta}\right)$*), then the Nash equilibrium profiles of the bimatrix Bayesian game* $\mathbf{\Gamma}_{Bayes}$ *with matrices* $\mathbf{A}(\alpha),\mathbf{B}(\beta)$ *for all* $\alpha=\overline{1,\kappa_1}$, $\beta=\overline{1,\kappa_2}$ *is the Bayes-Nash equilibria of the bimatrix informational extended game.*

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Modeling of oligopolistic competition using the evolution games theory

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Evolutionary games are quite distinct from ordinary games and from differential games, even though all three examine the dynamics of repeated strategic interaction. In an evolutionary game each individual chooses among alternative actions or behaviors whose payoff or fitness depends on the choices of others [1]. A natural interpretation is of essentially anonymous interactions, in which the individuals one deals with today are not recognizably the same as those dealt with yesterday. Another distinction is that detailed analysis of evolutionary games focuses on the distribution of behaviors in populations rather than on the behavior of rational individuals [2].
Consider a given industry composed of n firms, all of which sell a certain homogeneous product. Let the demand function of this product be given by a certain (strictly) decreasing function $P(\cdot)$, whose argument is the sum $x_1 + x_2 + ... + x_n$, where each x_i denotes the sales of firm $i = 1, 2, ..., n$. Let cost conditions be identical for each firm i and represented by a common (differentiable) cost function $C(\cdot)$ with $\frac{dC^2(\cdot)}{d(x_i)^2} > 0$.
Suppose firms choose their quantities simultaneously.
A symmetric Cournot-Nash equilibrium of this game $\{x^*\}$ is characterized by the inequality:

$$P(n\,x^*)x^* - C(x^*) \geq P((n-1)x^* + x)x - C(x),$$

for all $x \geq 0$.
On the other hand, competitive behavior requires that each firm $i = 1, 2, ..., n$ chooses a quantity x^w such that:

$$P(n\,x^w)x^w - C(x^w) \geq P(n\,x^w)x - C(x),$$

for all $x \geq 0$. That is, each firm takes the market-clearing price $P(n\,x^w)$ as given and maximizes profits.
It is shown that x^w is, however, an evolutionary stable strategy for the n-firm industry described. Specifically, if all other firms choose x^w, any single firm which chooses a different output always obtains a strictly lower profit than the other $n - 1$ firms.

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A fast numerical method for determining the limit matrix in Markov chains

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We consider a Markov chain L with finite set of states $\{x_1, x_2, \ldots, x_n\}$ and transition matrix $P = (p(x_i, x_j))_{i,j=\overline{1,n}}$. Let $q(x_i, x_j)$ be the probability with which the process L will occupy the state x_j after a large number of transitions when the initial state is x_i, $i, j = \overline{1, n}$. The matrix $Q = (q(x_i, x_j))_{i,j=\overline{1,n}}$ is called the limit matrix of the given system.

The computation of the limit matrix Q was also studied in [1] and [3], where polynomial algorithms were developed. The goal of this paper is the modification of the algorithm proposed in [3] for optimizing the running time complexity. The new algorithm consists from four steps:
1. determination of the characteristic polynomial $K_P(z) = |P - zI_n|$;
2. division of the polynomial $K_P(z)$ by $(z-1)^{m(1)}$, where $m(1)$ is the multiplicity of the eigenvalue $z_0 = 1$, obtaining the quotient $T(z)$;
3. computation of the matrix $R = T(P)$;
4. determination of the limit matrix Q by dividing the matrix R by the sum of the elements of an arbitrary its row.

For obtaining better performance, we must use one fast method at every step. Let $O(n^\omega)$ be the complexity of the fastest known matrix multiplication algorithm, i.e. Le Gall algorithm ([2]) with $\omega = 2.3728639\ldots$. So, it is recommended to use the $O(n^\omega)$ algorithm from [4] at the Step 1, $O(n \cdot m(1))$ Horner schema at the Step 2 and $O(n^{\max\{\omega+1/2,\ 3\}})$ algorithm from [5] at the Step 3, for obtaining the $O(n^3)$ complexity for the entire algorithm.

The developed algorithms were implemented using Wolfram Mathematica 10.1. The performance of the new algorithm is essentially increased, comparing with the algorithm developed in [3]. For example, running these two algorithms on the same computer for a 2500×2500 random transition matrix, we observed that the old algorithm obtained the limit matrix in 1350.73 seconds, but the new algorithm only in 49.689 seconds. This performance is due to $O(n^3)$ complexity of the new algorithm compared with $O(n^4)$ of the old algorithm, utilization of several Mathematica built-in optimized functions, compilation of the non built-in functions and automatical parallelization where is possible. Also, the maximum element of the absolute value of the difference between the obtained matrix and exact limit matrix is $1.9 \cdot 10^{-15}$ for the new algorithm and $1.5 \cdot 10^{-16}$ for the old algorithm.

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A stochastic approach for a path dependent Kolmogorov equation

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We provide a probabilistic representation of the viscosity solution for a functional parabolic partial differential equation. In order to do this we use the solution of a backward stochastic differential equation with time delayed–generators.

Models and methods for measurement of Exchange Rate Risk

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The currency changes can affect us, whether we are actively trading in the foreign exchange market, planning our next vacation, shopping online for goods from another country, or just buying food or other things imported from abroad. The value of a currency depends on factors that affect the economy such as trade, inflation, employment, interest rates, growth rate and geopolitical conditions. For the problem of Currency Exchange we can use the Game Theory and to construct a mathematical model for two participants (or more), and to analyze all parameters which can influence the results of this operations. In order to analyze this problem as a multistage interactive situation, in this paper we use the theory of extensive games and we construct a dynamical model. Every such situation can be represented as a strategic game of two persons (the seller and the buyer). In this case the outcome of the dynamical game can be determined applying the back tracking method using the best response-sets for every participant.
More complicated could be a model for measurement of exchange rate risk for a firm. In this case we could apply some statistical simulation. The three main types of exchange rate risk consider by (Shapiro, 1996; Madura, 1989) are: Transaction risk, Translation risk, Economic risk. After defining the types of exchange rate risk that a firm is exposed to, a crucial aspect in a firm's exchange rate risk management decisions is the measurement of these risks. At present, a widely used method is the value-at-risk (VaR) model. To calculate the VaR, there exists a variety of models. Among them, the more widely-used are: (1) the historical simulation, which assumes that currency returns on a firm's foreign exchange position will have the same distribution as they had in the past; (2) the variance- covariance model, which assumes that currency returns on a firm's total foreign exchange position are always (jointly) normally distributed and that the change in the value of the foreign exchange position is linearly dependent on all currency returns; and (3) Monte Carlo simulation, which assumes that future currency returns will be randomly distributed.

On Entropic Measures for Log-linear Semiparametric Regression Models with Poisson Response

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Stochastic modeling processes gain increasing value in terms of managing various phenomena. Ones of them are models with Poisson response. The paper identifies measures for the informational properties of log-linear semiparametric regression models with Poisson response in terms of conditional entropies: Shannon and -Renyi entropy. There are defined conditional entropies and calculated for different distributions of covariates. For conditional entropies a resubstitution kernel estimator is briefly presented. The issue of nonparametric estimation of the conditional quadratic Renyi entropy is addressed with extension of the Xu and Erdogmuns approach to random vectors. The paper closes with a simulation study focused on the performances of the new estimators. Based on their properties and on simulation results, the conclusion is that a new goodness-of-fit index for regression models with Poisson response is identified, as well as a new quantitative criterion for statistical modeling, based on conditional entropies.

Statistical analysis of monetary policy indicators variability

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This paper attempts to characterize the statistical indicators - quantitative measures - the statistical data we have available. The purpose of this paper is to present statistical indicators, primary and secondary, simple and synthetic, which is frequently used for statistical characterization of statistical series. We can thus analyze central tendency, and data variability, form and concentration distributions package data using analytical tools in Microsoft Excel that enables automatic calculation of descriptive statistics using Data Analysis option from the Tools menu. We also study the links which exist between statistical variables can be studied using two techniques of correlation and regression. From the analysis of monetary policy in the period 2003 - 2014 and information provided by the website of the National Bank of Romania (BNR) seems to be a certain tendency towards eccentricity and asymmetry of financial data series.

Determining of the distribution form for small volume samples

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It is important to know distribution law form of the studied random variables in the majority of the researches connected with use of probability theory methods and mathematical statistics. According to the classical probability theory the distribution law type on large volume samples

can be found by means of Pearson curves family.
Selecting such a curve from Pearson curves family, whose first four points coincide with sampling points determined by the experimental data, is the curve-fitting content for the probability distribution of the random variable.
It is necessary to consider that rather stable and effective distribution moments evaluations are achieved only at large volume samples. For example, the relative error of a selective mean square deviation is equal to $\sqrt{1/(2n-1,4)}$ (at n=5 makes 0,341 or 34,1%, and at n=10 makes @ S 0,232 or 23,2%). However, in the modern industry often meet such productions, which owing to technological restrictions can't give sample of rather large volume.
In this case it is proposed to reduce the error by searching estimates of small volume samples parameters by the method of point distributions.

Closed-form formula for S PDE for pricing in Heston framework

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In this paper we build a closed-form solution of S PDE (or Heston-S PDE), a PDE derived from Heston model (a stochastic extension of Geometric Brownian motion with a CIR process for stochastic volatility). Using an Ito like lemma, Heston obtain a PDE, but he introduce a new parameter from market (beta / lambda). Heston offers a closed solution based on characteristic function method. We use closed-form solution of Heston PDE and build solution for S PDE.

C++ code for S conjecture validation

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In this paper we build a C++ tool for a conjecture validation, in Popper sense. We say that conjecture, based on a new PDE derivation from Heston model with using a virtual derivative based on volatility, is correct, but we have not a proof (only derivation method that it seems reliable). Validation can be made only with comparison of: a) Monte-Carlo simulation (affected by discretization method and statistical method as confidence interval without any information about dimension of confidence interval); b) closed-form solution or other numeric solution of Heston PDE; and c) closed-form solution or other numeric solution of Heston-S PDE. Conjecture say that Heston-S PDE is better than Heston PDE for pricing derivatives. We used this tool on two sets of data: a) one with Feller condition respects, and b) one without Feller condition respects. Our conclusion is that we can say that conjecture is validated in Popper sense.

Comparative analysis of estimation methods of the real gross value added, in Romania through Cobb-Douglas production function

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This article describes a comparative analysis of the methods for estimating Cobb-Douglas production function. In the paper, the three models analyzed, in which two are static and one is dynamic, are solved by the linearization method and the logarithmic transcendental method. The data series which occur in patterns, are given by the real gross value added, regarded as output variable, and the tangible assets, respectively the average number of employees, regarded as in- put variables. The parameters of the models are determined using the least squares method (LSM), using Eviews. The comparative analysis of models refers both to capacity by estimate with small errors and to verify the statistical tests and ease of implementing these methods.

Stackelberg Equilibrium Set Computing in Mixed-Strategy Stackelberg Games

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The Stackelberg equilibrium set (**SES**) is described as a set of optimal solutions of an optimization problem obtained as a result of solving a sequence of optimization problems that reduces the graph of best response mapping of the last player to the **SES**. The problem of **SES** computing in multi-matrix finite mixed-strategy games (generalised Stackelberg games) is considered. A method for **SES** computing is exposed.

How common sense can be misleading in insurance?

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Some graphical representations of ruin probability computed mainly for Erlang type claims suggested an idea that intuitively seems to be true: if the first claims are small then the chance to get ruined is also small. However, for other claims this does not hold, as is shown by the counterexample mentioned here. Our conjecture is that if the claims follow a certain stochastic order, this statement is correct. We have proved it only for exponential random variables, which have the property that a mixture of them can be written as a convolution. This property proved to be essential, the negative binomial distribution functions have it also and they seem to satisfy the conjecture. But the problem remains open: what other types of claims enable us to order the ruin probabilities according to an increasing order of their arrival?

5. Algebra, Logic, Geometry (with applications)

Figuratrix foliations of a Cartan space

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A Cartan space is a manifold whose cotangent bundle of local coordinates (x, p) is endowed with a smooth regular Hamiltonian $K^2(x, p)$ which is positively homogeneous of degree 2 in momenta p. Then the cotangent bundle becomes a semi-Riemannian manifold with a metric G of Sasaki type.We prove that a) for a fixed $x = x_0$ the level hypersurfaces of the function $K(x_0, p)$ define a foliation which is totally umbilical and orthogonal with respect to G on the foliation spanned by the Liouville vector field in x_0 and b) the level hypersurfaces of the function $K(x, p)$ define also a foliation which is orthogonal on the foliation spanned by the Liouville vector field in (x, p) if and only if the Cartan space is Landsberg. The other natural foliations on the cotangent bundle are taken into consideration.
Mathematics Subject Classification 2010: 53C12,53C15

Topological medial right loops which are direct products of topological commutative groups

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In this paper was developed a new method of constructing topological medial right loops with using the special direct product of Abelian groups.
A topological groupoid is a Hausdorff space equipped with a jointly continuous binary operation.
A topological groupoid $(G, \cdot)$ is called *medial* if $xy \cdot zt = xz \cdot yt$, for any $x, y, z, t \in G$.
A groupoid $(G, \cdot)$ is called a quasigroup if for every $a, b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.
If, in addition, G is a topological space and $(\cdot)$ is continuous with respect to the topology of G, we say that $(G, \cdot)$ is a topological quasigroup.
If a medial quasigroup $(G, \cdot)$ contains an element e such that $x \cdot e = e$ for all $x \in G$, then e is called a right identity element of $(G, \cdot)$ and $(G, \cdot)$ is called a medial right loop.

The results we establish here are related to [1]-[5].

Theorem 1. *Let $(G, +)$ be a commutative topological group. The topological space $G \times G$ equipped with the continuous operation*

$$(x_1, y_1) \circ (x_2, y_2) = (x_1 + y_2 + x_2 + y_2, y_1 + y_2)$$

is a topological medial right loop. Moreover, the following statements are true:
1. If $(G, +)$ is compact, then $(G \times G, \circ)$ is also compact.
2. If $(G, +)$ is Hausdorff, then $(G \times G, \circ)$ is also Hausdorff.
3. If $(G, +)$ is completely regular, then $(G \times G, \circ)$ is also completely regular.

Theorem 2. *Let $(G,+)$ be a commutative topological group. The topological space $G\times G$ equipped with the continuous operation*

$$(x_1,y_1)\circ(x_2,y_2)=(x_1-y_2+x_2-y_2,y_1+y_2)$$

is a topological medial right loop. Moreover, the following statements are true:
1. If $(G,+)$ is compact, then $(G\times G,\circ)$ is also compact.
2. If $(G,+)$ is Hausdorff, then $(G\times G,\circ)$ is also Hausdorff.
3. If $(G,+)$ is completely regular, then $(G\times G,\circ)$ is also completely regular.

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About semireflexive subcategories

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In the category $\mathcal{C}_2\mathcal{V}$ of the vectorial topological locally convex Hausdorff spaces any nonzero reflective or coreflective subcategory is bireflective or bicoreflective. Let be $\mathbb{R}$ (respectively $\mathbb{K}$) the class of the nonzero reflective (respectively coreflective) subcategories, S the reflective subcategory of the spaces with weak topology, and $\widetilde{\mathcal{M}}$ - the coreflective subcategory of the spaces with the Mackey topology. For a reflector functor $l:\mathcal{C}_2\mathcal{V}\to\mathcal{L}$ (respectively a coreflective functor $k:\mathcal{C}_2\mathcal{V}\to\mathcal{K}$), we denote $\varepsilon\mathcal{L}=\{e\in\mathcal{E}pi\mathcal{C}_2\mathcal{V}\mid l(e)\in\mathcal{I}so\}$, (respectively $\mu\mathcal{K}=\{m\in\mathcal{M}ono\mathcal{C}_2\mathcal{V}\mid k(m)\in\mathcal{I}so\}$).

Let $\mathcal{K}\in\mathbb{K}$ and $\mathcal{L}\in\mathbb{R}$ be. It is said that $(\mathcal{K},\mathcal{L})$ forming a pair of conjugates subcategories, if $\mu\mathcal{K}=\varepsilon\mathcal{L}$. $(\widetilde{\mathcal{M}},S)$ is the smallest pair of conjugates subcategories (see [1]).

Let $\mathcal{L},\mathcal{R}\in\mathbb{R}$. $\mathcal{R}$ is called $\mathcal{L}^\varepsilon$-semireflexive (respectively $\mathcal{L}_\varepsilon$-semireflexive), if $\mathcal{R}$ is closed relative to $\varepsilon\mathcal{L}$ -subobjects (respectively $\varepsilon\mathcal{L}$-factorobjects). We note $\mathbb{R}^\varepsilon(\mathcal{L})$ (respectively $\mathbb{R}_\varepsilon(\mathcal{L})$) the class of all $\mathcal{L}^\varepsilon$-semireflexive subcategories (respectively $\mathcal{L}_\varepsilon$-semireflexive), and $\mathbb{R}^\varepsilon_\varepsilon(\mathcal{L})=\mathbb{R}^\varepsilon(\mathcal{L})\cap\mathbb{R}_\varepsilon(\mathcal{L})$.

For $\mathcal{L}\in\mathbb{R}$ and $\mathcal{K}\in\mathbb{K}$ we note $\mathbb{R}(\mathcal{L})=\{\mathcal{R}\in\mathbb{R}|\mathcal{R}\subset\mathcal{L}\}$, $\mathbb{R}(\mathcal{K})=\{\mathcal{R}\in\mathbb{R}|\mathcal{R}\subset\mathcal{K}\}$.

For a class $\mathcal{B}$ of bimorphisme of the category $\mathcal{C}_2\mathcal{V}$ we note $S_\mathcal{B}(\mathcal{A})$ (respectively $Q_\mathcal{B}(\mathcal{A})$) the full subcategory of all $\mathcal{B}$-subobjects (respectively $\mathcal{B}$-factorobjects) of objects of $\mathcal{A}$. Theorem. *Let $(\mathcal{K},\mathcal{L})$ a pair of subcategories of category A.*

1. *For any $\mathcal{R}\in\mathbb{R}$, $\mathcal{L}*_{sr}\mathcal{R}\in\mathbb{R}^\varepsilon_\varepsilon(\mathcal{L})$.*

2. *The mapping* $\varphi_1 : \mathcal{R} \mapsto \varphi_1(\mathcal{R}) = Q_{\varepsilon\mathcal{L}}(\mathcal{R})$, *defined on the elements of the class* $\mathbb{R}(\mathcal{K})$ *takes values in class* $\mathbb{R}_\varepsilon^\varepsilon(\mathcal{L}) : \varphi_1(\mathcal{R}) \in \mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$.

3. *The mapping* $\mathcal{R} \mapsto \overline{\psi_1}(\mathcal{R}) = \mathcal{K} \cap \mathcal{R}$, *defined on the elements of the class* $\mathbb{R}^\varepsilon(\mathcal{L})$ *takes values in class* $\mathbb{R}(\mathcal{K}) : \overline{\psi_1}(\mathcal{R}) \in \mathbb{R}(\mathcal{K})$.

4. *The mapping* $\psi_1 : \mathbb{R}_\varepsilon^\varepsilon(\mathcal{L}) \to \mathbb{R}(\mathcal{K})$, *where* ψ_1 *it is the restriction of the application* $\overline{\psi_1}$ *on the subclass* $\mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$ *is each invers of the mapping* φ_1*:* $\varphi_1 \cdot \psi_1 = 1$, $\psi_1 \cdot \varphi_1 = 1$.

5. *The mapping* $\mathcal{R} \mapsto \varphi(\mathcal{R}) = S_{\varepsilon\mathcal{L}}(\mathcal{R})$, *defined on the elements of the class* $\mathbb{R}(\mathcal{L})$ *takes values in class* $\mathbb{R}_\varepsilon^\varepsilon(\mathcal{L}) : \varphi(\mathcal{R}) \in \mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$.

6. *The mapping* $\mathcal{R} \mapsto \psi(\mathcal{R}) = \mathcal{L} \cap \mathcal{R}$, *defined on the elements of the class* $\mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$ *takes values in class* $\mathbb{R}(\mathcal{L}) : \varphi(\mathcal{R}) \in \mathbb{R}(\mathcal{L})$.

7. *The mapping* φ *and* ψ *are reciprocally reverse* $\varphi \cdot \psi = 1$, $\psi \cdot \varphi = 1$.

8. For any $\mathcal{R} \in \mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$ $(\psi_1(\mathcal{R}), \psi(\mathcal{R}))$ is a pair of conjugates subcategories of category $\mathcal{R}$.

9. $\mathbb{R}(\mathcal{K})$, $\mathbb{R}_\varepsilon^\varepsilon(\mathcal{L})$ and $\mathbb{R}(\mathcal{L})$ contain a own class of elements.

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Generalized convexity in a complex of multy-ary relations

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Let $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$ be a complex of multy-are relations, defined in the work [1]. In this complex every series of sequences $r_{t_1}^k, r_{t_2}^k, ...r_{t_s}^k$ of m-ary relation R^m with the properties:

a) $r_i^k \subset r_{t_1}^m$, $r_j^k \subset r_{t_s}^m$, $1 \leq k < m$;

b) $r_{t_p}^m \cap r_{t_{p+1}}^m \in R^l$, $k \leq l < m$, for any $1 \leq p \leq s-1$,

is called m-dimensional k-chain with the extremities in r_i^k, r_j^k and is denoted by ${}^kL^m\left(r_i^k, r_j^k\right)$, $1 \leq k < m \leq n+1$. The number s is called the length of this chain. We define the function of distance $d_k^m : R^k \text{ x } R^k \to N$, that for every two subsequences r_i^k, r_j^k with the same size k from $\mathcal{R}^{n+1}$ puts in correspondence a number equal to the length of the minim m-dimensional k-chain, $1 \leq k < m \leq n+1$ [2]. Thus defined the function of distance possesses the metric properties.

We denote by $\mathcal{P}(R^k)$ the family of all subsets from R^k, and by D^k the family of all sets $A \subset R^k$ with the property: for every two k-dimensional sequences $r_i^k \in R^k$, $r_j^k \in R^k$, the inclusion $\left\langle r_i^k, r_j^k \right\rangle_m \subset A$ is respected. The following assertions are true:

1. *The family* D^k *represents a convexity in* R^k, *namely possesses the properties:*

a) $R^k \in D^k$;

b) if $A_1, A_2 \in R^k$, *then* $A_1 \cap A_2 \in R^k$.

The sets of the family D^k are called (k,m)-convex sets.

2. *The application* $\varphi : \mathcal{P}\left(R^k\right) \to \mathcal{P}\left(R^k\right)$ *that for each element* $A \in \mathcal{P}\left(R^k\right)$ *puts in correspondence a minim* (k,m)*-convex set from* D^k, *that contains the set* A, *represents a convex hull, namely possesses the properties:*

a) $A \subseteq \varphi(A)$, *for any subset* $A \subset R^k$;

b) $\varphi(\varphi(A)) = \varphi(A)$, *for any subset* $A \subset R^k$;

c) $\varphi(A) \subseteq \varphi(B)$, *for any two subsets* $A, B \in \mathcal{P}\left(R^k\right)$ *that* $A \subset B$. It can be elaborated an iterative procedure to construct a convex hull for any subset of elements $A \subset R^k$, $1 \leq k \leq n$.

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About Extensions with First-Countable Remainders

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Any space is considered to be a non-empty completely regular T_1-space. If X is a dense subspace of a space Y, then Y is called an extension of X and the subspace $Y \setminus X$ is called remainder of X. An extension Y of a space X is called a p-extension if $Y \setminus X \neq \emptyset$ and there exist a metric space Z and a perfect mapping $g : Y \longrightarrow Z$ such that $g(X) \cap g(Y \setminus X) = \emptyset$.
An extension Y of a space X is called: an fc-extension if $Y \setminus X \neq \emptyset$ and the space Y has a countable base at any point $p \in Y \setminus X$; a one-point extension if $Y \setminus X$ is a singleton subset of Y.
Denote by $E_{fc}(X)$ the poset of all fc-extensions of the space X and by $S_{fc}(X)$ the poset of all one-point fc-extensions of the space X.
The present research was motivated initially by the Bel'nov's study of the poset $M(X)$ of metric extensions of a locally compact metric space X [2] and by the M.Henriksen, L.Janos and R. G. Woods study of the poset $S(X)$ of one-point metric extensions of a locally compact metric space X [3].
Using the notion of the extension traces from [1, 3], there are constructed all one-point fc-extensions of the spaces. We proposed new method of construction of all fc-extensions. Obviously, $E_{fc}(X) \neq \emptyset$ if and only if $S_{fc}(X) \neq \emptyset$. Moreover, a space X is not pseudocompact if and only if $E_{fc}(X) \neq \emptyset$.
Let X and Y be two spaces which are Lindelöf or X and Y are realcompact spaces in which the subspaces of points of locally compactness are Lindelöf. Then the posets $S_{fc}(X)$ and $S_{fc}(Y)$ are order-isomorphic if and only if the spaces $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic.

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Vanishing Traces of Frobenius

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The object of this note is to study Serre's lacunarity question of Frobenius traces in Artin-type representations. More general results can be found in [1], which is part of the author's doctoral thesis [2].
Let $\Gamma_F = \mathrm{Gal}(\overline{F}/F)$ be the absolute Galois group of a number field F. Let

$$\rho : \Gamma_F \to \mathrm{GL}_n(\mathbb{C})$$

be an irreducible n-dimensional Artin-type (i.e., $\mathrm{Im}(\rho)$ is finite) complex representation of Γ_F. Denote by $a_v = \mathrm{tr}\,\rho(\mathrm{Frob}_v)$ the trace of the image of the Frobenius element Frob_v at a finite place v of F. We are interested in the density $\delta(\Sigma)$ of the set

$$\Sigma = \{v \mid a_v = 0\}.$$

This question can be traced back to Serre, who studied the case of ℓ-adic Galois representations ρ in [3]. He gives a sharp upper bound of $1 - 1/n^2$ for $\delta(\Sigma)$. Serre also raised a similar question on the automorphic side, which was recently considered by Walji (cf. [4]) for $n \in \{2, 3\}$.
Once we assume that the adjoint action $\mathrm{Ad}(\rho)$ is irreducible we can provide an upper bound independent of the dimension n. Here $\mathrm{Ad}(\rho)$ is the composition of the natural projection of GL_n onto PGL_n with the (n^2-1)-dimensional adjoint representation.
The main result can be stated as follows:

Theorem 1. *Let $\rho : \Gamma_F \to \mathrm{GL}_n(\mathbb{C})$ be an irreducible $\mathbb{C}$-representation of the absolute Galois group Γ_F of a number field F. Suppose that the adjoint representation $\mathrm{Ad}(\rho)$ is irreducible. Then*

$$\delta(\Sigma) \leq \frac{1}{2}.$$

Under the assumption that Ad is irreducible, the theorem shows that for at least half of the primes the traces of Frobenius classes in finite Galois groups are nonzero. The author studies this problem more generally in [1], where he also constructs an infinite family of representations of finite groups with an irreducible adjoint action.

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Boundedness and Solutions of Topological Optimization Problems

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The class of compact spaces and the class of metrizable spaces play an important role in applications. As a rule, in applications there are important some specifically properties of these spaces. For this reason, many properties of compact and metrizable spaces are the bases of definitions new important classes of spaces.

By a space we understand a completely regular topological Hausdorff space. A subset L of a space X is bounded if and only if every continuous function on X is bounded on L.

Denote by $C(X)$ the Banach space of all bounded continuous functions $f : X \longrightarrow \mathbb{R}$ with the sup-norm $\| f \| = sup\{|f(x)|; x \in X\}$.

For any function $\psi : X \longrightarrow \mathbb{R}^{\infty} = \mathbb{R} \cup \{+\infty\}$ and $Y \subseteq X$ we put $inf(\psi) = inf\{\psi(x) : x \in X\}$, $m_X(\psi) = \{x \in X : \psi(x) = inf_Y(\psi)\}$ and $dom(\psi) = \{x \in X : \psi(x) < +\infty\}$. A sequence $\{F_n : n \in \mathbb{N}\}$ of subsets of a space X is called *a bounded sequence* if each sequence $\{x_n \in F_n : n \in \mathbb{N}\}$ is a bounded set in X and $\{F_n : n \in \mathbb{N}\} \neq \emptyset$. A sequence $\gamma = \{U_n : n \in \mathbb{N}\}$ of subsets of a space X is called *a* Π*-sequence* if the sequence γ is bounded in X and the sets $X \setminus U_n$ and U_{n+1} are completely separated in X for any $n \in \mathbb{N}$.

For a minimizing problem (X, f) consider the following conditions: (Q_1) The set $m_X(f) \neq \emptyset$; (Q_2) The sequence $\{m_{(X,n)}(f) = \{y \in X : f(y) < inf(x) + 2^{-n}\} : n \in \mathbb{N}\}$ is bounded; (Q_3) The sequence $\{m_{(X,n)}(f) : n \in \mathbb{N}\}$ is a Π-sequence; (Q_4) The sequence $\{m_{(X,n)}(f) : n \in \mathbb{N}\}$ is a Π-sequence and $m_X(f)$ is a singleton.

Let $f : X \to \mathbb{R}$ be a bounded below function, $Q \in \{Q_1, Q_2, Q_3, Q_4\}$ and $S_Q(f) = \{g \in C(X) : a\ minimizing\ problem\ (X, f + g)\ has\ property\ Q\}$. A minimizing problem $(X, f + g)$ is called a continuous perturbations of the minimizing problem (X, f). We examine the following questions:

$P1$. Under which conditions the set $S_Q(f)$ is non-empty?

$P2$. Under which conditions the set $S_Q(f)$ is dense in the space $C(X)$?

$P3$. Under which conditions the set $S_Q(f)$ contains a dense G_δ-subset of the space $C(X)$?

The communication is based on the joint work with Petar S. Kenderov and Julian P. Revalski (see [1, 2]).

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About absolutely closed topological quasigroups

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In this paper was developed a method of constructing absolutely closed topological quasigruops. We say that a topological quasigroup G is absolutely closed or is a complete topological quasigroup

if G is a subquasigroup of a topological quasigroup G', then G is closed in G'. The similar notion for topological groups was proposed by A.D.Alexandroff in 1940.

Theorem 1. *Let G be a subquasigroup of topological quasigroup G'. If G is Čech-complete, then G is closed in G'.*

Corollary 1. *A Čech-complete topological quasigroup is absolutely closed.*

Corollary 2. *A locally compact quasigroup is absolutely closed.*

Corollary 3. *A discrete quasigroup is absolutely closed.*

Open question. *Let G be a metrizable quasigroup. Is it true that G is a subquasigroup of a complete topological quasigroup?*

On Compact Subsets of Free Topological Algebras

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Any space is considered to be a T_0-space. We use the terminology from [1, 3, 4]. Let X be a topological space, and $\mathcal{F}$ be a family of subspaces of X. Following [2, 1], we will say that X is *compactly metrizable*, or that X *is jointly metrizable on compacta*, if there is a metric d on the set X such that d metrizes all compact subspaces of X.
Fix a signature $E = \cup\{E_n : n \in \omega\}$. The space E is discrete and countable. A class $\mathcal{V}$ of algebras with topologies is called *a quasi-variety* of topological E-algebras if:
(F1) the class $\mathcal{V}$ is multiplicative;
(F2) if $G \in \mathcal{V}$ and A is a subalgebra of G, then $A \in \mathcal{V}$;
(F3) every space $G \in \mathcal{V}$ is a T_2-space;
(F4) There exists a space $G \in \mathcal{V}$ which contains two distinct points.
A quasi-variety $\mathcal{V}$ of topological E-algebras is called *an ω-complete quasi-variety* of topological E-algebras if $G \in \mathcal{V}_\omega$ and $\mathcal{T}$ is a topology on G such that $(G, \mathcal{T})$ is a σ-compact completely regular topological E-algebra, then $(G, \mathcal{T}) \in \mathcal{V}_\omega$. Denote by $(F(X, \mathcal{V}), e_X)$, where $F(X, \mathcal{V}) \in \mathcal{V}$ and $e_X : X \to F(X, \mathcal{V})$ is a continuous mapping the free topological E-algebra of a space X in a class $\mathcal{V}$.
Theorem 1. *Let $\mathcal{V}$ be an ω-complete quasi-variety of topological E-algebras. Then for each completely regular space X:*
1. The mapping $e_X : X \longrightarrow F(X, \mathcal{V})$ is an embedding and $e_X(X)$ is a closed subspace of the space $F(X, \mathcal{V})$.
2. The algebra $F(X, \mathcal{V})$ is abstractly free.
3. If X is a k_ω-space, then $F(X, \mathcal{V})$ is a k_ω-space.

Theorem 1. *Let X be a completely regular μ-complete space. Then the space X is jointly metrizable on compacta if and only if the space $F(X, \mathcal{V})$ is jointly metrizable on compacta.*

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The Recognition of Parametrical Completeness in Pseudo-Boolean Algebras

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We examine the 5-valued pseudo-Boolean algebra $Z_5 = \langle\{0, \rho, \tau, w, 1\};\ \&, \vee, \supset, \neg\rangle$; where ρ and τ are incomparable elements and $0 < \rho < w < 1$; $0 < \tau < < w < 1$. The algebra $Z_3 = \langle\{0, w, 1\}; \&, \vee, \supset, \neg\rangle$ is a subalgebra of Z_5. We remember that the pseudo-Boolean algebra $\langle M; \Omega\rangle$, where $\Omega = \{\&, \vee, \supset, \neg\}$; here $\supset$ is relative pseudocompliment and $\neg$ is pseudocompliment. We say that the function f of algebra A can be parametrically expressed via a system of functions Σ of A, if there exists the functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via Σ using superpositions such the predicate $f(x_1, \ldots, x_n) = x_{n+1}$ is equivalent to the predicate $\exists t_1 \ldots \exists t_l \Big((g_r = h_1)\& \ldots \&(g_r = h_r)\Big)$ on the algebra A. The system Σ of pseudo-Boolean terms on the set of variables X is parametrically complete in $\langle M; \Omega\rangle$, if we can parametrically express the operations from Ω via functions expressed by terms over Σ. The function $f(x_1, \ldots, x_n)$ and $g(x_1, \ldots, x_k)$ are called permutable if the identity $f\Big(g(x_{11}, x_{12}, \ldots, x_{1k}), \ldots, g(x_{n1}, x_{n2}, \ldots, x_{nk})\Big) = g\Big(f(x_{11}, x_{21}, \ldots, x_{n1}), \ldots, f(x_{1k}, x_{2k}, \ldots, x_{nk})\Big)$ hold. The set of all functions from algebra A permutabled with a given function f is referred to us centralizator of the function f (denoted by $-\langle f\rangle-$) on algebra A. Let us define the function $f(p)$ on Z_5 as fallows:

$$f(0) = 0, f(\rho) = \tau, f(\tau) = \rho, f(w) = f(1) = 1.$$

Theorem 1. *A system Σ of pseudo-Boolean terms is parametrically complete in algebra Z_5 iff Σ is parametrically complete in subalgebra Z_3 and the system Σ is not included into the centralizator $-\langle f\rangle-$ on algebra Z_5.*

The Jacobsthal-Circulant Sequences and their Applications

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In this work, we define the generalized Jacobsthal-circulant sequence and the Jacobstah-circulant sequences of the first, second and third kind such that these sequences are obtained from the circulant matrix C_3 which are defined by the characteristic polynomial of the Jabosthal sequence. Then we obtain the relations between the elements of the sequences and generating matrices of the sequences. Also, we consider the cyclic groups which are generated by the generating matrices of the defined recurrence sequences and then, we examine the orders of these groups. Furthermore, we

extend the Jacobstah-circulant sequence of the second to groups and then, we study this sequence in finite groups. Finally, we obtain the length of the period of the Jacobstah-circulant sequence of the second in the binary polyhedral group $\langle 2, n, 2\rangle$ as applications of the results obtained.
2010 Mathematics Subject Classification: 11B50, 20F05, 15A36, 20D60

The Fibonacci p-Orbits and The Padovan p-Orbits of The Extended Triangle Groups

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For a p-tuple $(x_0, x_1, \cdots, x_{p-1}) \in X$, the Fibonacci p-orbit of the group G $F_p^{(x_0,x_1,\cdots,x_{p-1})}(G) = \{a_i\}$ is defined as follows [1]:

$$a_0 = x_0,\ a_1 = x_1,\ \cdots,\ a_{p-1} = x_{p-1},\ a_p = x_{p-1},\ a_{n+p} = a_{n-1} \cdot a_{n+p-1},\ \ n \geq 1.$$

For a p-tuple $(x_1, x_2, \cdots, x_p) \in X$, the Padovan p-orbit of the group G the Padovan p-orbit $Pap(G; x_1, x_2, \cdots, x_p) = \{a_i\}$ is defined as follows [2]:

$$a_0 = e,\ a_1 = x_1,\ a_2 = x_2, \cdots,\ a_p = x_p,\ a_{p+1} = e,\ a_{n+p+1} = a_{n-1} \cdot a_{n+p-1},\ \ n \geq 1.$$

In this work, we examine the behaviours of the lengths of the periods of the Fibonacci p-orbits and the Padovan p-orbits of the extended triangle groups $E(2, n, 2)$, $E(2, 2, n)$ and $E(n, 2, 2)$ for $n \geq 3$.
2010 Mathematics Subject Classification: 20F05, 20D60, 11B39

On Sequentiality of Function Spaces

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Any space is considered to be a completely regular T_1-space. We use the terminology from [1]. By $C_p(X, E)$ we will denote the space of all E-valued continuous mappings equipped with the pointwise convergence topology. Some similar assertions for $C_p(X, \mathbb{R})$ and $C_p(X, \mathbb{Z})$ were proved in [1, 2, 3, 4].
A family $\mathcal{A}$ of subsets of X is called an ω-cover (cover for finite subsets [1, 3] of X if for any finite subset $F \subseteq X$ there exists $U \in \mathcal{A}$ such that $F \subseteq U$. If $\xi = \{A_n : n \in \}$ is a sequence of subsets of X, then the set $\liminf \xi = \cup\{\cap\{A_k : k \geq n\} : n \in \mathbb{N}\}$ is called the lower limit of the sequence ξ.
Property γ_1 [1]: for any sequence $\{\eta_n : n \in \mathbb{N}\}$ of open ω-covers of X there exists a sequence $\xi = \{U_n \in \eta_n : n \in \mathbb{N}$ such that $\liminf \xi = X$.

Proposition 1. *Let E be a non pseudocompact space, a space X is non-empty, $indX = 0$ and $C_p(X, E)$ is a k-space. Then the space X has the property γ_1.*

Proposition 2. *Let E be a metrizable space, $|E| \geq 2$ and a space X has the property γ_1. Then $C_p(X, E)$ is a Fréchet-Urysohn space.*

Theorem 1. *Let E be a non compact metric space, X be a non-empty space and $indX = 0$. Then the following conditions are equivalent:*
(i) $C_p(X,E)^{\mathbb{N}}$ is a Fréchet-Urysohn space.
(ii) $C_p(X,E)$ is a sequential space.
(iii) $C_p(X,E)$ is a k-space.
(iv) X has the property γ_1.

Proposition 3. *Let E and X be non-empty spaces and the space of reals $\mathbb{R}$ be a closed subspace of E. If $C_p(X,E)$ is a k-space, then the space X has the property γ_1.*

Theorem 2. *Let E be a non trivial metrizable linear space and X be a non-empty space. Then the following conditions are equivalent:*
(i) $C_p(X,E)^{\mathbb{N}}$ is a Fréchet-Urysohn space.
(ii) $C_p(X,E)$ is a sequential space.
(iii) $C_p(X,E)$ is a k-space.
(iv) X has the property γ_1.

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Harmonicity with respect to Riemann extension

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Riemann extensions, defined by Patterson and Walker, are certain semi-Riemannian metrics on the total space of the cotangent bundle of a manifold endowed with a symmetric linear connection. They were generalized by Sekizawa to natural Riemann extensions. In the present paper, we study the relations between these semi-Riemannian metrics on one side and the harmonicity (in the sense of Garcia-Rio, Vanhecke, Vasquez-Abal) of a canonical almost para-complex structure that we construct here, on the other side. Our main result is a characterization of the harmonicity of the above almost para-complex structure.

On the geometry of the product metallic Riemannian manifold

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In this paper we discuss about properties of structures induced on a product manifold of metallic Riemannian manifolds and its submanifolds. The positive solution of the equation $x^2 - px - q = 0$ is called (p, q)- metallic number, for fixed positive integer values of the parameters p and q. These (p, q)-metallic numbers are members of the metallic means family (some important members of the metallic mean family are the golden mean obtained for p=q=1, the silver mean for p=2, q=1 and the bronze mean for p=3, q=1). A metallic structure on a Riemannian manifold is a structure defined as a polynomial structure determined by an (1, 1)- tensor field J which satisfies the above equation and the metric is J-compatible.
MSC 2010: 11B39, 53C15, 53C40.

$\mathcal{G}$-Sequential Continuity

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The purpose of this paper is to introduce we extend the concepts of $\mathcal{G}$-continuity and $\mathcal{G}$-sequential convergent in topological group to the generalized topological group. Also we introduce $\mathcal{G}$-sequential continuity and $\mathcal{G}$-sequential convergent to generalized topological groups and investigate some basic properties of this concepts.

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On the theory of finite groups of W_p-symmetry

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In the case of W_p-symmetry [1-4] the transformations of the qualities-"indexes", attributed to the points of geometrical figure, essentially depend on the choice of points. The discrete finite groups of W_p-symmetry with the initial group of substitutions P and the generating group of classical symmetry G, with the subset of generalised substitutions W', with the subgroup V of W-identical transformations and with the symmetry subgroup H, are subgroups of left standard wreath product of group P with the group G, accompanied with a fixed isomorphism $\varphi : G \to AutW$ (where W is the direct product of isomorphic copies of the group P which are indexed by elements of group G).

Any group of W_p-symmetry with the finite group W can be derived from its finite generating group G and group $W = \prod_{g_i \in G} P^{g_i}$ of multicomponent permutations by the following steps: 1) we find in W all subgroups V and subsets W', which are decomposed in left cosets by its subgroup V, and in G all proper subgroups H with the index equal to the power of set of all left cosets of W' by V and for which there is the isomorphism λ of factor-groups G_1/H and W_1/V_1 $(\lambda : G_1/H \to W_1/V_1$ by the rule $\lambda(Hg) = wV)$, where $G_1 \leq G$, $W_1 \leq DiagW$ and $V_1 = V \cap DiagW \leq W_1$; 2) we construct a generalized exact natural left quasi-homomorphism $\tilde{\mu}$ of the group G onto the set of all left cosets of W' by the subgroup V by the rule $\tilde{\mu}(Hg) = wV$ and which preserves the correspondence between the elements of factor-groups G_1/H and W_1/V_1 received as the result of isomorphism λ; 3) we combine pairwise each g' of Hg with each w' of $wV = \tilde{\mu}(g')$; 4) we define on the set of all these pairs the operation

$$g_i^{(w_i)} \circ g_j^{(w_j)} = g_k^{(w_k)},$$

where $g_k = g_i g_j$, $w_k = w_i^{g_j} w_j$ and $w_i^{g_j}(g_s) = w_i(g_j g_s)$.

If $V = w_0$, where w_0 is the unit of the group W, then the mapping $\tilde{\mu}$ is an ordinary exact natural left quasi-homomorphism. In this case, the universal method of deriving the groups of W_p-symmetry becomes more simple and similar to the method for deriving the semi-minor or pseudo-minor groups in dependence on W'.

Acknowledgement. *This work was partially supported by the project 15.817.02.26F.*

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Primitive Ideal Spaces of Semigroup $C*$ crossed Products

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Let (G, G_+) be a quasi-lattice-ordered group with positive cone G_+. Given a hereditary subsemigroup H_+ of G_+. In [1] we introduced a closed ideal I_{H_+} of the C^*-algebra B_{G_+}. Then we showed that the C^*-algebra $B_{(G/H)_+} \times_\beta G_+$ is realized as an induced C^*-algebra $\mathrm{Ind}_{H^\perp}^{\widehat{G}} \left(B_{(G/H)_+} \times_\tau (G/H)_+\right)$. In this paper, we prove the existence of the following short exact sequence of C^*-algebras

$$0 \to I_{H_+} \times_\alpha G_+ \to B_{G_+} \times_\alpha G_+ \to \mathrm{Ind}_{H^\perp}^{\widehat{G}} \left(B_{(G/H)_+} \times_\tau (G/H)_+\right) \to 0.$$

Which relates $B_{G_+} \times_\alpha G_+$ to the structure of $I_{H_+} \times_\alpha G_+$ and $B_{(G/H)_+} \times_\beta G_+$. Then we use our theorem to deduce information about the ideal structure of the C^*-algebra $B_{G_+} \times_\alpha G_+$.

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Tree trunk shape analysis using classical geometry

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Knowing the shape of the spindle trees is a challenge for more than 200 years and yet it remained a current matter that must keep up with technological development. This issue needs answers in real time. To explain and model the shape of the spindle, researchers use various geometrical bodies (neiloid, cone, paraboloid, cylinder) and shape equations. Solving the problems concerning cone or paraboloid is achieving satisfying results by the traditional approach to analytic geometry. In this paper I present, on the basis of data sets, how using classical geometry or equations may create standards that can approximate the shape of spindle trees (spruce, Picea Abies).

On pseudoautomorphisms of middle Bol loops

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A loop $(Q, \cdot)$ is called a middle Bol loop if every loop isotopic to $(Q, \cdot)$ satisfies the identity $(x \cdot y)^{-1} = y^{-1} \cdot x^{-1}$ (i.e. if the anti-autmorphic inverse property is universal in $(Q, \cdot)$) [1]. Middle Bol loops are isostrophes of left (right) Bol loops [2,3]. If $(Q, \cdot)$ is an arbitrary loop, $\varphi \in S_Q$ and $c \in Q$, then φ is called a left (resp. right, middle) pseudoautomorphism of $(Q, \cdot)$, with the companion c, if the equality $c \cdot \varphi(x \cdot y) = (c \cdot \varphi(x)) \cdot \varphi(y)$ (resp., $\varphi(x \cdot y) \cdot c = \varphi(x) \cdot (\varphi(y) \cdot c)$,

$\varphi(x \cdot y) = (\varphi(x)/c^{-1}) \cdot (c\backslash\varphi(y))$) holds, for every $x, y \in Q$, where c^{-1} is the right inverse of c. Below we'll denote by $PS_r^{(\cdot)}$ (respectively, $PS_l^{(\cdot)}, PS_m^{(\cdot)}$) the group of all right (resp. left, middle) pseudoautomorphisms of the loop $(Q, \cdot)$ [1,4,5]. A G-loop is a loop isomorphic to all its loop isotopes. It is known that a loop $(Q, \cdot)$ is a G-loop if and only if every element of Q is a companion of a left and of a right pseudoautomorphism [1].

Proposition 1. *$(Q, \cdot)$ be a right (respectively, left) Bol loop and $c \in Q$. A mapping $\varphi \in S_Q$ is a middle pseudoautomorphism of the loop $(Q, \cdot)$, with companion c, if and only if φ is a right pseudoautomorphism of the loop $(Q, \cdot)$, with companion c (resp., if and only if φ is a left pseudoautomorphism of the loop $(Q, \cdot)$, with companion c^{-1}).*

Proposition 2. *Let $(Q, \cdot)$ be a middle Bol loop with the unit e. The following statements hold: 1. If $x^2 = e$, for all $x \in Q$, then every $a \in Q$ is a companion for some right pseudoautomorphism of $(Q, \cdot)$; 2. If $a \in N_m^{(\cdot)}$ then $\tau = R_a^{-1} I_a^{-1} I$ is a right pseudoautomorfism with companion a^2.*

Proposition 3. *Let $(Q, \circ)$ be a middle Bol loop. The following statements hold: 1. If $(Q, \cdot)$ is the corresponding left Bol loop then $PS_m^{(\cdot)} = PS_l^{(\cdot)} = PS_r^{(\circ)}, PS_r^{(\circ)} = PS_m^{(\circ)}, \alpha in PS_l^{(\circ)} \Leftrightarrow I\alpha I \in PS_l^{(\cdot)}$; 2. ([4]) If $(Q, \cdot)$ is the corresponding righ Bol loop then $PS_m^{(\cdot)} = PS_r^{(\circ)}, PS_l^{(\cdot)} = PS_m^{(\circ)}, PS_r^{(\cdot)} = PS_r^{(\circ)}, \alpha \in PS_l^{(\circ)} \Leftrightarrow I\alpha I \in PS_r^{(\circ)}$.*

Theorem 1. *Let $(Q, \circ)$ be a middle Bol loop, $a, b \in Q$, and let $x \circ y = R_a^{-1} x \cdot L_b^{-1} y$, $\forall x, y \in Q$. Then: 1. $(Q, \circ) \cong (Q, \cdot)$ if and only if there exists a right pseudoautomorphism of $(Q, \cdot)$ with the companion $k = a^{-1}\backslash b^{-1}$; 2. $(Q, \circ) \cong (Q, \cdot)$ if and only if there exists a left pseudoautomorphism of $(Q, \cdot)$ with companion $k_1 = a^{-1}/b^{-1}$.*

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Some properties of the hyperfields extensions

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In the last years, some new applications of hyperfields and hyperrings in the number theory and geometry, done by A. Connes and C. Consani [1], M. Marshall [3], O. Viro [8] and others, asked new developments of the theory of algebraic hyperstructures. In our communication, we describe more possibilities of constructing hyperfields and hyperrings than we knew from papers written before by M. Krasner [2], C.M. Massouros [4,5], A. Nakassis [6] , M. Stefanescu [7] and others.

Moreover we give conditions for some extensions of hyperfields to extend special hyperfields which play an important role in the theory of hyperfields.

Definition 1. *A hyperfield* $(\mathbf{R}, +, *)$ *is a nonempty set* $\mathbf{R}$ *endowed with a hyperaddition* $+ : \mathbf{R} \times \mathbf{R} \longrightarrow \mathcal{P}(\mathbf{R})$ *and a binary multiplication* $*$ *such that:*

(1) $(\mathbf{R}, +)$ *is a canonical hypergroup i.e.*

   (a) $x + y = y + x, (x + y) + z = x + (y + z)$*;*

   (b) there exists a neutral element 0 *in* $\mathbf{R}$ *such that* $0 + x = x + 0 = x$*, for all* x *in* $\mathbf{R}$*;*

   (c) for any x *in* $\mathbf{R}$ *there is a unique element* $-x$ *in* $\mathbf{R}$ *such that* $0 \in x + (-x)$ *and if* $x \in y + z$ *then* $z \in x - y$ *, for all* x, y, z *in* $\mathbf{R}$*;*

(2) $(\mathbf{R}, *)$ *is a monoid with* 1 *different from* 0.

(3) *The multiplication is bidistributive with respect to hyperaddition.*

(4) *For all* x *in* $\mathbf{R}$*,* $x * 0 = 0 * x = 0$.

(5) *Each nonzero element has an inverse with respect to multiplication.*

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About congruences and commutators modular model

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In an axiomatic language in [1] the concept of congruence was defined on algebraic system. In particular, if the signature of the system does not contain predications symbols, that is a model, that definition applies be available for model. In this paper the definition of congruence is specified on a model and show for a model some applications related to obtaining new models but and isomorphic (or omomorphic) relations between them, which easy to check. Modular models is also specified the definition of the commutator of two congruence showed in [2] for universal algebra, but also in [3] for algebraic system. Based on this concept new models are obtained: abelian models; nilpotent models, i.e. models with finite strings descendant (or ascendant) of the central commutators; rezolubile models, i.e. models with finite strings descendant of the central commutators. Some examples are brought.

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6. Mathematical Modeling

Vibration Control of Airfoil in Hypersonic Flow by Variational Control Algorithms

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In this paper, the vibration of the plunging displacement in hypersonic flow is controlled by using the variational optimal control. Firstly, the flutter motion equations of a two-dimensional wing are obtained by piston theory of hypersonic flow and energy method. This system includes both structural and aerodynamics nonlinearities. And then, the flutter speeds of the system are investigated. Variational optimal control is designed in the next part of this study. This controller provides minimizing the vibration of plunging displacement and control of the pitching angle. Airfoil model with the controller and airfoil model without the controller are simulated simultaneously by the computer program. Figures of the pitching angle and the plunging displacement are shown in the conclusion part of this paper. The results of this study are compared with uncontrolled model's results and it is obvious that the controller is satisfactory.

The stable sets in a complex of multi-ary relations

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In a complex of multi-ary relations K^n we denote by $\Gamma(Q)$ the neighborhood of abstract quasisimplex $Q \in K^n$ [1]. A set of abstract quasisimplexes possessing the property $\Gamma(Q) \cap F = \emptyset$, for any quasisimplex $Q \in F$, is called internally stable set.
We define the unequivocal application $g : K^n \to K^n$ such that for any m-dimensional quasisimplex Q^m of K^n is true relation:

$$g(Q^m) = min\{N_0 \setminus g(\Gamma(Q^m))\}.$$

This application is called Grundy function defined on the complex and is used to examine some combinatorial games on discrete structures [2].
Grundy function is used to study the properties of internal stable sets in the complex of multi-ary relations. It is determined that:

1. The set of quasisimplexes $Q \in K^n$ for which the Grundy function receives the same value represent a internal stable set in K^n.

2. Minimal number of internal stable sets which form a coverage of the complex of multi-ary relations is equal to $\gamma \in N_0$ if and only if the following equality is true:

$$\max_{Q \in K^n} g(Q) = \gamma - 1.$$

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Transitive orientation of graph using B-stable subgraphs

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Transitively orientable graphs offer solutions for some theoretical problems [3] and have many practical applications [1], etc. This class of graphs can be studied using different structures, such as implication classes [2], stable subgraphs [3]. Based on definition of stable subgraph we define a specific class called B-stable sugraph.

Definition 1. *Graph $F = (X_F; U_F)$ is called B-stable subgraph of the undirected graph $G = (X; U)$ if F is stable subgraph of G and for every stable subgraph M of G one of the following conditions is satisfied:*

1. $X_F \cap X_M = \emptyset$;
2. $X_F \subseteq X_M$.

Theorem 1. *If F is a B-stable subgraph of the graph $G = (X; U)$ and $x \in X_G \setminus X_F$ is a vertex adjacent to the set X_F, then for every transitive orientation $\overrightarrow{G}$ only one of the following relations holds:*

1. $[x, y] \in \overrightarrow{U_G}, \forall y \in X_F$;
2. $[y, x] \in \overrightarrow{U_G}, \forall y \in X_F$.

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Fuzzy Modelling for Extraction of Total Polyphenols from Mandarin Leaves

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Free radicals called reactive oxygen species (ROS) are known to be harmful to DNA and cell membranes. Photochemicals containing antioxidants are scavengers of free radicals, thus reducing these harmful effects. *Citrus unsiu* Marc, known as Satsuma Mandarin, is the most common mandarin species grown in Turkey. Polyphenolic substances, especially flavonoids, are plentiful in Satsuma Mandarin. Flavonoids have been the subject of numerous studies because of their antioxidant, antimicrobial, antitumor, antiviral and anti-inflammatory properties. Optimization of the extraction process in terms of yield is of great importance due to its economic profitability. The time consuming and expensive nature of traditional extraction methods led researchers to investigate new alternatives. One of the new methods, homogenizer-assisted extraction causes the plant cell to swell up within the solvent and burst spectacularly. This level of efficiency lowers the amount of solvent needed in the extraction process, thus showing the method to be more economic. In this study, solid/solvent ratio, mixing rate of homogenizer and extraction time selected as experimental factors which affect the extraction yield and feasibility of the process. Thus, determination of optimal operating conditions for an extraction method is a must for scale-up applications of the process. We compare response surface methodology (RSM) and fuzzy model in mandarin leaf extraction rich in polyphenols through homogenizer.

An Approach for Grapefruit Leaf Extraction through Homogenizator: Response Surface Methodology versus Fuzzy Model

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Turkey is in the 7th place in grapefruit (*Citrus paradisi* Macf.) production growing more than 240,000 tons in recently. Grapefruits are mostly used as in fruit juice industry and consumed as fruit. After the consumption of citrus; large amounts of peel, leaf, the pulp portion remains called as bio-waste which containing almost the half of total fruit weight. Bio-wastes of grapefruit contain higher sources of vitamin C, phenolic acid such as ferulic acid and coumaric acid and flavonoid such as naringin and hesperidin which are well-known essential nutrients with antioxidant properties compared to edible parts. Extraction process is a must to convert bio-waste into useful products. Optimization of the extraction process in terms of yield is of great importance due to the high energy need of the extraction process which contains 70 % of the overall process. For the last decades, the suitable extraction method has been investigated to get phytochemical compounds from the cheapest bio-waste with higher efficiency and lesser time consumption. One of the novel extraction technologies, homogenizer-assisted extraction causes the plant cell to swell up within the solvent and burst spectacularly. This level of efficiency lowers the amount of solvent, thus showing the method to be more economic. Although the effectiveness of the extraction method is necessary, other process parameters such as solid/solvent ratio and mixing rate of homogenizer

affect the extraction yield and feasibility of the process. Therefore, determination of optimal operating conditions for an extraction method is a must for scale-up applications of the process. In this study, we compare response surface methodology (RSM) and fuzzy model in grapefruit leaf extraction rich in polyphenols via homogenizer.

About equations for electrical polarization in some non-linear anisotropic optical crystals

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Revised second-order nonlinear equations (in both real and complex form) are considered for the electrical polarization in an anisotropic crystal under laser radiation. The temporal-spatial description of electro?magnetic vectorial field, based on the wave equation and bilinear form for electrical susceptivity is analyzed. The spectral behavior of a laser beam as a result of photon interaction with crystal is systematized.

Mathematical Modeling for the problems of dynamics and stability of pipeline

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This article devoted to the problem of the dynamics and dynamic stability of the pipeline, which are a hollow rod with the fluid (gas) runs inside it. The strainless pipeline on plane xOy are according to segment $[0, l]$ on x–axis. The fluid velocity is denoted by U and its direction coinciding with the direction of the x–axis. We use $w(x, t)$ to denote elastic displacement of single point of pipeline towards the y–axis. Similarly, $u(x, t)$ denote the one displacement towards the x–axis.
Taking into account only w – oscillations towards the y–axis, pipeline dynamics is described by the following equation

$$(m_0 + m_*)\,\ddot{w} + \left(\frac{EJ}{\rho}\right)'' + m_*\frac{U^2}{\rho} + 2m_*U\left(\arcsin w'\right)^{\cdot} + N\frac{w''}{\sqrt{1+(w')^2}} - \\ -\Theta_0 w''\Psi - \Theta_* w''\dot{\Psi} + \alpha\dot{w}'''' - \beta\ddot{w}'' + f(x,t,w,\dot{w}) = 0, \tag{1}$$

here, ρ and Ψ are given by

$$\frac{1}{\rho} = \frac{w''}{(1+(w')^2)^{\frac{3}{2}}}, \quad \Psi = \int_0^l \sqrt{1+(w')^2}\,\mathrm{d}x - l \tag{2}$$

The apostrophe and dot symbols denotes partial derivatives with respect to the variable x and y. When w' is negligible, the equation (1) approximates to

$$(m_0+m_*)\,\ddot{w}+\left[EJw''\left(1-\frac{3}{2}(w')^2\right)\right]''+m_*U^2w''\left[1-\frac{3}{2}(w')^2\right]+Nw''\left(1+\frac{1}{2}(w')^2\right)+ \\ +2m_*U\dot{w}'\left[1+\frac{1}{2}(w')^2\right]-\frac{1}{2}\Theta_0w''\int_0^l(w')^2\,\mathrm{d}x-\Theta_*w''\int_0^l w'\dot{w}'\,\mathrm{d}x+ \\ +\alpha\dot{w}''''-\beta\ddot{w}''+f(x,t,w,\dot{w})=0 \tag{3}$$

Investigation of stability of the pipeline carried out by two different approaches. The first one based on deriving the Lyapunov functional for some particular cases of the equation (3). The second approach involves the construction of solutions of the equation (3) using Galerkin method. In this case, $w(x,t)$ is sought as

$$w(x,t)=\sum_{k=1}^{n}w_k(t)g_k(x), \tag{4}$$

here, $\{g_k(x)\}_1^\infty$ – is complete system on $[0,l]$ of basis functions, which are satisfy to the consolidation conditions of pipeline endpoints. Taking into account both w – displacement of single point of pipeline towards the y–axis and u – displacement towards the x–axis, pipeline dynamics is described by the following system of equation

$$\begin{cases}(m_0+m_*)u_{tt}-EA_0\left(u_x+\frac{1}{2}w_x^2\right)_x+\gamma_0u_{xxt}+g(x,t,u,u_t)=0\\ (m_0+m_*)w_{tt}-EA_0\left[w_x\left(u_x+\frac{1}{2}w_x^2\right)\right]_x+EJ_0\left(w_{xx}\delta^3\right)_{xx}+\alpha_0w_{xxxxt}-\\ -\beta_0w_{xxtt}+m_*U^2w_{xx}\delta^3+2m_*U\phi w_{xt}+f(x,t,w,w_t)=0\\ \delta=\frac{1}{\sqrt{1+2\left(u_x+\frac{1}{2}w_x^2\right)}},\quad \phi=\frac{1}{\sqrt{1-w_x^2}}\end{cases} \tag{5}$$

$$\begin{cases}(m_0+m_*)u_{tt}-EA_0\left(u_x+\frac{1}{2}w_x^2\right)_x+\gamma_0u_{xxt}+g(x,t,u,u_t)=0\\ (m_0+m_*)w_{tt}-EA_0\left[w_x\left(u_x+\frac{1}{2}w_x^2\right)\right]_x+EJ_0\left(w_{xx}\left[1-3\left(u_x+\frac{1}{2}w_x^2\right)\right]\right)_{xx}+\alpha_0w_{xxxxt}-\\ -\beta_0w_{xxtt}+m_*U^2w_{xx}\left[1-3\left(u_x+\frac{1}{2}w_x^2\right)\right]+2m_*U\left(1+\frac{1}{2}w_x^2\right)w_{xt}+f(x,t,w,w_t)=0\end{cases} \tag{6}$$

For this system investigation of stability carried out by two approaches. The former based on deriving the Lyapunov functional for some particular cases of the equation system (6). The second one involves the construction of solutions of the equation (6) using Galerkin method. In this case, functions $w(x,t)$ and $u(x,t$ are sought as

$$w(x,t)=\sum_{k=1}^{n}w_k(t)g_k(x),\;\; u(x,t)=\sum_{k=1}^{n}u_k(t)f_k(x) \tag{7}$$

here, $\{g_k(x)\}_1^\infty$, $\{f_k(x)\}_1^\infty$ – are complete systems on $[0,l]$ of basis functions, which are satisfy to the consolidation conditions of pipeline endpoints.

We assume that the external influences $f(x,t,w,w_t)$, $g(x,t,u,u_t)$ applied at time t immediately, or with a delay in time: $f=f\left(x,t,w\left(x,t-\tau_1\right),w_t\left(x,t-\tau_2\right)\right)$, $g=g\left(x,t,u\left(x,t-\tau_3\right),u_t\left(x,t-\tau_4\right)\right)$. By means of designed functional Lyapunov type, stability theorems was formulated and analytical stability conditions for the parameters of the mechanical system and different types of an initial conditions was founded. A mathematical software package was developed to solve this problems. This package allows to find an approximate numerical solution of differential equation for describing pipeline model and plot a stability region appropriate to both sufficient and necessary stability conditions.

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Modelling of the adsorption of fluorine onto diatomite

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The investigation on the equilibrium and kinetics of fluorine adsorption from water with fluoride ion concentration of up to 20 mg/L has been carried out on a modified diatomite. The classical Lengmuire, Henry, and Freundlich equations of adsorption were used for description of experimental data of fluorine adsorption on diatomite. The constants and parameters of these equations were determined. Based on the adsorption equilibrium constants and maximum adsorption capacity the calculation of fluorine adsorption isotherms has been made. The obtained results demonstrate that the Freundlich and Henry mathematical models of adsorption are the best for describing the experimental data of fluorine adsorption on diatomite ($R^2 = 0,9373$ and $0,9358$ respectively).
The following adsorption kinetics equations were used for description of the process of fluorine removal from water and examinations of the controlling mechanism of the sorption of F^- ions on diatomite: the pseudo - first order Lagergren [1], pseudo - second order Mackay and Ho [2], and intraparticle diffusion model of Weber-Morris [3]. The Elovich equation has also been successfully applied in an aqueous solution to describe adsorption and desorption reactions [4],

$$a_t = a_m(1 - e^{-kt}), a_t = \frac{k_2 a_m^2 t}{1 + k_2 a_m t} \tag{1}$$

$$a_t = k_d t^{0.5} + C, \theta_t = \frac{1}{\beta}\ln(\alpha\beta) + \frac{1}{\beta}\ln(t) \tag{2}$$

Here a_m and a_t are the amounts of F- ions adsorbed at equilibrium (mg / g) and at time t (min), respectively, k_1 is the rate constant of the pseudo-first order kinetics, (min^{-1}), k_2 is the rate constant of the pseudo-second order kinetics, (g / mg min), k_d is the intraparticle diffusion rate constant, (g / mg $\text{min}^{0.5}$), C is a constant that gives an idea about the effect of boundary layer thickness α and β are constants, t-is the time of adsorption, and θ_t is the surface coverage. The parameters of the convenient models K_1, K_2, K_d, and α, β, were found from the linear plots $\ln(a_m - at)$ versus t; t/a_t versus t; a_t versus $t^{0.5}$, and θ_t versus $\ln(t)$ (using the values of the intersection points and the slopes of the lines). It is shown that high values of the correlation coefficients ($R^2 = 0,9983$) of the pseudo - second order model indicate that this model very well

fits the experimental data of fluorine adsorption on diatomite. This gives evidence of the chemical character of the interaction of fluorine with the active adsorption centers on the sorbent surface. It is also noted that the mechanism of fluorine removing is controlled by the intraparticle diffusion and superficial adsorption.The estimated values of the kinetic constants were used to solve the mathematical model that describes the temporal variation of the amount of fluoride ions adsorbed on the diatomite in the aqueous medium with a low content of fluorine up to 20 mg F- /L. For the model of kinetic of pseudo second order adsorption the analytical equation has the following form:

$$a_t = \frac{0,00789t}{1 + 0,03376t}$$

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7. Computer Science

Error estimation of the adaptive Backward Differentiation Formula

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In this work we will consider the first-order Initial Value Problem (IVP). When we are solving systems of stiff ordinary differential equations by nu- merical integration, it is important to use an accurate algorithm which has good stability properties. Many researches have been focused on the gen- eration of efficient algorithms for the numerical integration of stiff systems and some of them have been based on backward differentiation formulae (BDF), due to its good stability properties. In this work we will focus on the second order accurate and unconditionally stable BDF (BDF2). We have implemented an adaptive BDF2 using the local truncation error as the error estimation. We have done it in two different ways. The first one, by imple- menting an adaptive BDF2 that uses a constant step size advancing formula, and the second one, using an adaptive step size advancing formula. This second approach results more efficient than the one that uses the constant step size advancing formulae.

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Time Series Analysis and Prediction Based on Fuzzy Logic

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This paper presents a new way of implementing the forecasting process using a neural network and a fuzzy controller. At the first stage of the process, the neural network performs the forecasting based on internal parameters, and then the fuzzy controller adapts the results to the changes of the external parameters. The main problem of neural networks in forecasting is the impossibility to foresee the changes in the external parameters. This problem is solved by the fuzzy controller, which reacts to any change in the external parameters and adjusts the forecasted data provided the neural network.

Processing of ambiguous and contradictory rules in a fuzzy controller

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For the design of complex technical systems, fuzzy methods have often proved to be useful. The knowledge obtained from experts can be expressed as a set of rules. However, in most fuzzy controllers, the expressed rules are not always precise enough and they may not consider all combinations. Therefore, a good construction of linguistic rules for the fuzzy controller are very important. To increase the knowledge base, the linguistic rules are collected from more than one expert, however this will increase the probability of the mentioned rules contradiction with each other. In this paper, a new method of designing a fuzzy controller is proposed. Due to the characteristic of the weighting in the proposed network, the proposed fuzzy controller can be operated successfully despite the occurrence of rule contradictions.

Numerical scheme for water flow on real topography with porosity and friction influences

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We report on a numerical scheme for a system of shallow water equations modelling the flow on a landscape. Besides the gravitational force, we also take into account the influence of the soil friction as well as the drag of the vegetation on the water flow. We describe the numerical algorithm based on finite volume method and applied to a regular hexagonal network of cells. The way this scheme works is illustrated through numerical applications with synthetic and real terrain data (from some Romanian valleys)

The formalization of various internet portals and sites as expert systems

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The aim of this work is to demonstrate the correctness of the following concept - most of modern Internet portals, online forums, and, in generally, any Internet sites, where are certain discussions (debates) on the resolution of certain problem situations (setting instrument drivers, discussion ambiguities in physical problems, discussion of games, etc.) can be formalized in the frames of Expert Systems theory.

Methods of processing unstructured texts

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Most of the existing researches and applications of natural language processing are made in English. As only 28.6% of Internet users speak English [1], the construction of resources and tools in languages other than English is a growing need. For this reason the European Commission has initiated a number of projects in support of the technologization of European languages other than the English language. This policy of promoting multilingualism through information technologies is continued, fact which can is proven by the priorities set out in the Framework Programme 7 (currently HORIZON 2020 Programme).
Analyzing various natural language processing applications one can find that, ultimately, the overwhelming majority of them are based on computational linguistic resources. The more voluminous and comprehensive are these resources, the more exact and qualitative results are obtained. Romanian language begins to emerge as one of the significant languages in what concerns informatics resources and technologies applied to them. Therefore, the actual problem remains to automate the process of filling computational linguistic resources for Romanian [2].
The combination of methods from the field of information technology with the ones from linguistics would resolve the problem of computational linguistics, namely structured information retrieval of unstructured texts.
The aim of this article is to present our approach in the elaboration of the system for processing unstructured text data in order to create structured data output as computer linguistics resources by means of lexicon of markers.

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The interval method for solving fuzzy multi-criteria fractional transportation problem

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In this paper is presented an interactive solving approach for the multi-objective fractional transportation problem with fuzzy cost coefficients and time minimizing criterion. The approach is based on interval presentation of each cost functions coefficients. By finding of the probabilistic parameter of belonging of coefficients of objective functions to their variation intervals for every criterion, we can find iteratively the corresponding set of efficient solutions for the multiple criteria

transportation model for every value of parameter, structured by the time minimizing criterion. Thus, we could solve the multi-criteria transportation problem of fuzzy type for any value of the probabilistic parameter of belonging, that is in fact one of stochastic Sbottleneck T type problem. In other words, we have obtained one significant result, that any decision-making situation described by the proposed model can be predicted by its time and cost characteristics. The proposed algorithm, being tested on several examples, has proved to be quite efficient.

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UML and Coloured Petri nets in modeling of logistics systems

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One of the most evident features of modern companies' functioning is the wide use of various innovations, such as information and communication tools and technologies, new approaches to marketing and logistics system in enterprise management, etc. On the one hand, innovations are contributing to more efficient work of enterprises, the rapid achievement of desired results. On the other hand, their implementation may require some changes in the functioning of the company, in the logistics system business processes particularly. Modeling that can be performed by various tools such as Petri nets and UML occupies one of basic places in the management of logistics business processes.

The model of logistic business processes is offered to discussion. The project includes an UML diagrams that were built by means of object-oriented Modelio 3.2 design environment [1]. Simulation and verification of coloured Petri nets are performed using the CPN Tools package [2]. The model of logistics business processes' system was developed according to the specific requirements of functionality. Namely, processing subsystem (system) of orders for the sale and/or delivery of goods at the company was provided within the system.

The logistics business processes' system includes components (activities) that are representing the chain of goods' implementation and delivery operations. Each option activity was built by means of UML diagrams [3] for the purpose of modeling and analysis of material and information flows and further detailing of procedural transactions' features within the system of logistics business processes.

Primarily the analysis of requirements to the logistics business processes' system with the help of UML and colored Petri nets allows errors identifying at the design stage. However, ready to the implementation algorithm was obtained and it's correct functioning was proved formally. The proposed methods allow getting verified design of the logistics business processes' system without errors or duplication flows and thus increase its effectiveness and efficiency.

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8. Education

Elemente de combinatorică şi cursul de Teoria Probabilităţilor

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Combinatorica joacă un rol deosebit la rezolvarea unor probleme actuale în teoria numerelor, teoria grafurilor, logica matematică, informatică, etc. Un rol fundamental îl are combinatorica pentru teoria probabilităţilor, în special, la etapa de iniţere, în cazul experimentelor aleatoare cu mulţmi de evenimente elementare cel mult numărabile.
În ultimul timp în instituţiile de învăţământ superior s-a micşorat numărul orelor pentru cursul Teoria probabilităţilor şi Statistică matematică, chiar şi la facultăţile de Matematică şi Informatică. Astfel, unor teme din combinatorică (principiul fundamental, permutări şi combinări cu repetiţii, principiul includerii şi excluderii, scheme cu urne, metoda traiectoriilor, funcşia generatoare etc.) profesorul nu le poate acorda suficient timp, şi aceasta creează dificultăşi serioase la rezolvarea problemelor, îndeosebi a celor cu tematică discretă.
Pentru a corecta situaţia creată, Departamentul Matematici Aplicate, Universitatea de Stat din Moldova, a propus în planul de studii al anului I, pentru specialităţile Matematică şi Matematică Aplicată, cursul Combinatorică şi scheme de probabilitate discreteă. Considerăm că astfel se creează o bază bună pentru cursul Probabilităţi şi statistică care se predă la anul II.

Cabinetul de matematică în sistemul activităţii didactice. Specificul lecţiilor desfăşurate la cabinetul de matematică.

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We understand by "Laboratory of Mathematics" in the broader sense, a structure and method for making the connection university-research-production. Learning mathematics at this stage appears as a social necessity, and education must meet this need to find methods and means to ensure the optimization of mathematics learning. Laboratory of mathematics must have this designation.

High potential for logical-mathematical intelligence

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In this paper we propose to realize a scientific incursion in logical-mathematical intelligence by clearly sense on giftedness, characteristics that define high potential for logical-mathematical intelligence and forms of educational intervention for a maximal development potential. If Piaget is the initiator of research genesis of intelligence, Gardner added a new vision of intelligence, multiplicity. Intersecting research of two authors, it outlines a scientific understanding of logical-mathematical intelligence with direct implications on differentiated curriculum.

Interdisciplinary activities for the acquisition of competences in geometry in the first course of engineering degrees

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This work is focused on the subjects of "Algebra and Geometry" and "Graphical Expression", which are basic subjects on the majority of the degrees of engineering. The aim of this work is to try to correct the difficulty that many students find to relate concepts of different subjects. The approach of interdisciplinary activities favours the autonomous learning of the student, making easy its extrapolation to other situations. This process has consisted of the consensus of common matters to both subjects, design of interdisciplinary activities of learning, generation of didactic material, implantation of the collaborative learning methodology and the evaluation of the overall process. The analysis of the project reflects that the methodology used is positive and that it has increased the self-learning capacity of the students, although the self-assessment activities have to be improved.

Building interactive figures and simulations with mathematical software

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The use of mathematics in engineering has a big impact on the analysis and the design of processes and products. Nowadays there are powerful computational tools to do graphs and simulations, as well as to do calculations fast and efficiently. The software Mathematica, used for educational and professional purposes, is one of these. This software is a symbolic calculus program with many graphical possibilities that allow the resolution of a variety of engineering problems and its analysis from a graphical point of view. It is also possible to make simulations without too much difficulty by varying the problem data, once it has been solved.
This work explains the project entitled *Visualizing and moving on the Mathematics* developed with students of Civil Engineering and Energy and Mining Engineering, with the purpose of making easy the use of computational tools related to mathematics. These tools result helpful as a support of their study, and many applications oriented to improve students' spatial vision are shown. This initiative expects that students learn the correct utilisation of the instructions of Mathematica, being able to detect engineering problems which can be solved by it, and formulating and solving them.

Folosirea strategiilor anticipative în rezolvarea problemelor de geometrie în ciclul gimnazial

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Lucrarea prezintă una dintre strategiile importante ale predării-învăţării geometriei în ciclul gimnazial, strategiile anticipative.
În prima parte sunt prezentate o serie de aspecte teoretice privind modul de abordare a strategiilor anticipative, care au o importanţă deosebită în organizarea operaţiilor rezolutive.
De asemenea sunt evidenţiate cele trei serii de strategii: strategii anticipativ-exploratorii, strategii anticipativ-rezolutive şi strategii anticipativ-executive.
Cea de-a doua parte este consacrată folosirii strategiilor amintite în cazul rezolvării problemelor de geometrie plană, plecând de la o ipoteză generală, ce constituie ideea de plecare şi care determină o zonă numită zona căutării euristice.
Pa baza unei probleme de geometrie de clasa a VII-a se elaborează o schemă ce pune în evidenţă cele trei serii de strategii anticipative şi mai ales aspectele caracteristice ale posibilităţilor individuale parcurse în rezolvarea problemei.
Lucrarea se încheie cu câteva concluzii privind modul de folosire a strategiilor anticipative în rezolvarea problemelor de geometrie plană în ciclul gimnazial şi cu o serie de aspecte formative ale acestor strategii.

Unele aspecte psiho-pedagogice utilizate în procesul de predare-învăţare-evaluare a cursului universitar ,,Tehnici de programare"

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În acest articol se abordează mai multe aspecte didactice care ţin de predarea cursului universitar Tehnici de programare. Tehnicile de programare au un rol important în formarea culturii informaţionale, în dezvoltarea capacităţii de a gândi creativ şi a elabora algoritmi optimi în scopul obţinerii soluţiilor eficiente a problemelor examinate. Tradiţional în procesul de predare-învăţare a cursului respectiv se folosesc un număr limitat de instrumente didactice care presupun aplicarea tehnologiilor informaţionale moderne. Astfel, de exemplu, la predarea tehnicilor de programare, profesorii din instituţiile de învăţământ superior, se bazează tradiţional pe utilizarea limbajelor de programare: C/C++, Java, Turbo Pascal, etc. Dar, luând în consideraţie progresul tehnologiilor informaţionale, procesul de predare-învăţare a tehnicilor de programare poate fi completat, destul de eficient, cu noi strategii şi instrumente didactice, care ţin de implementarea tablei interactive cu utilizarea soft-ului SMART Notebook. Din acest punct de vedere, este foarte important ca profesorul să folosească metode active şi stimulative care să-i solicite pe studenţi să-şi îndeplinească sarcinile de lucru şi să atingă obiectivele propuse [1], [2].
Dezvoltarea gândirii critice, creşterea abilităţilor studenţilor de a înţelege ţi a utiliza gândirea vizuală este facilitată şi de folosirea materialelor şi resurselor didactice care pot coopta atenţia şi

stimula gândirea creativă prin intermediul paletei de culori [3]. Astfel de instrumente didactice, după cum confirmă unele cercetări [4], contribuie la eficientizarea procesului de învăţare, sporind viteza de memorare cu peste 40%, intensificând motivaţia studentului pentru învăţare, accentuând atenţia, stimulând imaginaţia, crescând acuitatea de percepere şi precizia.
Un alt studiu [5], [6] desfăşurat pe parcursului unui an de specialiştii de la Universitatea Massachusetts a ajuns la următoarea concluzie: *gândirea vizuală în culori stimulează învăţarea utilă pe termen lung şi capacitatea de rezolvare a problemelor.* În contextul experimentului desfăţurat de noi s-a aplicat modelul didactic *vezi înainte de a auzi.* Acesta este un model revizuit al modelului elaborat de Mary Lynn Manns, University of North Carolina, Asheville [7]. Pentru a înţelege esenţa acestei metode scoatem în evidenţă ideile lansate Mary Lynn Manns şi care stau la baza acestui model: *Studenţii găsesc cu dificultate convertirea noţiunilor pe care le aud în clasă la predarea lecţiei în abilităţi pe care ei le pot folosi în afara clasei. De regulă, îşi reamintesc mai puţin ceea ce au auzit decât ceea ce au văzut sau au experimentat. Abordarea ,,aude înainte de a vedea" este extrem de abstractă şi poate face, pentru studenţi, foarte dificilă aplicarea noţiunilor teoretice în practică. De aceea, este necesar să li se creeze oportunitatea de a vedea şi experimenta un nou concept înainte de a auzi despre el. Astfel studenţii sunt încurajaţi să înregistreze în memorie şi să reflecteze la ceea ce se întâmplă atunci când sunt implicaţi în învăţare.* De exemplu:
Vede: cum are loc completarea stivei (soluţiei) pentru problema permitărilor aplicând tehnica backtracking; fiecare nivel în stivă poate fi reprezentat printr-un dreptunghi în care profesorul (studentul) va scri cu cerneală digitală valoarea depusă în stivă; se va utiliza aţa opţiune ca clonarea obiectelor; profesorul poate face comentarii, poate atenţiona studenţii la o anumită etapă de completare a stivei, şi anume, la respectarea condiţiilor de validare pentru ai conduce spre scrierea corectă a tuturor subalgoritmilor în special a subalgoritmului recursiv (backtratcking) care returnează soluţia problemei.
Aude: urmărind atent câteva exemple propuse (rezolvate) la etapa "**vede**", şi "aude" comentariile profesorului, astfel, poate asigura solidificarea noilor concepte care au fost introduse prin exemplele propuse. Se poate face în permanenţă referire la ceea ce au făcut studenţii în timpul experienţelor realizate în faza "**vede**".
Aţa dar, implementând tabla interactivă, ca un instrument didactic de înaltă performanţă, extrem de puternic şi util, în procesul de predare-învăţare a tehnicilor de programare, se poate pune un accent deosebit pe aşa elemente ca vizibilitatea, atractivitatea, interactivitatea, şi vizualizarea etapelor de rezolvare a problemelor. În aşa mod, conceptul didactic de "alfabetizarea vizuală" (visual literacy) devine nu numai popular dar şi eficient în procesul de predare-învăţare. În contextul respectiv, la studenţi se dezvoltă gândirea vizuală care contribuie convingător la consolidarea abilităţilor ce ţin de tehnicile de programare.
În concluzie, avantajele utilizării SMART Notebook-ului se referă la: reutilizarea, de mai multe ori, a informaţiei scrise şi graficilor construite anterior; folosirea eficientă în procesul de predare-învăţare a paletei de culori; dinamismul şi atractivitatea în explicarea unor detalii, considerate tradiţional dificile, care ţin de tehnicile de programare; economisirea timpului în procesul didactic, etc.

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Funcţii generatoare şi şiruri recurente

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Funcţiile generatoare[1] reprezintă un mijloc foarte eficient pentru rezolvarea problemelor de numărare a unor obiecte sau clase de obiecte. Metoda are o vechime de cîteva sute de ani, şi o putem întîlni în lucrările lui Newton, Bernoulli, Euler, Gauss, Riemann la demonstrarea unor rezultate remarcabile.
Aplicarea funcţiilor generatoare are la bază o idee foarte simplă. Fie $\{a_n\}_{n\geq 0}$- un şir de numere reale. *Funcţia generatoare ordinară* a şirului $\{a_n\}_{n\geq 0}$ este seria formală de puteri

$$F(x) = \sum_{n=0}^{\infty} a_n x^n,$$

seria numindu-se formală, deoarece, în general, nu suntem interesaţi de convergenţa ei. Numerele a_n se numesc *coeficienţii funcţiei generatoare ordinare.*
Exemplificăm rezolvarea recurenţelor, utilizînd noţiunea de funcţiie generatoare a şirului în baza următorului exemplu.
Exemplu. Să se afle termenul de rang n al şirului recurent $a_0 = 4$,$a_1 = 7$,$a_{n+2} = 5a_{n+1} - 6a_n$.
Rezolvare. Egalitatea $a_{n+2} = 5a_{n+1} - 6a_n, n \geq 0$, se înmulţeşte la x^n şi apoi sumăm:
$a_{n+2}x^n = 5a_{n+1}x^n - 6a_n x^n$, $\sum_{n=0}^{\infty} a_{n+2}x^n = 5\sum_{n=0}^{\infty} a_{n+1}x^n - 6\sum_{n=0}^{\infty} a_n x^n$. Notăm $F(x) = \sum_{n=0}^{\infty} a_n x^n$, atunci $\sum_{n=0}^{\infty} a_{n+1}x^n = \frac{F(x)-a_0}{x}$, $\sum_{n=0}^{\infty} a_{n+2}x^n = \frac{F(x)-a_0-a_1 x}{x^2}$.
Se obţine ecuaţia $\frac{F(x)-4-7x}{x^2} = 5\frac{F(x)-4}{x} - 6F(x)$, din care obţinem pentru $F(x)$: $F(x) = \frac{4-13x}{1-5x+6x^2} = \frac{4-13x}{(1-2x)(1-3x)} = \frac{A}{1-2x} + \frac{B}{1-3x}$

Coeficienţii A şi B se determină din egalitatea $4-3x = A(1-3x) + B(1-2x)$: $A = 5, B = -1$.
$F(x) = \frac{5}{1-2x} + \frac{-1}{1-3x} = 5\sum_{n=0}^{\infty}(2x)^n - \sum_{n=0}^{\infty}(3x)^n = \sum_{n=0}^{\infty}(5\cdot 2^n - 3^n)x^n$.
Deoarece $F(x) = \sum_{n=0}^{\infty} a_n x^n$, obţinem:

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty}(5\cdot 2^n - 3^n)x^n, \longrightarrow a_n = 5\cdot 2^n - 3^n$$

Răspuns. $a_n = 5\cdot 2^n - 3^n$

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The design of educational software - a paradigm shift

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In the last 15 years, the efforts to introduce and use educational programs in the Romanian educational system have not the expected success. Tacitly recognized, the failure might be due to the inertia of system, to lack of good preparation of human resources and inadequate and insufficient material resources. But the real cause is the quality of software realized. These, besides the inadequacy of the curriculum, proved accentuated defective design in psycho-pedagogical terms.
The users, teachers in undergraduate education, with a good training in specialty and psycho-pedagogical, have been the decisive opponents of the demarche for implementation of the used programs. One of the deficiencies consistently evidenced was the poor teacher virtual representation in the scenario of teacher learning programs. The core of the training process is the teacher, with arsenal of teaching methods and procedures. In most of training programs, the teacher was substituted by computer interface elements, specific to Windows operating systems, that present eventually, the familiarity apparent advantage.
Elimination of this important shortcoming in designing of learning programs, can not be achieved only through a major paradigm shift in the design of teaching scenarios afferent of learning programs. The new paradigm involves bringing of the teacher image in foreground of training process, in a teaching scenario similar that the classic scenario, but which make performance by using the digital resources. The presence of teacher must be discrete, but continuous and decisive in key moments of the class.
The paper aims to present the elements of teaching scenario which produce the paradigm shift in the design of educational software.

The Role of Mathematics Education in Shaping Contemporary Personality Profile

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The choise for a specific learning path influences our way of thinking, feeling and behaving. Even the most ancient civilisations integrated elementary mathematics in their formal education. Mathematical fields of arithmetic and geometry were present in the structure of classical education of medieval Europe, in the Renaissance, during the industrial revolution and mathematics education gained progressively the status of an independent field of research.
Present paper analyzes the impact of mathematics education as a long-therm investment that streghtens the individual development and prepares students for their lives as active members of society in the perspective of a continuum personality growth in the contemporary information age.

Mathematics and Science Assessment for the 6^{th} grade - Use of Assessment Data

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Student assessment is an essential tool for monitoring and improving the teaching-learning-assessing process. We present a national testing strategy in Romania, designed to measure what the learners of the 6th grade are able to do with the content of mathematics, physics and biology already learnt and to what degree the learners connect those contents.
With the increasing importance given to standardized international and national testing, in 2014, we implemented in Romania a national testing program using a unique test combining elements from mathematics, physics and biology, for 12-13 year-old students. We present the stages of implementing this testing. We defined six interdisciplinary competencies [1] and we used a three-parameter logistic model [2] to describe the link between the students–performance in tests and their corresponding abilities in mathematics, physics and biology. Instead of being graded, each test is evaluated based on clustered codes, providing individual feedback regarding each student (e.g., the curricular area where further work is needed – personalized learning plans) as well as clustered feedback at class, school, regional and national level, identifying the areas where adjustment would increase the effectiveness of the teaching process – personalized teaching plans.
The use of Mathematics&Science Assessment data as well as national reports and surveys are essential in order to improve the quality of instruction and to support new policy developments.
Mathematics Subject Classification 2010: 97D60, 97B20 General education.

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The Kumon Method in Mathematics and Informatics for Gymnasiums

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The Kumon Method is a very popular technique for teaching children. Today, it is applied in more than 40 countries and regions worldwide. The key element in this method is to empower children to become self-learners. Each student progresses at his or her own pace through an individualized program of worksheets carefully planned by the Instructor.
We intend to elaborate a didactic technique based on key points of the Kumon method and apply it in Mathematics and Informatics for gymnasiums.

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Rezolvarea numerică a unor probleme tipice, bazate pe calculul integralei definite

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În literatura de specialitate se descriu pe larg aşa metode de integrare numeric C cum ar fi formulele de cuadratură a trapezelor, dreptunghiurilor şi Simpson. Însă aplicarea lor în practică este destul de puţin vizată. În lucrare se analizează probleme, bazate pe aplicarea calculului integralei definite, rezolvate prin metode numerice.
Sunt cercetate unele probleme, care se rezolvă prin aplicarea calculului integralei definite:

1 Determinarea drumului parcurs de un punct;

2 Determinarea poziţiei punctului;

3 Determinarea volumului de lucru;

4 Determinarea forţei de presiune;

5 Determinarea numărul π.

Sunt descrise şi rezolvate mai multe exemple concrete, aplicînd metodele de cuadratură a trapezului, a dreptunghiurilor, a parabolelor şi Gauss. Se cercetează convergenţa aplicării acestor metode. În concluzie utilizarea unui sistem de sarcini concrete în prezentarea materialului teoretic faciliteaz C perceperea informaţiei de către studenţi.
Lucrarea este destinată în special elevilor şi studenţilor la calculul integralelor definite în cazul soluţionării unor probleme concrete.

Metoda euristică de optimizare a mulţimii dependenţelor funcţionale la proiectarea bazelor de date

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La proiectarea bazelor de date relaţionale nu trebuie de uitat, că relaţiile bazei de date trebuie să ilustreze o stare a unei ramuri a lumii reale. Acest lucru înseamnă că toate atributele relaţiilor impun restricţii, în conformitate cu cerinţele din ramura de subiect şi bunul simţ. Proiectantul bazei de date, ştiind semantica atributelor şi studiind ramura de subiect, formulează restricţii care trebuie să fie satisfăcute de relaţiile unei baze de date adevărate. O clasă importantă de astfel de restricţii sunt dependenţele funcţionale care ocupă un rol important în teoria proiectării bazelor de date relaţionale. Transformarea mulţimii dependenţelor funcţionale, în general, este o problemă de inferenţă. Articolul oferă o modalitate vizuală de manipulare a dependenţelor funcţionale, bazată pe prezentarea matricială a funcţiilor booleene.

Unele puncte de vedere referitor la istoria şi metodologia matematicii

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Istoria oricărei ştiinţe, matematica nefiind excepţie, poate fi tratată, expusă, pe de o parte, din diferite puncte de vedere, cum ar fi: cronologic, regional, evoluţionist, al genezei, perindării noţiunilor şi ideilor; pe de altă parte, al legăturilor şi influenţei diverselor ramuri ale ştiinţelor exercitate asupra celei studiate. În lucrarea dată se propune o privire referitor la Istoria matematicii pornind din a) *perioadele* de dezvoltare a ei; b) conexiunea cu Teoria informaţiei (în sensul cel mai larg al acestui cuvânt), conform stării lucrurilor de la începutul sec. XXI când *informatizarea* capătă un caracter global. În studiul dat vom considera iniţierea unei noi perioade din punct de vedere genetico-metodologic. Din aceste considerente se propune următoarea periodizare în dezvoltarea matematicii. **Perioada I** - *de constituire a noţiunilor fundamentale legate de cunoaştere ca atare* facultăţi ale vitalităţii de la reflecţie @ S reacţie până la recepţionarea, memorizarea şi prelucrarea informaţiei, lucru continuat de toate ştiinţele prin modalităţile respective. Un rol deosebit, la începuturi, revine modalităţilor de codificare (prin diverse simboluri) a informaţiei. Codificarea *sonoră*, pe o scară largă, a informaţiei implică apariţia limbilor, deci şi a umanoizilor. Un punct crucial a fost momentul înţelegerii că *bătrânii*, deşi nu pot, dar *ştiu*, *cunosc*, şi cunoştinţele lor pot fi folosite - rolul efectiv al *memoriei externe*, începându-se astfel cultul învăţătorului, înţeleptului cu toate cele ce urmează. Prin mil. IV-III î.e.n. se constituie şi noţiunile de bază ale matematicii. **Perioada II** *perioada matematicii practice (algoritmice)* triumful calculului numeric (Babilonia) iniţieri geometrice (Egipt). Punct crucial - descoperirea *scrisului*, alt tip de memorie externă. **Perioada III** din sec.VII-VI î.e.n. - perioada *matematicii teoretice*, meritul Greciei Antice, în centrul atenţiei fiind pusă fundamentarea, demonstrarea propoziţiilor considerate ce se putea efectua numai prin metoda axiomatică. **Perioada IV** – sec. XIV-XVII e. n. este perioada formării *limbajelor matematice* bazat pe un simbolism specializat (un prototip al limbajelor de programare). Rolul de bază îi revine lui F. Viete (1540-1603), perfectările şi finalizările lui R. Descartes (1596-1650). *Perioada V* începe în s. XVII când obiectul central de studii devine noţiunea de *funcţie* şi reprezintă constituirea analizei matematice prin exploatarea limbajului matematic şi a sistemelor de coordonate, cu aplicaţiile ei fenomenale, fapt ce a adus la o înflorire generală a ştiinţelor naturale şi a tehnicii. *Perioada VI*, deceniile II-III ale sec. XIX – teoria structurilor axiomatice, perioadă în care *metoda axiomatică, din mijloc de fundamentare a unor teorii deja dezvoltate,obţine şi modalitatea de generare a teoriilor noi.* Situaţii de acest tip au apărut aproape în acelaşi timp în geometrie şi algebră. În geometrie N. I. Lobacevski (1792-1856) şi J. Bolyai (1802-1860) au creat prima geometrie neeuclidiană adăugând la axiomele geometriei absolute negaţia axiomei paralelelor. În algebră E. Galois (1811-1832) a introdus noţiunile de grup şi corp pentru a finaliza:wq problema rezolvării în radicali a ecuaţiilor algebrice. Astfel, apar diferite structuri geometrice, algebrice, topologice etc. Începând cu cea de a doua jumătate a sec. XX se conturează o nouă **Perioadă a VII** cea care poate fi numită a *structurilor globale* ca cele de algebre universale, categorii etc., completate de caracterul globalizator al informaticii.

9. Supplement

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Applications over the Negative-Binomial Model

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In the frame of the Negative-Binomial Model, we consider a Bayesian approach using Beta and Pearson type VI as priors. Taking them into consideration in terms of posterior densities, we shall reach the closed-form integration. From there on, an expansion of polynomial type for the gamma function is introduced and further, different parameters of interest to re-parameterize the model are taken into account. In order for the recorded observations not to be biased, when data are recorded according to a certain stochastic model, the recorded observations will not have the original distribution unless every observation is given an equal chance of being recorded. We also introduce a weighted distribution, with a new positive parameter. An apriori Gamma distribution for the new parameter c exists, and given that, the new formed distribution is also of Gamma type. For this, the line of Lio is considered. Comparisons with the maximum likelihood and moment method estimates are performed. The Bayesian estimates for the parameters of interest are analyzed via mean squared error and variance through computer simulation. In conclusion, the Bayesian method for the Negative-binomial model provides an alternative to the maximum likelihood approach. Unlike the maximum likelihood estimates and the moment method estimates, the Bayes estimation produces values in the feasible regions of parameters. By using the proposed sampling procedure, the Bayesian approach for the Negative-binomial Model can be implemented successfully in real life applications.

Acknowledgement. *This work was supported by the strategic grant POSDRU/187/1.5/S/155559* **Competitive multidisciplinary doctoral research at European level (CdocMD)** *cofinanced by the European Social Found within the Sectorial Operational Program Human Resources Development 2007 - 2013.*
Ioana Dănilă was supported by the strategic grant POSDRU/187/1.5/S/155559 ***Competitive multidisciplinary doctoral research at European level (CdocMD)*** *cofinanced by the European Social Found within the Sectorial Operational Program Human Resources Development 2007 - 2013.*

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